The Economics of Family Influence and Inequality
(Adapted from Becker and Tomes (1986) and from Becker's lectures)

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Becker's Human Capital

- Human Capital is special because:
  it has an inextricable link to the human being and nonpecuniary elements affect its choice and returns
- Human Capital is a fundamental component of productivity
- Human Capital also provides a natural economic link between parents and children
- Like capital, it is durable; up-front costs have downstream returns
- Human capital has distinctive features:
  (a) Cannot be bought and sold (no slavery)
  (b) No market for its stocks (only its flows)
  (c) Cannot be used as collateral (lending problems)

Basics

- Self-productivity (skill begets skill), but some models of capital have this feature
- Empirical facts about Human Capital
  - Positive correlation between education, health, training, good diet, adequate use of contraception, marriage stability, and adaptation to technology
  - **Complementarity** is a general feature of it, in its use and in its production (components of human capital reinforce each other)
  - Parents convey advantages and affect social mobility

Single Child Adult Model

- Basic assumptions
  - Representative family with one adult and one child
  - Two periods, childhood and adulthood
  - Denote as t the period in which child and parent overlap
  - Do not overlap as adults
  - Parent makes decisions in t. At the beginning of $t+1$ the parent dies and the child becomes an adult.
  - We drop "t" subscript until it is useful (later in notes)

Resource Allocation

- Parent decides how to allocate exogenous income Y_p into
  - Consumption C_p
  - Investment in child I_c

$$\underbrace{C_p}_{\text{parental consumption}} + \underbrace{I_c}_{\text{investment in children}} = \underbrace{Y_p}_{\text{parental income}} . \quad (1)$$

- Child has no way to repay parent for the investment made in t.
- Model interesting only if there is something about the child that the parent values (e.g., via paternalism, altruism, desire to preserve genes).
- **Exercise:** Distinguish paternalism from altruism.

Parental Utility

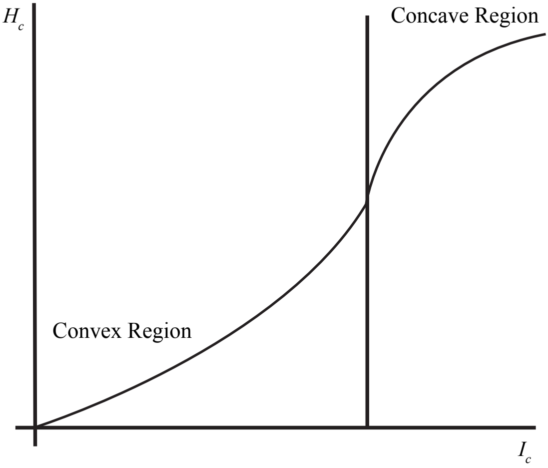
$$V_p(Y_p) = \overbrace{U(C_p)}^{\substack{\text{utility of} \\ \text{parent from} \\ \text{consumption}}} + \overbrace{\underbrace{a}_{\substack{\text{altruism} \\ \text{(in one} \\ \text{interpretation)}}} \underbrace{V_c(Y_c)}_{\substack{\text{child's} \\ \text{utility?}}}}^{\substack{\text{utility of} \\ \text{child for} \\ \text{parent}}} \tag{2}$$

- There are multiple interpretations of $V_c(Y_c)$: could be parent's perception of what is good for the child. In this case "*a*" is not altruism – it is a measure of concern for what they think is good for the child.
- C_p is parental consumption, Y_c is child's income *during adulthood*, and "*a*" is a measure of *direct altruism* (under one interpretation)
- Assume (2) is concave in its arguments, twice differentiable and satisfies Inada conditions.
- $\left(\lim_{C_p \to 0} \frac{\partial U(C_p)}{\partial C_p} \to \infty \text{ and } \lim_{Y_c \to 0} \frac{\partial V_c(Y_c)}{\partial Y_c} \to \infty \right)$

Technology of Skill Formation

- $H_c \equiv f(I_c)$ denote child's human capital in relevant region.
- Strictly concave in parental investment and twice differentiable:
- $\frac{\partial H_c}{\partial I_c} > 0$, $\frac{\partial^2 H_c}{\partial I_c^2} < 0$.
- Concavity (eventually) comes from limiting factors.

Figure 1: Human Capital Production Possibilities



The Rental Rate of Human Capital

- Let W be the rental rate of human capital (payment per unit human capital).
- This rate is (i) common across households; (ii) taken as parametric by parents as part of their market environment.
- $Y_c = WH_c$
- So far the only reason why earnings differ among children is because of human capital—which investment in children, I_c, creates.
- This is a Smithian vision and is very Beckerian.
- All differences among people are self- (or societally- or parentally-) generated.
- We are all equal at birth.

The Problem of the Parent

$$\max_{I_c, C_p} \{V_p(Y_p) = U(C_p) + aV_c(Y_c)\} \tag{3}$$

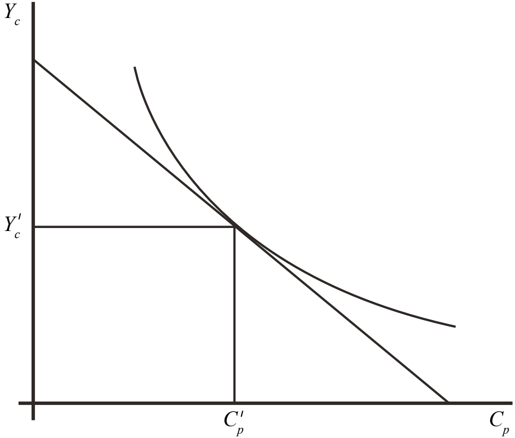
s.t.

$$C_p + I_c = Y_p \tag{4}$$

$$Y_c = WH_c \tag{5}$$

$$H_c = f(I_c) \tag{6}$$

Figure 2: Solution to the Single Child Unisex Adult Model



Optimality

- The first order condition is

$$\frac{\partial U(C_p)}{\partial C_p} \geq a\frac{\partial V_c(Y_c)}{\partial I_c} = a\frac{\partial V_c}{\partial Y_c} W \frac{\partial f}{\partial I_c} \tag{7}$$

- May have a corner solution ($I_c = 0$) if parents a=0 or value of child low, or income low (Assuming $f(0) = 0$ and $a \neq 0$, the Inada condition on V_c rules out a corner solution for I_c)
- $W\frac{\partial f}{\partial I_c} \equiv (1 + r_I)$ is the marginal rate of return of investment in the child, I_c.
- Rewrite (7) in interior equilibrium as

$$\frac{\partial U(C_p)}{\partial C_p} = a\frac{\partial V_c(Y_c)}{\partial Y_c}(1 + r_I). \tag{8}$$

- Assuming Inada conditions, (4), (5), (6) and (8) characterize the equilibrium assuming that the marginal utilities of C_p and Y_c are bounded away from zero.

Comparative Statics

- Consider a change in Y_p on the demand for I_c and C_p
- Observe that $C_p + I_c = Y_p$

$$C_p + I_c = Y_p \tag{9}$$

$$(*) \quad (aW)\frac{\partial f}{\partial I_c}\frac{\partial V_c(Y_c)}{\partial Y_c} - \frac{\partial U(C_p)}{\partial C_p} = 0$$

Totally differentiate $(*)$ to obtain

$$(aW)\frac{\partial^2 f}{\partial I_c^2}\left(\frac{\partial V_c(Y_c)}{\partial Y_c}\right) dI_c \tag{10}$$

$$+(aW^2)\left(\frac{\partial f}{\partial I}\right)^2\left(\frac{\partial^2 V_c(Y_c)}{\partial Y_c^2}\right) dI_c$$

$$-\frac{\partial^2 U(C_p)}{\partial C_p^2} dC_p = 0.$$

Observe that

$$dC_p = dY_p - dI_c \tag{11}$$

Substitute out dC_p

$$\Bigg[\overbrace{aW\frac{\partial^2 f}{\partial I_c^2}\quad\frac{\partial V_c(Y_c)}{\partial Y_c} + aW^2\left(\frac{\partial f}{\partial I}\right)^2\frac{\partial^2 V(Y_c)}{\partial Y_c^2}}^{<0 \text{ (concavity)}}\Bigg]dI_c$$

$$= \overbrace{\left(\frac{\partial^2 U(C_p)}{\partial C_p^2}\right)}^{<0 \text{ (concavity)}}dY_p$$

From strict concavity we get

$$\frac{\partial I_c}{\partial Y_p} > 0.$$

Comparative Statics

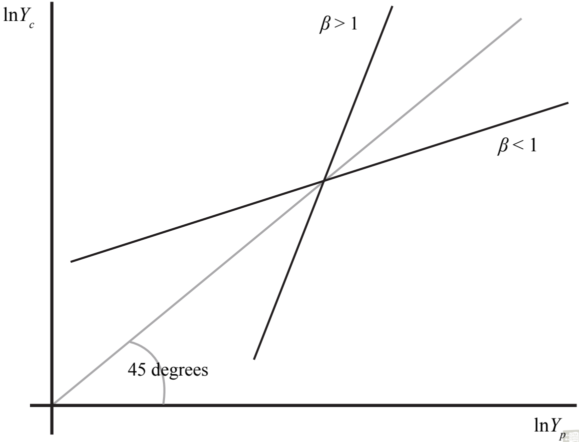
- Concavity and additive separability: guarantee that C_p and I_c are normal
- An increase in Y_p generates a standard income effect
- The income effect increases investment in children: links income across generations, takes us to the study of *intergenerational income mobility*

- Economists study this link and call it intergenerational income mobility

$$\ln Y_c = \alpha + \beta \ln Y_p + \epsilon_c. \quad (12)$$

- ϵ_c are "shocks" that affect children. ϵ_c is assumed uncorrelated with Y_p
- β = IGE (Inter Generational Elasticity)
- Higher β, the less social mobility in the society (children's status tied to that of parent's).
- β is a policy relevant parameter as it is the causal effect of a 1% increase on Y_p has on Y_c
- If $\beta < 1$ there is regression to the mean
- If $\beta > 1$ there is regression away the mean
- Observe that if $\epsilon_c = 0$ and $\beta > 0$, ranks of child and parent the same across generations in their respective generational distributions.

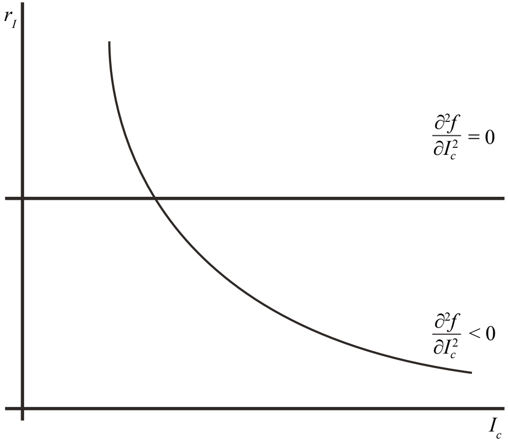
Figure 3: The IGE, Regression “to” and “away” the Mean



Returns to Investment

- A special case: the so-called "AK" model in macroeconomics which in our context is $H_c = QI_c$
  - Q is a constant common across families
  - $(1 + r_I) = WQ$
  - "Markets" are working perfectly
  - Returns to investment in the child are the same across different levels of parental income
- More general case: $\frac{\partial^2 f}{\partial I_c^2} < 0$, recall that $(1 + r_I) = W\frac{\partial f}{\partial I_c}$
  - "Price effect:" further investment decreases the marginal return to investment
  - A rich parent finds it less beneficial (at the margin) to invest in the child
  - The fact that rich parent cannot lend to a poor parent introduces an inefficiency

Figure 4: The Return to Investment under Different Production Functions



Summarizing

$$\frac{\partial I_c}{\partial Y_p} > 0 \therefore \frac{\partial \ln Y_c}{\partial \ln Y_p} = \beta > 0$$

Two natural questions

- Is there regression to the mean?
  - The empirical evidence for the US finds $\beta \cong .4$
- What are the implications of β for inequality in a generation?
  - Let the variance be a measure of inequality in a generation
  - Assuming stationarity $\mathrm{var}(\ln Y_p) = \mathrm{var}(\ln Y_c)$, $\mathrm{var}(\epsilon_p)\mathrm{var}(\epsilon_c)$
  - Suppose $\epsilon_c \perp\!\!\!\perp Y_p$ (this says ϵ_c is statistically independent of Y_p and hence is uncorrelated)
  - $\mathrm{var}(\ln Y_c) = \frac{\mathrm{var}(\epsilon_c)}{1-\beta^2}$
  - As $\beta \to 1$ inequality in a generation explodes
  - Causal story: from β to inequality
  - This is not the usual "Gatsby curve" story

- Allow a child's human capital production to depend on parental human capital: $H_c = f(I_c, H_p)$
- H_p is given in t and we do not explicitly model how it gets there

$$\begin{aligned} \frac{\partial f}{\partial H_p} &> 0 \\ \text{Complementarity}: \quad \frac{\partial^2 f}{\partial H_p \partial I_c} &> 0. \end{aligned} \tag{13}$$

Complementarity

- Property (13) is crucial.
- Differences in the production function across families are based on the inputs and not on the technology.
- Now, however, the productivity of investment for the technology differs because H_p differs.
- This affects the opportunities available to children.

Revisit our analysis:

$$\max_{I_c, C_p}\{V_p(Y_p) = U(C_p) + aV_c(Y_c)\} \tag{14}$$

s.t.

$$C_p + I_c = Y_p \tag{15}$$

where $Y_c = Wf(I_c, H_p)$.

What is the effect of a change in H_p on I_c and Y_c?

First order condition is (7) (with equality) or rewritten as (8).

$$\frac{\partial U(C_p)}{\partial C_p} = a\frac{\partial V_c(Y_c)}{\partial Y_c} W \frac{\partial f}{\partial I_c}$$

Totally differentiate (but now remember that f has an additional argument).

$$(**) \quad \frac{\partial^2 U(C_p)}{\partial C_p^2} dC_p - \left\{ a\frac{\partial V_c(Y_c)}{\partial Y_c} W \frac{\partial^2 f}{\partial I_c^2} + a\frac{\partial^2 V_c}{\partial Y_c^2}\left(W\frac{\partial f}{\partial I_c}\right)^2 \right\} \cdot dI_c$$

$$= \left\{ a\frac{\partial^2 V_c(Y_c)}{\partial Y_c^2} W^2 \frac{\partial f}{\partial I_c}\frac{\partial f}{\partial H_p} + a\frac{\partial V_c(Y_c)}{\partial Y_c} W \frac{\partial^2 f}{\partial I_c^2 H_c} \right\} \cdot dH_p$$

$$dC_p + dI_p = WdH_p$$

Suppose we compensate for the higher parental income flow arising from greater H_p (i.e., impose tax $dT = WdH_p$). Then $dC_p + dI_p = 0$. Thus we have that we can rewrite $(**)$ as

$$-\left\{\overbrace{\frac{\partial^2 U(C_p)}{\partial C_p^2}}^{<0} + \overbrace{a\frac{\partial V_c(Y_c)}{\partial Y_c} W \frac{\partial^2 f}{\partial I_c^2}}^{<0} + \overbrace{a\frac{\partial^2 V_c(Y_c)}{\partial Y_c^2}\left(W\frac{\partial f}{\partial I_c}\right)^2}^{<0}\right\} \cdot dI_c$$

$$= \left\{\underbrace{\overbrace{a\frac{\partial^2 V_c(Y_c)}{\partial Y_c^2} W^2 \frac{\partial f}{\partial I_c}\frac{\partial f}{\partial H_p}}^{<0}}_{\substack{\text{diminishing} \\ \text{marginal utility}}} + \underbrace{\overbrace{a\frac{\partial V_c(Y_c)}{\partial Y_c} W \frac{\partial^2 f}{\partial I_c \partial H_p}}^{>0}}_{\substack{\text{productivity} \\ \text{effect}}}\right\} dH_p$$

The income compensated effect of an increase in H_p is thus ambiguous. The term on the left hand side is positive (from concavity). The first term on the right hand side is negative. It is more negative the more steeply the diminishing marginal utility of expenditure on Y_c. For high income families it is more negative than for poor families.

This effect arises because of the higher H_p (compensating for any effect of income). The higher is $H_c = f(I_c, H_p)$, i.e., for the same investment, the child has more H_c and hence Y_c. It's like manna from heaven falling on the family in the form of a costless increase in Y_c (Remember H_p is treated as determined outside the model). *Ceteris paribus*, the family will reallocate less funds to I_c and spend more on C_p. Remember, their total budget is kept constant. This effect is analogous to the improvement in quality of a good in standard consumer theory. *Ceteris paribus*, families will invest less because manna has fallen from heaven on the child, so families need to invest less to get the same child quality.

The second term is positive. This is because of complementarity $\frac{\partial^2 f}{\partial I_c \partial H_p} > 0$. Children from families with higher H_p make more productive investments. If we add back the income effect so now $(dT = 0)$, this is a force toward more C_p and more I_c and hence Y_c.

But notice that even if $I_C \downarrow$, $Y_c \uparrow$. The intuition is that parents are better off and their children are too.

Exercise: Prove this claim.

Efficiency vs. Equity

- When $H_c = f(I_c)$ there is no trade-off between efficiency and equity
  - As I_c increases, $(1 + r_I)$ diminishes
- When $H_c = f(I_c, H_p)$ things are not so simple
  - It is more efficient to invest in the human capital of the child with a relatively higher parent's human capital
  - "Matthew Effect"
  - But diminishing marginal utility may offset this effect

Figure 5: Investment and its Return without Parental Human Capital in Technology

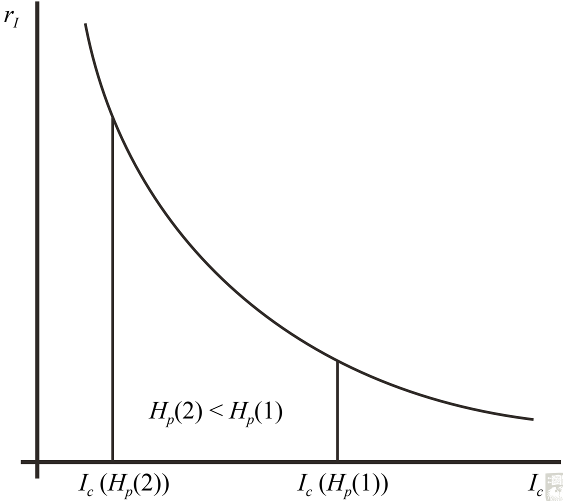
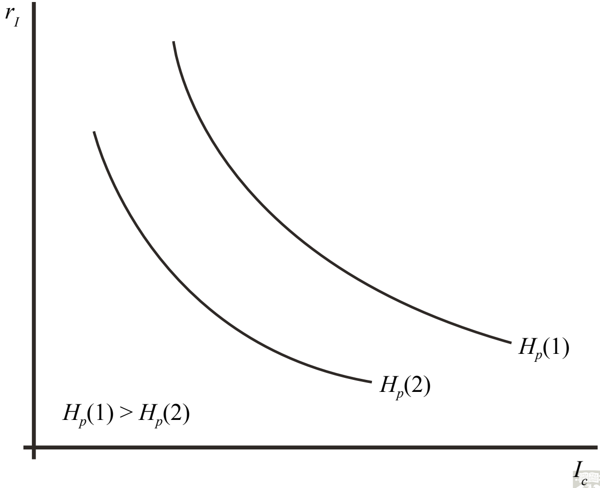


Figure 6: Investment and its Return with Parental Human Capital in Technology



- Adding innate ability to the model is straightforward. Let A_c denote ability (of child) and note we take them as fixed outside the model. In this extension,

$$H_c = f(I_c, H_p, A_c) \tag{16}$$

with $\frac{\partial H_c}{\partial A_c} > 0$.

- If parental human capital and child ability are complements it is straightforward to show (by the same argument as before) that we cannot sign $\frac{\partial I_c}{\partial A_c}$ because the diminishing marginal utility ("manna") effect is negative while the complementarity (productivity) effect is positive.
- Thus, it is not possible to sign the *direct effect* of ability on investment in the general case.

The Indirect Effect of Ability on Investment

- From the model, it is easy to see that $\frac{\partial Y_c}{\partial A_c} > 0$.
- **Exercise:** Show this.
- If it is also the case that $\frac{\partial Y_p}{\partial A_p} > 0$. Due to $\frac{\partial H_p}{\partial A_p} > 0$, an abler parent invests more in his child through an effect of A_p on income, an *indirect effect*.
- As in the case of greater parental human capital, it is more efficient to invest in a relatively more able child.
- Thus, efficiency and equity may go in opposite directions when ability affects human capital production.

- Extend the model to a case where there is a market for physical capital.
- If parental utility satisfies standard Inada conditions, $I_c > 0$ in the last sections.
- Now, another condition needs to hold. Let r_k denote the rate of return to capital.
- Thus, in order for $I_c > 0$ it has to be the case that $r_I \geq r_k$.
- When capital is available, child's income is

$$Y_c = WH_c + (1 + r_k)K_c \tag{17}$$

where K_c is the capital the parent gives for his child in t.

- Hence, the parental budget constraint is

$$C_p + I_c + K_c = Y_p. \tag{18}$$

while the utility is still the same as before.

Exercise: Extend the analysis to consider the case where parents have two children. One is more able than the other ($A_1 > A_2$).

- Analyse:
  (a) The investment in each child
  (b) The capital transferred to each child
  (c) The income of each child
- Assume that the are equally altruistic toward each child and children have the same preferences.
- Thus the preference function is

$$U(C_p) + aV_c(Y_{c_1}) + aV(Y_{c_2}) \tag{19}$$

$$Y_p = I_{c_1} + I_{c_2} + C_p + K_1 + K_2 \tag{20}$$

where K_1 and K_2 are the capital given child one and child two and investments are similarly subscripted

$$H_{c_1} = f(I_{c_1}, H_p, A_1) \qquad H_{c_2} = f(I_{c_2}, H_p, A_2) \tag{21}$$

where H_p is like a public good.

Physical Capital over Time

- Once physical capital is introduced, the next natural question is to ask if it is possible to generalize the model to multiple periods and add features such as parental bequests.
- Let parents have a middle (m) and an old (o) age.
- Let B_c denote the bequest the parent leaves to the child and B_p the bequest the parent receives from the grandparent.
  - The reason why a bequest is necessary is because the parent decides how much physical capital to acquire in his middle age, K_m, which pays a return in his older age.
  - He decides how much of it to transfer to his child through B.
  - This is slightly different from the model before, in which part of the transfer to the child is explicit capital.

Optimality with Multiple Periods

- The multiple period budget constraints are

$$\text{Middle age} \quad C_m + I_c + K_m = Y_p + \overbrace{B_p}^{\text{from their parent}} \tag{22}$$

$$C_o + \underbrace{B_c}_{\text{to their child}} = (1 + r_k)K_m \tag{23}$$

- This assumes bequests come at the end of life
- The utility function is,

$$V_p(Y_p + B_p) = U(C_m) + \delta U(C_o) + \delta a V_c(WH_c + B_c) \tag{24}$$

where $\delta > 0$ is a discount factor.

- The optimality conditions are as usual and the main remark is that if $r_k = r_I$ then $B_c \geq 0$ while if $r_k < r_I$ then $B_c = 0$.
    - That is, if the investment in the child, I_c, is more productive than investing in physical capital then there is no bequest because there is no initial investment in physical capital to accumulate it for the the older age period (when the parent bequests).

- Becker and Tomes (1986) extend these ideas
- The main components of the model are the same but there are generalizations on functional forms of production functions and transmission processes
- They build on their previous work, Becker and Tomes (1979), and assume innate ability follows an stochastic, linear autoregressive process. For person i, ability in generation t is A^i_{t-1}

$$A^i_t = \eta_t + hA^i_{t-1} + \upsilon^i_t \tag{25}$$

  where υ^i_t is genetic ("luck")
- This mechanically links the family across multiple generations, with the strength of family persistence controlled by h
- Generations are linked through ability and through family income

Their model has the following feature. Take the IGE equation (12) but write it more generally. Let t be a generation.

$$lnY_{t+1} = \alpha + \beta lnY_t + A_{t+1}. \quad (26)$$

Ability determines earnings of the child, and it is the "error term" (There may be other factors but abstract from them for now).

Let $A_{t+1} = \eta + hA_t + \upsilon_{t+1}$ where the υ_t are mutually independent and uncorrelated. (26) comes from a model where parents are credit constrained and β is derived from a model where children do not have access to credit markets to finance their investment. (So parents cannot leave debts to children).

Notice that lnY_t and A_{t+1} are correlated as long as $h \neq 0$. (Why? Show this.) So least squares applied to (26) is biased.

Exercise: Derive the bias.

We can write (substituting (25) into (26) to solve out A_{t+1} in terms of A_t):

$$lnY_{t+1} = (\alpha + \eta) + \beta lnY_t + hA_t + \upsilon_{t+1}. \tag{27}$$

From (26) lagged one period,

$$lnY_t - \alpha - \beta lnY_{t-1} = A_t.$$

Substituting into (27)

$$lnY_{t+1} = (\alpha + \eta - h\alpha) + (\beta + h) \ln Y_t - h\beta Y_{t-1} + \upsilon_{t+1}.$$

Notice further thatβ and h are not separately identified.

Notice that if $\beta > 0$ and $h > 0$, "the effect" of income of generation $t - 1$ on the income of generation $t + 1$ is negative! (Show).

Exercise: Add a regressor Z_t to equation (26).

$$lnY_{t+1} = \alpha + \beta lnY_t + \gamma Z_t + A_{t+1}.$$

Assume that Z_t is uncorrelated with A_{t+1}.

Question. If $Z_t \neq Z_{t-1}$, show how β and h are separately identified.

Question. If $Z_t = Z_{t-1}$, show that they are not identified.

Exercise: Holding ability constant, what is the effect of experimentally changing Y_{t-1} (i.e., randomly assigning it) on Y_{t+1}? Would a policy of changing the income of grandparents (by exogenous transfers) raise the income of grandchildren? (Consider two cases—one where $H_c = f(I_c)$ and one where $H_c = f(I_c, H_p)$.)

- These ideas are used in their article in a nonlinear model
- They propose the following earnings function for generation t

$$Y_t = WH_t + L_t, \tag{28}$$

where Y_t is earnings and L_t is labor market luck.

- The production function for child's human capital in time t is generalized to depend positively of three inputs:
  - parental investment, $I_{c,t-1}$;
  - public expenditures, S_{t-1};
  - ability, A_t.
- The general form of the production function for human capital is

$$H_t = f\left(I_{c,t-1}, S_{t-1}, A_t\right). \tag{29}$$

- Ability is assumed to be a complement of both parental investment and public expenditures. That is,

$$\frac{\partial^2 H_t}{\partial j \partial A_t} > 0 \tag{30}$$

for $j = I_{c,t-1}, S_{t-1}$.

- Marginal return of parental investment:

$$\frac{\partial Y_t}{\partial I_{c,t-1}} = Wf_I \equiv 1 + r_{I,t}(I_{c,t-1}, S_{t-1}, A_t, W).$$

Generalized Earnings Autoregression

- Parents equate marginal returns to their utility of private consumption to their perceived returns from expenditure on the child.
- To study the solution define as $r_{k,t}$ the return to physical capital in time t and define the optimal investment as $I^*_{c,t-1}(A_t, s_{t-1}, r_{k,t})$ such that $r_{I,t} = r_{k,t}$.
- Using $I_{c,t-1}(\cdot,\cdot,\cdot)$, (28), and (29) we can write earnings as

$$\begin{aligned} Y_t &= f(I_{c,t-1}(A_t, S_{t-1}, r_{k,t}), S_{t-1}, A_t, W) + L_t \\ &= \phi(A_t, S_{t-1}, r_{k,t}, W) + L_t \\ \phi_A &= f_{I_{c,t-1}} \frac{\partial I_{c,t-1}}{\partial A} + f_A > 0 \end{aligned} \tag{31}$$

- Substitute in (25) and obtain the following

$$Y_t = G\left(Y_{t-1}, L_{t-1}, \upsilon_t, h, S_{t-1}, S_{t-2}, r_{k,t}, r_{k,t-1}, \eta_t, W\right) + L_t. \tag{32}$$

- Parental earnings only enter as a proxy for ability

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A Special Case: Linear function ϕ (w.r.t. A)

- If G is additively separable it is possible to summarize the complex process in (32) as follows:

$$\begin{aligned} Y_t &= c_t + \alpha_t \phi_A + hY_{t-1} + L_t^* \\ L_t^* &= L_t - hL_{t-1} + \phi_A \upsilon_t \\ c_t &= c(S_{t-1}, S_{t-2}, h, r_{k,t}, r_{k,t-1}, W) \end{aligned} \tag{33}$$

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- To introduce imperfect access to capital, **if parents face financial constraints**, optimal investment depends on parental earnings, Y_{t-1}, altruism, a, and uncertainty on child's (and future generations) luck, τ_t:

$$I^*_{c,t} = g^*\left(A_t, S_{t-1}, Y_{t-1}, \tau_t, a, W\right) \tag{34}$$

where all the arguments enter positively in g^*.

- By assumption, an increase on Y_{t-1} implies an increase in investment.

The Multi-generational Earnings Process

- We can write an equation analogous to (31) as follows

$$Y_t = \phi^* \left(A_t, Y_{t-1}, a, S_{t-1}, \tau_t, W \right) + L_t. \tag{35}$$

- Similarly, as in (32) we can write earnings as

$$Y_t = G \left(Y_{t-1}, Y_{t-2}, I_{t-1}, \upsilon_t, a, \eta_t, S_{t-1}, \epsilon_{t-1}, S_{t-1}, \tau_t, W \right) + L_t. \tag{36}$$

- Parental earnings have two effects in this context: a direct effect through earnings *per se* and an indirect effect through the transmission of ability.
- Moreover, the grandparent's earnings effect on child's income, holding parent's earnings fixed is:

$$\frac{\partial Y_t}{\partial Y_{t-2}} = -h\phi^*_{Y_{t-2}} \left(\frac{\phi^*_{A_t}}{\phi^*_{A_{t-1}}} \right) > 0$$

Why is the Grandparent Earnings Coefficient Negative?

Interpretation:

- An increase in the earnings of grandparents lowers the earnings of grandchildren's keeping parent's earnings and grandchildren's earnings and luck constant.
- From (31), the total effect of an increase in grandparent earnings on parent earnings is positive; denote this effect by ϕ_Y^*.
- In order to hold parent earnings fixed while raising grandparent earnings, parent earnings must be reduced by ϕ_Y^* through endowments.
- The reduction in parent endowments by ϕ_Y^* is then passed on the the child as an endowment reduction of size $\phi_Y^* h$.

- In summary, the negative partial derivative of grandparent earnings on child earnings indicates that shocks to persistent endowments are a more powerful force than transitory earnings shocks in the economics of the family.
- The linear representation of this relationship is a linear approximation of (36)

$$Y_t \approx \eta_t' + \left(\phi_Y^* + h\right) Y_{t-1} - \phi_Y^* h Y_{t-2} + L_t^* \tag{37}$$

where η_t' is a function of α_t.

Becker, G.S. (1991). *A Treatise on the Family*, chapters 6 and 7. Cambridge: Harvard University Press.

Education: A Traditional Approach to Human Capital (Becker, Econ 343)

- A traditional analysis of human capital formation is the study of investment in education by a high-school graduate young adult who takes human capital as given and decides whether or not to go to college.
- The young adult arrives to this decision period, or *node*, with existing human capital H_0, take as exogenous and attributable to past decisions of the child, the parent, or both. Let T denote the number of periods remaining in the child's life once he/she reaches the college attendance decision node.

- Suppose the child has perfect foresight over two earnings streams:

$$Y_{H,t,i} \quad t = 0, \ldots, T \text{ :high-school graduate stream} \tag{38}$$

$$Y_{C,t,i} \quad t = 0, \ldots, T \text{ :college graduate stream.} \tag{39}$$

  Let t_c be the number of years college requires, τ be the full cost of college tuition and other psychic costs and $\Delta_t \equiv Y_{C,t} - Y_{H,t}$ is the expected difference in earnings between a college and high school graduate in period t.
- Importantly, during the first t_c years $Y_{C,t}$ may be lower than $Y_{H,t}$ because college students may have less time to work than high-school (already) graduates.

- Individuals choose to go to college if,

$$\sum_{t=0}^{T} R_t \Delta_t \geq \tau \tag{40}$$

where R_t is the discount factor $[= (1+r)^t]$. Δ_t is the "high-school college" wage differential.

- Thus, the decision depends on
  (i) the time college takes to complete, t_c;
  (ii) the duration of the life span, T; tuition, τ; and,
  (iii) the wage differential at each age, Δ_t.

- Additional considerations, such as the probability of dying, can be added to the model.
- For example, let $(1 - m_t)$ the probability of surviving last period and note that $m_{t'} = 1$ if $m_t = 1$ for $t' > t$.
- Then, the young adult attends college if and only if,

$$\sum_{t=0}^{T} R_t \Delta_t (1 - m_t) \geq \tau. \tag{41}$$

- Abilities at the age (A) at which the decision node is reached may also be important to this decision. Ability may enter as follows:

$$\frac{\partial t_c(A, H_0)}{\partial j} < 0 \tag{42}$$

$$\frac{\partial \tau(A, H_0)}{\partial j} < 0, \quad j \in \{A, H_0\} \tag{43}$$

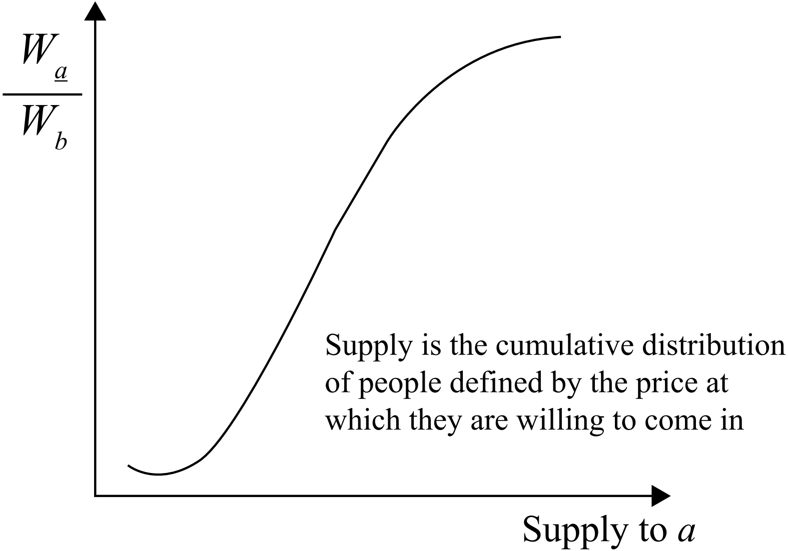
- Abler and more endowed people, holding the rest of the components of the model constant, graduate faster (e.g., repeat less courses) and find it cheaper to study (e.g., scholarships).

Specialization and the Division of Labor

- In the Roy model, people have different talents with different prices so that agents enter the sector in which they have a comparative advantage.
- The problem with this, (according to Becker) is that skills are not given, they are produced.
- This is consistent with his Adam Smith vision of the world that we are all born alike and education and socialization is what separates us.
- Talents and abilities depend on investment and opportunities.

- Investing in a smaller set of skills instead of a widespread set can make for a higher rate of return.
- One of the important characteristics of investment in human capital is that the rate of return tends to increase with the time put into it.
- So, activities that take a lot of investment usually are not part time (like schooling).
- This implies that there are increasing returns to investment in human capital with specialization.

- Note that the elasticity of supply is largely determined by heterogeneity in the population.
- If agents are not identical, supply will be very elastic.
- People with high abilities enter at low prices and people with low ability need a higher price to enter.


$\frac{W_a}{W_b}$
Supply is the cumulative distribution of people defined by the price at which they are willing to come in
Supply to a

- Suppose initially that one task is needed to produce one product.
- We will address the general problem of m tasks (where m can be very large).
- All agents are the same (equally able and with the same level of initial human capital), and all have access to a competitive market in financing human capital.
- Each task takes some investment, so that the more time an agent invests, the more skilled he becomes.

- t: the amount of hours available.
- Individual must divide time between investing in a task (h) and working (l).
- $t = l + h$.
- Complementarities assumed away initially.
- The production function of human capital:

$$H = g(h) \tag{44}$$
$$g' > 0, g'' \leq 0 \tag{45}$$

- Notice that in a one period model, the concept of "capital" is ambiguous.
- Task output:

$$Y = lH. \tag{46}$$

- The agent only cares for Y.
- Goal: maximize Y subject to equation (44) and $t = l + h$.
- FOC's:

$$H = \lambda \tag{47}$$

$$g'(h)l = \lambda. \tag{48}$$

- Thus

$$l = \frac{g(h)}{g'(h)}. \tag{49}$$

Example:

- $g = ch^{\theta}$ for $0 < \theta \leq 1$.
- Solution:

$$I^* = \frac{1}{1+\theta} t \text{ and } h^* = \frac{\theta}{1+\theta} t. \tag{50}$$

- If $\theta = 1$, then $I^* = h^* = \frac{1}{2}$. As $\theta \downarrow$, $h^* \downarrow$.
- In the general case

$$Y = \frac{[g(h)]^2}{g'(h)}. \tag{51}$$

- For the example, $g = ch^{\theta}$,

$$Y = \frac{c}{\theta}\left(\frac{\theta}{1+\theta}\right)^{1+\theta} t^{1+\theta}. \tag{52}$$

- (51) shows clearly *increasing returns to scale* in h and hence t.
- Second case, (52), $0 < \theta \leq 1 \Rightarrow$ that Y grows more than proportionally with time.
- If $\theta = 1$, then $Y = \frac{ct^2}{4}$.
- This is consistent with learning by doing.

Two Tasks

- Suppose two tasks are required to produce a final output.
- $t = l_1 + l_2 + h_1 + h_2$,
- $Y_i = l_i H_i$, and
- $H_i = g_i(h_i)$ for $i = 1, 2$.
- Total output:

$$Q = \mathrm{Min}\left[Y_1, Y_2\right].$$

- Leontief production function.
- The agent has to produce both tasks.

- Solve this in stages.
- For a fixed t_i (time allocated to i), maximize Y_i as before:

$$l_i^* = \frac{g(h_i^*)}{g'(h_i^*)} \text{ and } h_i^* = t_i - l_i^* \tag{53}$$

- For $g = c_i h_i^{\theta}$

$$l_i^* = \frac{1}{1+\theta_i} t_i \text{ and } h_i^* = \frac{\theta_i}{1+\theta_i} t_i. \tag{54}$$

- Optimal t_i?
- It must be true that $Y_1^* = Y_2^*$, thus in our example,

$$Y_1^* = \frac{c_1}{\theta_1} \left(\frac{\theta_1}{1+\theta_1} \right)^{1+\theta_1} t^{1+\theta_1} = \frac{c_2}{\theta_2} \left(\frac{\theta_2}{1+\theta_2} \right)^{1+\theta_2} t^{1+\theta_2} = Y_2^*. \tag{55}$$

- If $\theta_1 = \theta_2 = \theta$,

$$c_1 t_1^{1+\theta} = c_2 t_2^{1+\theta}. \tag{56}$$

- If $c_1 = c_2$, divide the t's equally; as $c_i \uparrow$, $t_i \downarrow$.
- $c_1 = c_2 = c$ and $\theta = 1$.

$$t_i = \frac{t}{2}; Y_i = \frac{c}{4}\left(\frac{t}{2}\right)^2 = \frac{ct^2}{16} = Q. \tag{57}$$

- For the case of m tasks,

$$Q = \text{Min}\left[Y_1, \ldots, Y_m\right] \tag{58}$$

$$Y_i^* = \frac{[g_i(h_i^*)]^2}{g'(h_i^*)} = \lambda \text{ for all } i \tag{59}$$

$$\sum t_i^* = \sum (l_i^* + h_i^*) = t. \tag{60}$$

- For example, when $\theta = 1; c_i = c \Rightarrow g_i = ch_i^* = \frac{1}{2}ct_i^*$,

$$Q^* = \frac{ct^2}{4m^2} \text{ since } t_i^* = \frac{t}{m}. \tag{61}$$

- Even if $\theta = 1$, as m gets large, agent productivity in producing final output is reduced if the same time is spread over more tasks.
- We have increasing returns to scale.
- By having to reduce time for each task, task agents reduces productivity.
- The solution is to *trade and specialize*.

- This results in teams, groups of specialized individuals working together to produce output.
- By forming a team, agents get rid of the m^2 disadvantage.
- Suppose there are m tasks and n team members.
- Each person has time t available.
- Each puts all his time into one task:

$$Y_i^* = \frac{c}{\theta}\left(\frac{\theta}{1+\theta}\right)^{1+\theta} t^{1+\theta}. \tag{62}$$

- If $\theta = 1$, $Y_i^* = \frac{ct^2}{4}$.
- Total production:

$$Q^* = \frac{ct^2}{4}. \tag{63}$$

- Must be shared among the members of the team.
- Suppose we divide it equally, **[Question: Why would we?]** then each agent j would get:

$$Q_j^* = \frac{Q^*}{m} = \frac{ct^2}{4m} > \frac{ct^2}{4m^2}. \tag{64}$$

- In any case, it is better than what the agent would get if he had to do all the tasks by himself, but it is not as good as the case of just one task.
- **[Question: Is a technology with more tasks a burden?]**
- In the no trade case, output per worker is falling at a rate of m^2.
- In the second case, it falls at a rate of m.
- That is, there is a cost to having a lot of tasks, but the cost is linear while the return is quadratic.

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Division of Labor in the Household

- Take for example a couple of equally productive people.
- Will there be a gain in the division of labor?
- Yes, if there are tasks with some complementarity, and there are gains to investment.
- Think of household and market with similar production functions to the ones we have.
- Now, $m = 2$ and $n = 2$.

- It is more efficient to have one member work in the market and the other in the household.
- Why have women historically been the ones in the household?
- This is a challenging question.
- It is not implied by the model of investment in human capital *per se*.
- There are two possible explanations:
  1. Discrimination against women in the market.
  2. Intrinsic difference, women may have absolute advantage, but relatively more productivity in the household.
     (c_i may differ between men and women.)

- We don't need a lot of either to explain this phenomenon.
- Segregation occurs very easily (to avoid discrimination or to benefit from biological differences).

The Division of Labor and the Extent of the Market

- Suppose there are m tasks where m is very large.
- They are independent.
- Huge gains from specialization.
- But, the market may be limited and prevent specialization.
- One interpretation is, given we have just one product, that the extent of the market is given by the number of available workers (N).

- Let N be the number of identical workers, then:

Cases

1. If $N > m$ and $Q = min\{Y_1, ..., Y_m\}$, how many people would produce each task?
   (a) The number of teams is $\frac{N}{m}$. E.g., if we have m = 1000 and N = 2000 we have two teams with m members each.
   (b) In a competitive market, everyone ends up earning the same. Hence division of labor is not limited by the extent of the market.

2. Suppose $N < m$ and $Q = min\{Y_1, ..., Y_m\}$.
    - There are not enough people to go around.
    - $\frac{m}{N} = s$ tasks per person.
    - In this case, the division of labor is limited by the extent of the market.
    - E.g., a very small town might have a surgeon, while a city like Chicago might have a heart surgeon, a brain surgeon, etc.
    - Suppose $g = ch^{\theta}$ in all activities.
    - An agent puts time t_i into the i^{th} task.
    - $Y_i^* = kt_i^{1+\theta}$ where $t_i = \frac{t}{s}$.
    - Total output would be:

$$Q^* = Y_i^* = k\left(\frac{t}{m}\right)^{1+\theta} N^{1+\theta}. \tag{65}$$

    - Per capita income would be: $q^* = \frac{Y_i^*}{N} = k\left(\frac{t}{m}\right)^{1+\theta} N^{\theta}$.

- Agents don't want to do a little bit of each task because of increasing returns due to specialization.
- So, when $m > N$, per capita income rises with N.
- When $N = m$, per capita income doesn't grow anymore.
- Notice that having more teams actually increases competition even when there are increasing returns.

Coordination Costs

- Now, if it were really true that the division of labor is limited by the extent of the market, no more than one person would be doing any single task in a small town.
- However, this does not make accord with reality, so there must be something else limiting the division of labor other than the extent of the market.
- One possibility: coordination costs.
- We need coordinators (entrepreneurs of various forms).
- Analogous to Coase's transaction costs idea, when there are specialists, they need coordination assuming fixed proportions.
- Notice that the size of a team is not given, *a priori*, it is a function of the degree of specialization.

- Let n be the number of members of any one team.
- Then, the number of two-way interactions is $n(n-1)$.
- Three-way: $n(n-1)(n-2)$; etc.
- Let $c(n)$ be the coordination costs per member of the team so that $\frac{\partial c(n)}{\partial n} > 0$.
- Then, net per capita income is

$$I(n) = Y(n) - c(n). \tag{66}$$

- We have a different maximization problem than before.
- Choose n to maximize per capita income.
- In this way, even when $N > m$, the size of the team can be smaller than m.
- This raises another question—why is it that there may be more than one team even when $m > N$?
- The FOC of the problem of maximizing $I(n)$ is:

$$\frac{\partial I(n)}{\partial n} = 0$$
$$\Rightarrow \frac{\partial Y(n)}{\partial n} - \frac{\partial c(n)}{\partial n} = 0 \tag{67}$$

- SOC:

$$\frac{\partial^2 Y(n)}{\partial n^2} - \frac{\partial^2 c(n)}{\partial n^2} < 0.$$

Cases

1. $\frac{\partial Y(n)}{\partial n} > \frac{\partial c(n)}{\partial n}$ for all $n \leq N \leq m$. Then, $n = N \leq m$ so that everyone is in the same team and it's only the extent of the market limiting specialization.
2. $\frac{\partial Y(n)}{\partial n} < \frac{\partial c(n)}{\partial n}$ for all $n > 1$. Then, there would be no specialization.
3. $1 < n^* < N$.
   - There will be some specialization $(n^* > 1)$ but not to the extreme of being market limiting.
   - E.g., if $m = 10000$ and $N = 5000$, it might appear that the extent of market is limiting, but if $n^* = 1000$ there will be 5 teams.
   - In fact, even a case with two teams would not lead to market-limiting specialization.

Example:

$$I = cn^{\theta} - \lambda n^{\beta} \tag{68}$$

$$\beta > \theta > 0. \tag{69}$$

- $n^* = \left(\frac{c\theta}{\lambda\beta}\right)^{\frac{1}{\beta - \theta}}$.
- $n^* \uparrow c, \downarrow \lambda$.
- Higher λ means higher costs, less specialization and smaller teams.

Population Density D:

- Coordination costs may depend on other factors, e.g. population density—$\lambda(D)$ with $\lambda' \leq 0$.
- This provides an explanation for the existence of cities; not only do they provide bigger markets, their high population density reduces costs.

- So $I^* = I^*(\lambda, \beta, \theta)$ with $\frac{\partial I}{\partial \lambda} < 0$.
- Also, assume (consistent with evidence that) $\frac{\partial I^*}{\partial D} = \frac{\partial I^*}{\partial \lambda}\frac{\partial \lambda}{\partial D} > 0$.
- Population density reduces coordination costs, and hence raises per capita income.
- That is, both the division of labor and per capita income should be higher in cities.
- Why is there such a big city size effect found in the data?
- Let P be population.

- If $D = \alpha P \Rightarrow \lambda(D) = \lambda(\alpha P)$ so that $\frac{\partial D}{\partial P} \leq 0$.
- This is a force why per capita income might be rising in level of population.
- That is, even if density is not proportional to population, per capita income might rise with the level of population, as long as

$$\frac{\partial I^*}{\partial P} = \frac{\partial I^*}{\partial \lambda} \frac{\partial \lambda}{\partial D} \frac{\partial D}{\partial P} > 0 \tag{70}$$

- This is a source of increasing returns and it is consistent with competition.
- As the population grows, the number of teams might change.
- An increase in density to D_2 results in an increase in both n and I.

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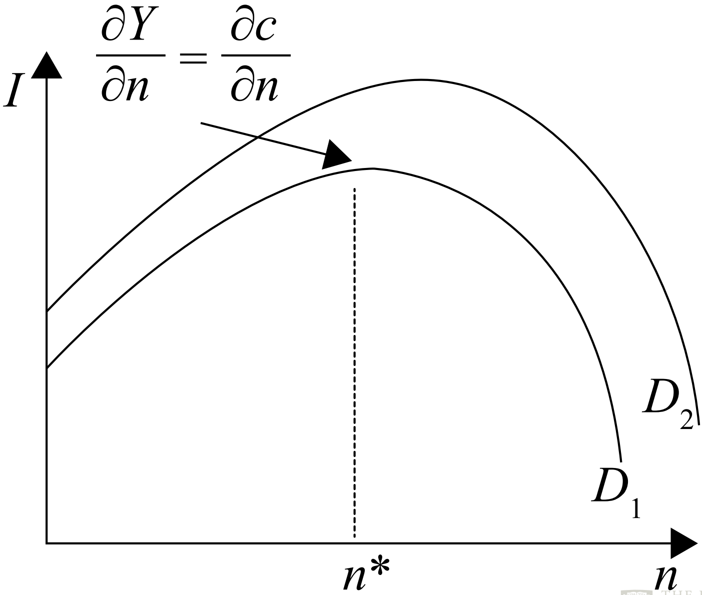
$\frac{\partial Y}{\partial n} = \frac{\partial c}{\partial n}$
I
D2
D1
n*
n

Adding Human Capital

- We assumed that each person started with a certain amount of human capital (general human capital).
- Why does general human capital interact with individual human capital?
- It seems that the more general skills an agent has, the more productive in human capital will be in each task.

$$Y = Y(H, n) \tag{71}$$

- H: human capital.
- Suppose $Y = cn^{\theta} H^{\gamma}$, and $\frac{\partial Y}{\partial H} > 0$ (i.e., $\gamma > 0$) and $\frac{\partial^2 Y}{\partial H \partial n} > 0$.
- So, an increase in H for any degree of specialization raises productivity.

- What about effects on coordination costs?
- Since we don't know, we will assume there is no effect of H on coordination costs λ.
- What is the effect of $H \uparrow$?
- FOC as before: now give people more H.
- Add more to the endowment H: what is the effect on n?

$$\frac{\partial I}{\partial n} = \frac{\partial Y}{\partial n} - \frac{\partial c}{\partial n} = 0 \tag{72}$$

$$\frac{\partial^2 I}{\partial n \partial H} = \frac{\partial^2 Y}{\partial n \partial H} + \frac{\partial^2 Y}{\partial n^2}\frac{\partial n}{\partial H} - \frac{\partial^2 c}{\partial n^2}\frac{\partial n}{\partial H} = 0 \tag{73}$$

which implies (using compact notation)

$$\frac{\partial n}{\partial H} = \frac{Y_{nH}}{c_{nn} - Y_{nn} > 0}. \tag{74}$$

- The optimal division of labor rises with the level of human capital.
- If $I = cn^{\theta} H^{\gamma} - \lambda n^{\beta}$,
- $n^{*} = \left(\frac{c\theta}{\lambda\beta} \right)^{\frac{1}{\beta-\theta}} H^{\frac{\gamma}{\beta-\theta}}$
- Like what we had before times an interaction term with general human capital.

$$I^{*} = K \lambda^{-\frac{\theta}{\beta-\theta}} H^{\frac{\gamma\beta}{\beta-\theta}}. \tag{75}$$

- Where K is a positive constant.
- **Question: What is K?**

- So, both per capita income and n rise in H.
- Then, an increase in H by 1% increases I by more than γ since $\frac{\beta}{\beta-\theta} > 1$.
- That is, not only a direct effect of H (given by γ).
- But also that we are increasing the degree of specialization.
- So, an increase in H should lead to greater specialization and greater per capita income.
- As economies become more specialized, coordination becomes harder.

Can we look at this as a growth model

- H increases both n and I increase.
- But, it is not only that specialization is induced by growth in H but also that growth in H might be induced by growth in specialization.
- The rate of return on H is given by

$$\frac{\partial I}{\partial H} = \frac{K\gamma\beta}{\beta - \theta} H^{\frac{\gamma\beta}{\beta-\theta}-1}. \tag{76}$$

- This shows the relationship between marginal product and H.
- Now, if $\frac{\gamma\beta}{\beta-\theta}=1$, there is no increase or decrease of the marginal return with H.
- Notice that we can assume $\gamma<1$ (diminishing returns) and still get growth.
- That is, we get endogenous nondiminishing returns via the interaction with specialization.

Health as Human Capital

- The objective of this section is to analyze health as a dimension of human capital.
- Individuals can invest in the stock of health capital in order to improve it.
- We consider a two period model.
- In each period $t = 1, 2$, the individual decides how much time to work, l_t, and how much to consume, C_t.
- We interpret his stock of human capital, $s(H)$, as his probability of surviving to the second period, where H is now interpreted as the stock of health capital at the end of the first period, and s satisfies $\frac{\partial s(H)}{\partial H} > 0$, $\frac{\partial^2 s(H)}{\partial H^2} < 0$.

- There is a strictly convex cost to invest on health, $g(h)$, and individuals are endowed with a unit of time each period.
- Thus, the budget constraint is

$$C_1 + s(H)\frac{C_2}{(1+r)} + g(H) = W(1-l_1) + \frac{s(H)(1-l_2)}{1+r} \quad (77)$$

- Where W is the wage and r is the discount rate.
- Assuming $\beta = \frac{1}{1+r}$, individuals choose C_1, l_1, C_2, l_2 to maximize

$$U(C_1, l_1) + \frac{1}{1+r}s(H)U(C_2, l_2). \quad (78)$$

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The optimality conditions are

$$\frac{\partial U(C_1, l_1)}{\partial C_1} = \frac{\partial U(C_2, l_2)}{\partial C_2} \tag{79}$$

$$W_2 \frac{\partial U(C_1, l_1)}{\partial l_1} = W_1 \frac{\partial U(C_2, l_2)}{\partial l_2} \tag{80}$$

$$\frac{\partial U(C_1, l_1)}{\partial l_1} = W_1 \frac{\partial U(C_1, l_1)}{\partial C_1} \tag{81}$$

$$\frac{\partial U(C_1, l_1)}{\partial l_1} = W_2 \frac{\partial U(C_1, l_1)}{\partial C_2} \tag{82}$$

$$\frac{1}{1+r} \frac{\partial s(H)}{\partial H} \frac{1}{\o}(C_2 - W_2 l_2) = \frac{\partial g(H)}{\partial H} - \frac{1}{1+r} \frac{\partial s(H)}{\partial H} W_2. \tag{83}$$

- Condition (83) assumes that utility is homogeneous of degree ø.
- **Question.** Define what this means.

- The first four conditions are standard.
- The fifth condition is new.
- The left-hand side states that greater health increases the probability of surviving to the next period, which increases utility through additional consumption, but also has a utility cost through the disutility of labor.
- The right-hand side is the difference between the marginal cost of health ($\frac{\partial g(H)}{\partial H}$) and the marginal increase in "full" income due to working an additional period.

Multiple Periods

- Consider a similar framework with an extra period and note that adding extra periods is analogous.
- Let s_i and S_i be, respectively, the conditional and unconditional probabilities of surviving to time $t = i$ for $i = 1, 2, 3$.
- Thus, $S_1 = s_1, S_2 = s_1 s_2, S_3 = s_1 s_2 s_3$.
- The conditional probabilities are assumed to be strictly concave in h.
- The budget constraint is

$$S_1 C_1 + \frac{S_2 C_2}{1+r} + \frac{S_3 C_3}{(1+r)^2} = S_1 W_1 (1 - l_1) + \frac{S_2 W_2 (1 - l_2)}{(1+r)} + \frac{S_3 W_3 (1 - l_3)}{(1+r)^2}. \tag{84}$$

- The parent maximizes

$$S_1 U(C_1, l_1) + S_2 \frac{1}{1+r} U(C_2, l_2) + S_3 \frac{1}{(1+r)^2} U(C_3, l_3) \tag{85}$$

- and the "relevant to health" first order condition is

$$\begin{aligned} \frac{\partial S_1(H)}{\partial H} U(C_1, l_1) + \frac{1}{1+r} \frac{\partial S_2(H)}{\partial H} U(C_2, l_2) + \frac{1}{(1+r)^2} \frac{\partial S_3(H)}{\partial H} U(C_3, l_3) = \lambda \frac{\partial g(H)}{\partial H} \\ + \lambda \left[\frac{\partial S_1(H)}{\partial H} W_1(1-l_1) + \frac{\partial S_2(H)}{\partial H} \frac{W_2(1-l_2)}{(1+r)} + \frac{\partial S_3(H)}{\partial H} \frac{W_3(1-l_3)}{(1+r)^2} \right] \end{aligned} \tag{86}$$

- where λ is the Lagrangian multiplier of the optimization problem.

- The main lesson from this model is that health, H, “helps” to increase the unconditional probability of every period.
- Thus, spending in $t = 1$ increases the probability of surviving to periods $t = 1, 2, 3$.
- The optimal decision is to invest in earlier periods in H as to increase the unconditional probability of surviving to the rest of the periods and obtain more utility flows.