# The Measurement of Educational Inequality: Achievement and Opportunity


Francisco H. G. Ferreira

(The World Bank and IZA)

Jérémie Gignoux

(Paris School of Economics)


OECD (Paris), 5 April 2012

# Plan of the talk

1. Motivation
2. (Some related literature)
3. Data
4. Measuring inequality in educational achievement
5. Measuring inequality of opportunity in education
6. I.Op. and covariates: descriptive correlations
7. Conclusions

# 1. Motivation

- The advent, in the last decade, of test-based cognitive achievement surveys that are applied consistently across countries represents a major opportunity for understanding international differences in educational performance.
  - Including differences (across countries) in the inequality of opportunity for a good education.
  - PISA, TIMSS, PIRLS, IALS
- But certain features of data collection and analysis create comparability problems which do not appear to be widely understood.

# 1. Motivation

- This paper aims to provide a set of statistically robust international comparisons of:
  - Inequality in educational achievement
  - Inequality in educational opportunity (I.Op.)
- That account for:
  - The implications of test-score standardization for cardinal and ordinal equivalence of inequality measures;
  - PISA sample selection biases
- In particular, the proposed measure of IOp:
  - Relates naturally to the mobility and I.Op. literatures
  - Is cardinally insensitive to standardization
  - Is additively decomposable
  - Relies on a comprehensive set of background variables
- The analysis is for all 57 countries in the PISA 2006 round.

# 3. Data

- Programme of International Student Assessment (PISA) 2006.
  - Third round
  - All 57 countries
  - 15 year-olds in grades 7 or higher
  - IRT-corrected and standardized test scores in mathematics, reading and science.
  - Also contains information on schools and on family background, including:
    - Gender, father’s and mother’s education, father’s occupation, language spoken at home, migration status, access to books at home, durables owned, cultural items owned, school location.

# 3. Data

- *Item response theory* and the standardization of scores in PISA:
  - IRT is essentially a statistical technique to try and account for heterogeneity in the difficulty of test items on the basis of the observed distribution of responses (and an assumption about the underlying distribution of ability)
  - IRT generates a distribution of corrected scores $x_{ij}$, with an indeterminate metric. PISA standardizes the world distribution of this variable as follows:

$$y_{ij} = \hat{\mu} + \frac{\hat{\sigma}}{\sigma}\left(x_{ij} - \mu\right)$$

# 3. Data

- Ancillary data sets (used to correct for PISA sample selection):
  - Brazil's PNAD 2006
  - Indonesia's SUSENAS 2005
  - Mexico's ENIGH 2006
  - Turkey's HBS 2006

# 4. Measuring Inequality in Educational Achievement

1. **The Standardization Issue:**
   - Zheng's (1994) theorem: No inequality measure that satisfies symmetry, continuity and the transfer axiom can satisfy both:
     - Scale-invariance $I(y) = I(\lambda y), \lambda > 0$
     - Translation-invariance $I(y) = I(y + a), a \neq 0$

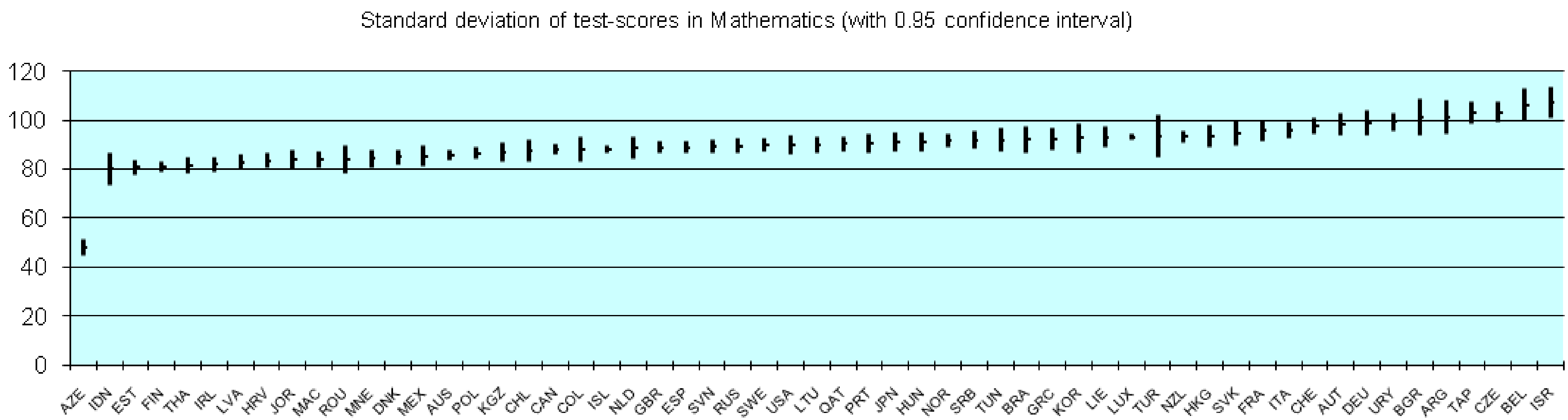
- Remark 1: No meaningful inequality index yields a cardinally identical measure for pre- and post-standardization distributions of the same test scores.
- Remark 2: Some common measures are not even ordinally equivalent, including the Gini and the Theil index:

$$G_j^y = \frac{\mu_j^x \hat{\sigma}}{\mu_j^y \sigma} G_j^x$$

- Remark 3: The variance is ordinally invariant to standardization:

$$V_j^y = \left(\frac{\hat{\sigma}}{\sigma}\right)^2 V_j^x$$

# 4. Measuring Inequality in Educational Achievement



# 4. Measuring Inequality in Educational Achievement

## 2. The PISA sample selection issue:

- PISA samples are designed to be representative of all 15 year-olds enrolled in grades 7 or higher, in any educational institution.
- If evasion and repetition are correlated with student characteristics that affect test performance, then sample is NOT representative of universe of 15 year olds.
- Particularly problematic for LDCs:

**Table 2: PISA Sample Coverage: Analysis for four developing countries**

| | Brazil | Indonesia | Mexico | Turkey |
|---|---|---|---|---|
| Expanded 15 year-old populations, using PISA data and weights | | | | |
| Total population of 15-year-olds | 3 390 471 | 4 238 600 | 2 200 916 | 1 423 514 |
| Total enrolled population of 15-year-olds at grade 7 or above | 2 374 044 | 3 119 393 | 1 383 364 | 800 968 |
| Weighted number of students participating to the assessment | 1 875 461 | 2 248 313 | 1 190 420 | 665 477 |
| Coverage rate of the population of 15-year-olds, from PISA | 55,3 | 53,0 | 54,1 | 46,7 |
| Total missed children | 44,7 | 47,0 | 45,9 | 53,3 |
| Composition of those not covered by PISA samples | | | | |
| Out-of-school children | 10,2 | 25,5 | 24,1 | 21,6 |
| Delays of more than two years | 19,8 | 0,9 | 13,1 | 22,2 |
| PISA sampling issues | 14,7 | 20,6 | 8,8 | 9,5 |

Source: PISA 2006 surveys; PNAD 2006 for Brazil, Susenas 2005 for Indonesia; ENIGH 2006 for Mexico, and HBS 2006

# 4. Measuring Inequality in Educational Achievement

- Two (non-parametric) approaches to correct for sample selection, using ancillary HH surveys:

  1. Selection on observables (gender, M. Ed., F. Oc.)

     Replace $$f_j(y) = \iiint \Phi_j(y, X)dX = \iiint g_j(y|X)\phi_j(X)dX$$

     With $$f_j^{SO}(y) = \iiint \Phi_j(y, X)dX = \iiint g_j(y|X)\psi_j(X)\phi_j(X)dX$$

     Where $$\psi_j(X) = \frac{\phi_j(X|s = HH)}{\phi_j(X|s = PISA)}$$

  2. Allowing for selection on unobservables, under an (extreme) assumption of no common support:

     In each cell, give $\sup\left[\Psi_j(X) - 1, 0\right]$ the lowest observed grade in the cell.

# Figure 3: Distribution of standardized Turkish reading test scores under three alternative assumptions about selection into PISA participation

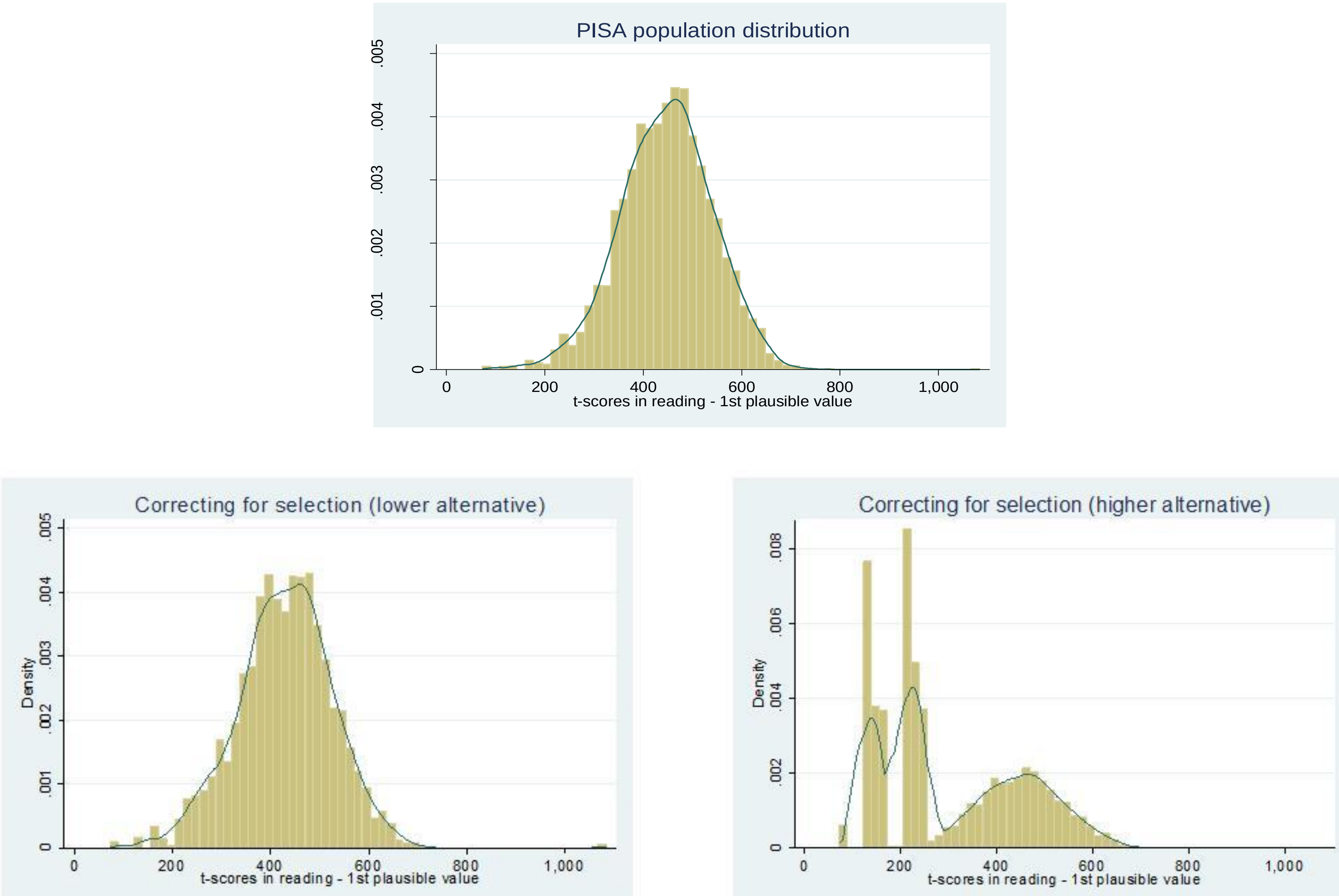


**Table 3: Inequality of Achievement and Opportunity in Low-Coverage Countries: sensitivity to different assumptions on selection into the PISA sample**

| | | PISA population without any correction | | | Correction assuming *selection on observables* | | | Correction assuming *strong selection on unobservables* | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | Reading | Math | Science | Reading | Math | Science | Reading | Math | Science |
| TURKEY | | | | | | | | | | |
| | Inequality (SD) | 92.90 | 93.24 | 83.20 | 98.38 | 91.43 | 82.58 | 155.67 | 134.04 | 121.61 |
| | | 2.75 | 4.32 | 3.14 | | | | | | |
| | IOp | 0.251 | 0.241 | 0.249 | 0.250 | 0.236 | 0.250 | 0.327 | 0.320 | 0.326 |
| | | 0.026 | 0.033 | 0.032 | | | | | | |
| BRAZIL | | | | | | | | | | |
| | Inequality (SD) | 102.46 | 92.02 | 89.28 | 102.86 | 90.44 | 86.75 | 179.82 | 146.68 | 146.17 |
| | | 3.34 | 2.65 | 1.93 | | | | | | |
| | IOp | 0.268 | 0.318 | 0.286 | 0.265 | 0.309 | 0.262 | 0.404 | 0.404 | 0.385 |
| | | 0.020 | 0.005 | 0.021 | | | | | | |
| MEXICO | | | | | | | | | | |
| | Inequality (SD) | 95.68 | 85.27 | 80.70 | 95.63 | 85.02 | 79.18 | 196.85 | 162.79 | 136.99 |
| | | 2.27 | 2.16 | 1.47 | | | | | | |
| | IOp | 0.278 | 0.261 | 0.271 | 0.267 | 0.242 | 0.255 | 0.256 | 0.250 | 0.228 |
| | | 0.024 | 0.002 | 0.024 | | | | | | |
| INDONESIA | | | | | | | | | | |
| | Inequality (SD) | 74.79 | 80.01 | 70.06 | 71.03 | 76.27 | 65.74 | 130.56 | 135.89 | 112.79 |
| | | 2.39 | 3.18 | 3.26 | | | | | | |
| | IOp | 0.250 | 0.237 | 0.220 | 0.218 | 0.200 | 0.181 | 0.274 | 0.261 | 0.261 |
| | | 0.038 | 0.042 | 0.045 | | | | | | |

# 5. Measuring Inequality of Educational Opportunity

- Ex-ante approach to inequality of opportunity (Checchi & Peragine, 2010; Ferreira and Gignoux, 2011):

1. Partition the population of test-takers into ***types*** (circumstance-homogeneous groups): $\Pi = \{T_1, T_2, \ldots, T_K\}$
   - Gender, father's and mother's education, father's occupation, language spoken at home, migration status, access to books at home, durables owned, cultural items owned, school location.
2. I. Op. is a measure of differences in the opportunity sets faced by these different types.
3. Value opportunity set: mean achievement
4. Construct smoothed distribution (Foster and Shneyerov, 2000)
5. Calculate inequality of opportunity as ratio of inequality in the smoothed distribution to total inequality: $$\theta_r = \frac{I\left(\left\{\mu_i^k\right\}\right)}{I(y)}$$

# 5. Measuring Inequality of Educational Opportunity

- Interpretation: $\vartheta_r$ is a lower-bound measure of inequality of opportunity
  - Omitted circumstances cannot lower it.
- In practice, when the number of types is large, the non-parametric decomposition is hampered by imprecision in the estimation of each cell mean.
- Alternative: estimate a linear reduced-form version of the model:

$$y = f(C, E, u)$$
$$E = g(C, v)$$

- As: $y = C\psi + \varepsilon$
- Note that the parametrically smoothed distribution is $\tilde{\mu}_i = C_i\hat{\psi}$
- And compute $\theta_r^P = \dfrac{I(\tilde{\mu}_i)}{I(y)} = \dfrac{Var(C\hat{\psi})}{Var(y)}$

# 5. Measuring Inequality of Educational Opportunity

- Related to the converse of (origin-independence) mobility.
- In the classic Galton regression:

$$y_t = \beta y_{t-1} + \varepsilon_t$$

$$R^2 = \frac{Var(\beta y_{t-1})}{Var(y_t)} = \frac{Cov^2(y_{t-1}, y_t)}{Var(y_t)Var(y_{t-1})} = \rho_{t,t-1}^2$$

- $y_{t-1}$ is unobserved, but family background vector ***C*** is observed.

$$y_t = C_t{}'\psi + \eta_t$$

- Measure IPI by:

$$\theta = \frac{Var\left(C_t\hat{\psi}\right)}{Var(y_t)}$$

# 5. Measuring Inequality of Educational Opportunity

- ## Properties of IOp (continued):

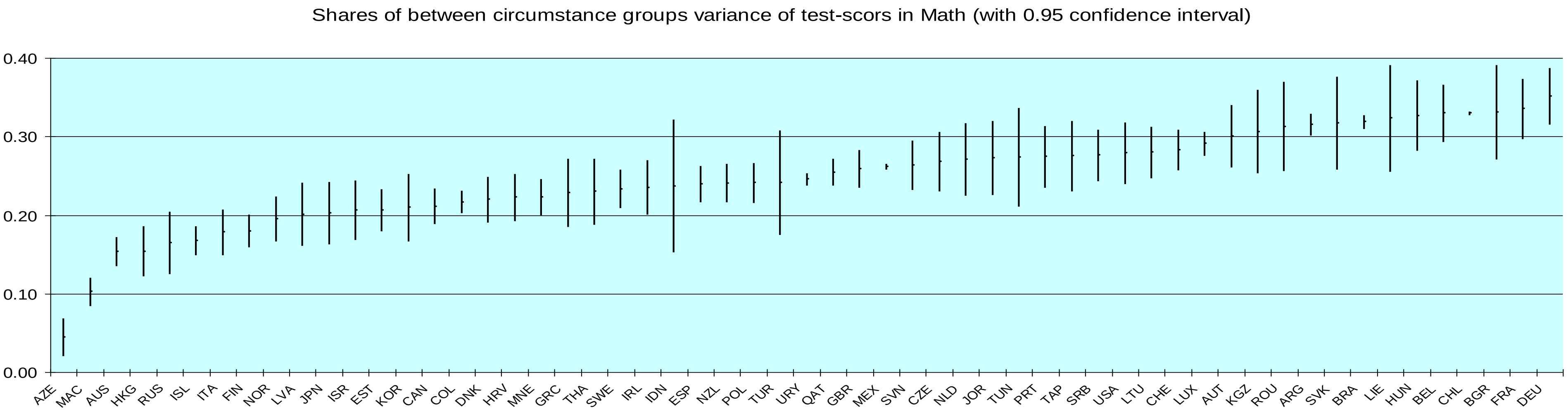
2. Cardinally invariant in the standardization
3. Additively decomposable into circumstance-specific components:

$$\theta = \sum_j \theta^j = \sum_j (\operatorname{var} y)^{-1} \left[ \beta_j^2 \operatorname{var} z_j + \frac{1}{2} \sum_k \beta_k \beta_j \operatorname{cov}(z_k, z_j) \right]$$

4. Uses information on a broader set of circumstances than usual measures. Recall:
   i. Gender
   ii. father's education
   iii. mother's education
   iv. father's occupation
   v. language spoken at home
   vi. migration status
   vii. access to books at home
   viii. durables owned
   ix. cultural items owned
   x. school location

# 5. Measuring Inequality of Educational Opportunity

**Figure 2: Inequality of Educational Opportunity: countries ranked by share of variance explained by background factors.**




Asia; Nordic countries, Italy

US, UK, Japan

LAC, much of continental Europe

Range (exc. Azerbaijan): 10.2% to 35.1%

# 5. Measuring Inequality of Educational Opportunity

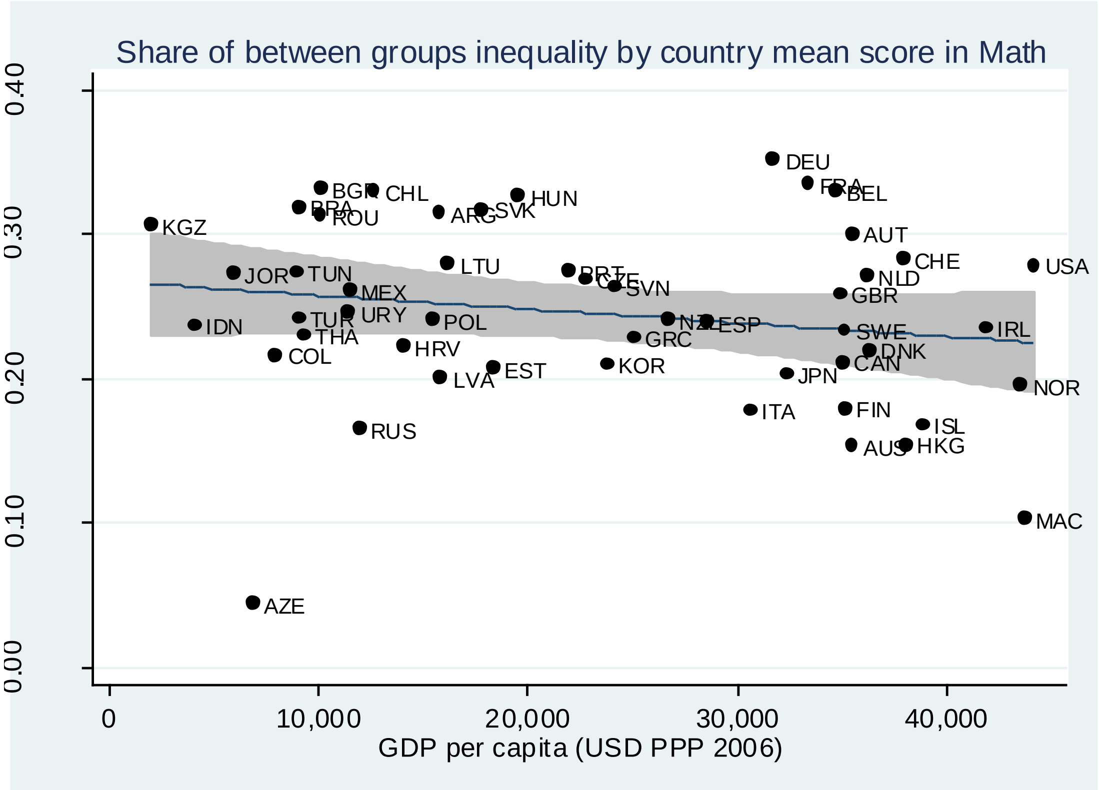
**Table 5: Partial shares of the total variance in mathematics scores: decomposing IPI into individual components**

| | Total | Gender | Father's education | Mother's education | Father's occupa-tion | Area type | Language at home | Immi-gration status | Number of books | Durables | Cultural posses-sions |
|---|---|---|---|---|---|---|---|---|---|---|---|
| *Latin America* | | | | | | | | | | | |
| Argentina | 0.315 | 0.004 | 0.014 | 0.026 | 0.024 | 0.022 | 0.000 | 0.003 | 0.079 | 0.114 | 0.029 |
| Brazil | 0.318 | 0.009 | 0.019 | 0.024 | 0.027 | 0.014 | 0.005 | 0.001 | 0.025 | 0.184 | 0.011 |
| Chile | 0.330 | 0.021 | 0.016 | 0.055 | 0.050 | 0.026 | 0.001 | 0.000 | 0.068 | 0.060 | 0.033 |
| Colombia | 0.216 | 0.017 | 0.009 | 0.015 | 0.014 | 0.014 | 0.003 | 0.000 | 0.049 | 0.085 | 0.010 |
| Mexico | 0.261 | 0.003 | 0.001 | 0.025 | 0.018 | 0.074 | 0.014 | 0.002 | 0.033 | 0.077 | 0.014 |
| Uruguay | 0.245 | 0.005 | 0.013 | 0.047 | 0.029 | 0.006 | 0.000 | 0.000 | 0.056 | 0.059 | 0.030 |
| *North America & Oceania* | | | | | | | | | | | |
| Australia | 0.153 | 0.008 | 0.007 | 0.009 | 0.044 | 0.002 | 0.000 | 0.000 | 0.055 | 0.011 | 0.016 |
| Canada | 0.211 | 0.008 | 0.029 | 0.011 | 0.035 | 0.017 | 0.003 | 0.000 | 0.078 | 0.013 | 0.018 |
| New Zealand | 0.241 | 0.005 | 0.036 | 0.016 | 0.036 | 0.003 | 0.000 | 0.000 | 0.074 | 0.034 | 0.037 |
| United States | 0.279 | 0.004 | 0.014 | 0.018 | 0.062 | 0.013 | 0.000 | 0.003 | 0.122 | 0.036 | 0.010 |

# 6. IOp and covariates: descriptive correlations

When outliers are excluded, there is a weak negative correlation between IOp and GDP per capita, significant at the 10% level.

**Figure 5: Intergenerational transmission of educational inequality and GDP per capita.**



# 6. IOp and covariates: descriptive correlations

The negative association between IOp and the primary share of public expenditure on education is always significant for reading, and becomes significant for all subjects when excluding outliers and including basic controls.

**Table 6: Coefficients on the primary share of public education expenditure in regressions of IPI on that variable; with and without controls.**

| | Reading | | Math | | Science | |
|---|---|---|---|---|---|---|
| No controls | | | | | | |
| All countries | -0.00217*** | (0.00092) | -0.00077 | (0.00112) | -0.00152 | (0.00105) |
| Excluding outliers | -0.00300*** | (0.00078) | -0.00113 | (0.00101) | -0.00172* | (0.00101) |
| Controlling for GDP and public expenditure in education per pupil | | | | | | |
| All countries | -0.00197** | (0.00087) | -0.00013 | (0.00120) | -0.00103 | (0.00113) |
| Excluding outliers | -0.00184*** | (0.00072) | -0.00181* | (0.00102) | -0.00185* | (0.00108) |

*Notes: Regression coefficients of the share of public expenditure in education allocated to the primary level. Dependent variable: IPI in the subject at column header. Standard errors in parentheses. Where indicated, outliers are identified using the method proposed by Besley, Kuh and Welsch (1980). Data source: UNESCO Institute for Statistics database; ***/**/*: significant at 1/5/10%.*

# 6. IOp and covariates: descriptive correlations

IOp is consistently and significantly positively associated with tracking, measured as the share of technical and vocational enrollment in secondary schools.

**Table 7: Coefficients on tracking in regressions of IPI on that variable; with and without controls.**

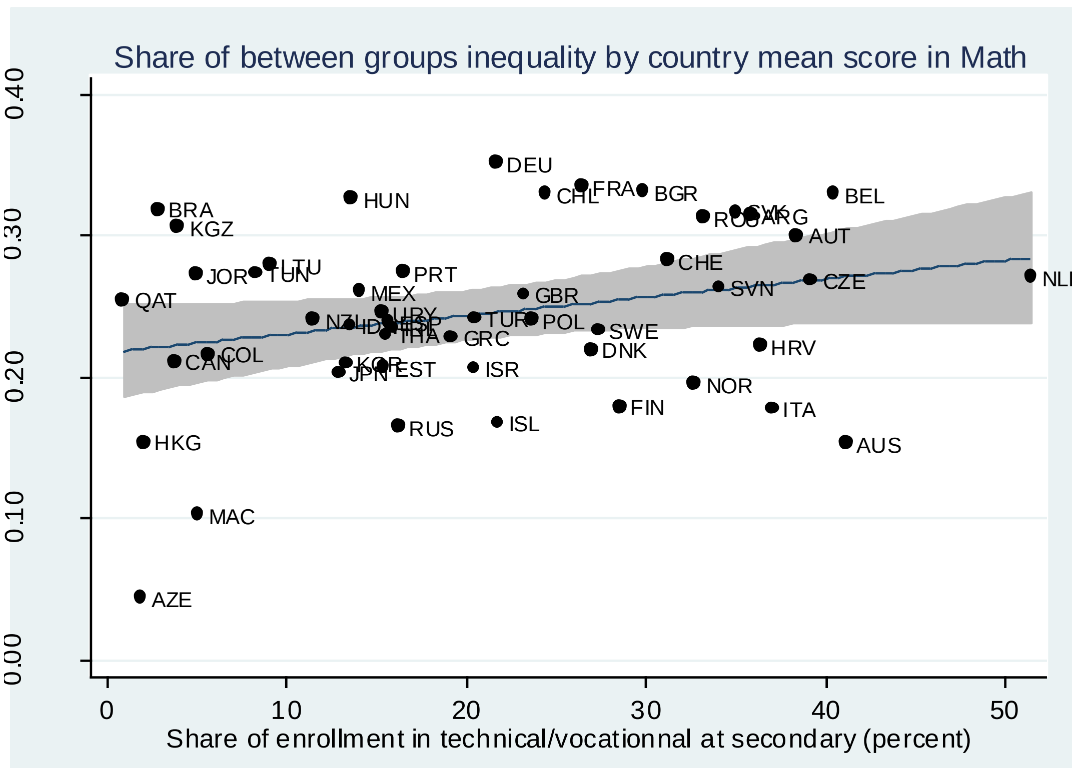
| | Reading | | Math | | Science | |
|---|---|---|---|---|---|---|
| <u>No controls</u> | | | | | | |
| All countries | 0.00106* | (0.00059) | 0.00130* | (0.00070) | 0.00179*** | (0.00063) |
| Excluding outliers | 0.00158** | (0.00060) | 0.00109* | (0.00062) | 0.00160*** | (0.00059) |
| <u>Controlling for GDP and public expenditure in education per pupil</u> | | | | | | |
| All countries | 0.00148*** | (0.00057) | 0.00173*** | (0.00074) | 0.00214*** | (0.00068) |
| Excluding outliers | 0.00090* | (0.00047) | 0.00175*** | (0.00065) | 0.00205*** | (0.00067) |

*Notes: Regression coefficients of tracking (measured as the share of technical and vocational enrollment at the secondary level). Dependent variable: IPI in the subject at column header. Standard errors in parentheses. Where indicated, outliers are identified using the method proposed by Besley, Kuh and Welsch (1980). Data source: UNESCO Institute for Statistics database; ***/**/*: significant at 1/5/10%.*

# 6. IOp and covariates: descriptive correlations

IOp is consistently and significantly positively associated with tracking, measured as the share of technical and vocational enrollment in secondary schools.

**Figure 7: Intergenerational transmission of educational inequality and tracking.**



# 7. Conclusions

- Data sets such as PISA are a hugely valuable source of information on the distribution of cognitive achievement.
- But the standardization of test scores and sampling frame issues require caution in defining and interpreting measures of educational inequality.
  - Unlike the Gini or the Theil, the simple variance is ordinally invariant to standardization.
  - In countries where PISA coverage rates are low, sample selection biases could lead to substantial underestimates of inequality (of achievement or opportunity).

# 7. Conclusions

- Our lower-bound measure of inequality of educational opportunity is the share in the variance of test scores "explained" by ten pre-determined personal and family circumstances.
  - In the 57 countries of the PISA 2006 this share ranges from 10% to 39%, depending on country and subject.
  - In some countries, selection correction can raise the share by up to eight percentage points.
  - Cultural and economic endowments appear to account for most of the effect of family background.
  - IPI is pos. associated with early tracking, and neg. associated with the primary share of public educational expenditures.
  - IPI is particularly high in some Western European countries (e.g. France, Germany, Belgium) which are not usually thought of as particularly opportunity-unequal. They are also high in LAC.