Economic Models for Social Interactions

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Introduction

Social Life **and** Economics

- "The outstanding discovery of recent historical and anthropological research is that man's economy, as a rule, is submerged in his social relationships. He does not act so as to safeguard his individual interest in the possession of material goods; he acts so as to safeguard his social standing, his social claims, his social assets. He values material goods only in so far as they serve this end." (Polanyi, 1944)
- "Economics is all about how people make choices. Sociology is all about why they don't have any choices to make." (Duesenberry, 1960)

Where do Social Interactions Appear?

Phenomena

- Labor markets
  - Career Choices
  - Retirement
- Fertility
- Health
- Education Outcomes
- Violence

Mechanisms

- Peer effects
  - Stigma
- Role models
- Social Norms
- Social Learning
- Social Capital?

Questions

- What are appropriate tools for modelling social interactions?
- Models of social interactions: Social norms, group membership, peer effects.

- Describe the peer effects. What goes on at the micro level?
- What are the aggregate effects of interaction on social networks?

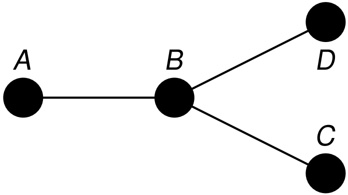
Plan

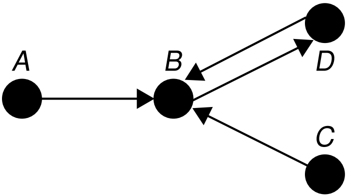
- Network Science
- Labor Markets — Weak and Strong Ties
- Peer Effects and Complementarities — Games on Networks
- Matching and Network Formation
- Social Capital
- Social Learning
- Diffusion

Network Science

Graphs

A directed graph $\mathcal{G}$ is a pair (V, E) where V is a set of vertices, or nodes, and E is a set of Edges. In a directed graph, an edge is an ordered pair (v, w) of vertices, meaning that there is a connection **from** v **to** w. In an undirected graph, an edge is an unordered pair of vertices.



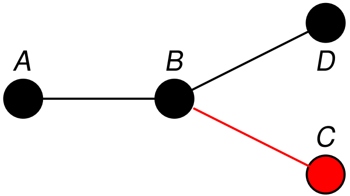


$$V = \{A, B, C, D\}$$
$$G = \{(A, B), (B, C), (B, D)\}$$

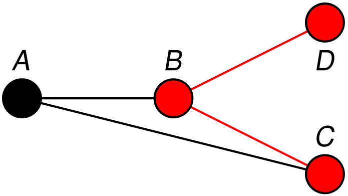
$$V = \{A, B, C, D\}$$
$$G = \{(A, B), (C, B), (B, D), (D, B)\}$$

The degree of a node in an undirected graph $\mathcal{G}$ is $\#\{w : (v, w) \in E\}$.

A path of $\mathcal{G}$ is an ordered list of nodes $(v_0, \ldots, v_N)$ such that $(v_{n-1}, v_n) \in E$ for all $1 \leq n \leq N$. A geodesic is a shortest-length path connecting v_0 and v_n.



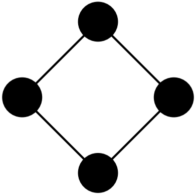
$\deg C = 1.$

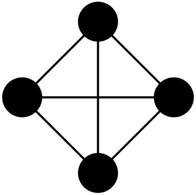


(C, B, D)

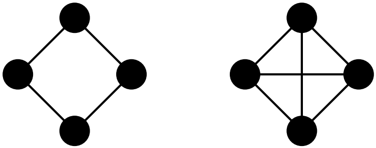
Graphs

A subset of vertices is connected if there is a path between every two of them. A component of $\mathcal{G}$ is a set of vertices maximal with respect to connectedness. A clique is a component for which all possible edges are in E.



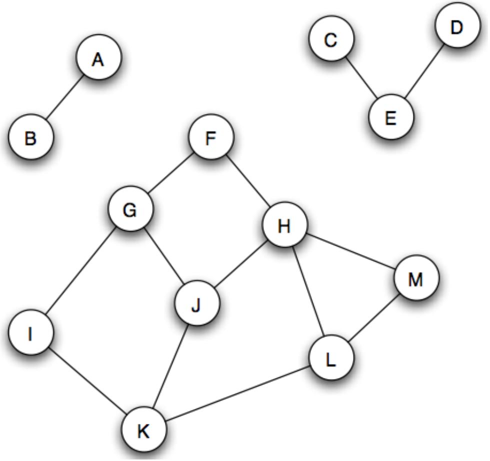


A graph $\mathcal{G}$ has a matrix representation. A adjacency matrix for a graph (V, E) is a $\#V \times \#V$ matrix A such that $a_{vw} = 1$ if $(v, w) \in E$, and 0 otherwise. A weighted adjacency matrix has non-zero numbers corresponding to edges in E.



$$\begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \end{pmatrix}$$

Graphs



- 3 Components, $\{A, B\}$, $\{C, D, E\}$, $\{F, \ldots, M\}$.
- Min degree $= 1$.
- Max deg $= 4$.
- Diam Comp. $3 = 3$.
- Degree Dist. $1 : 4/13,\ 2 : 4/13,\ 3 : 4/13,\ 4 : 1/13$.

Common Network Measurements

- Graph diameter — maximal geodesic length.
- Mean geodesic length.
- Degree distribution.
- Clustering coefficient — the average (over vertices) of the number of length 2 paths containing i that are part of a triangle. (Measures degree of transitivity.)
- Component size distribution

Some Social Networks

Network	Type	n	m	z	ℓ	α	$C^{(1)}$	$C^{(2)}$	r
film actors	undirected	449 913	25 516 482	113.43	3.48	2.3	0.20	0.78	0.208
company directors	undirected	7 673	55 392	14.44	4.60	–	0.59	0.88	0.276
math coauthorship	undirected	253 339	496 489	3.92	7.57	–	0.15	0.34	0.120
physics coauthorship	undirected	52 909	245 300	9.27	6.19	–	0.45	0.56	0.363
biology coauthorship	undirected	1 520 251	11 803 064	15.53	4.92	–	0.088	0.60	0.127
telephone call graph	undirected	47 000 000	80 000 000	3.16		2.1			
email messages	directed	59 912	86 300	1.44	4.95	1.5/2.0		0.16	
email address books	directed	16 881	57 029	3.38	5.22	–	0.17	0.13	0.092
student relationships	undirected	573	477	1.66	16.01	–	0.005	0.001	−0.029
sexual contacts	undirected	2 810				3.2			

n – # nodes, m – # edges, z – mean degree,
l – mean geodesic length, α – exponent of degree dist.,
$C^{(k)}$ - clustering coeff.s, r degree corr. coeff.

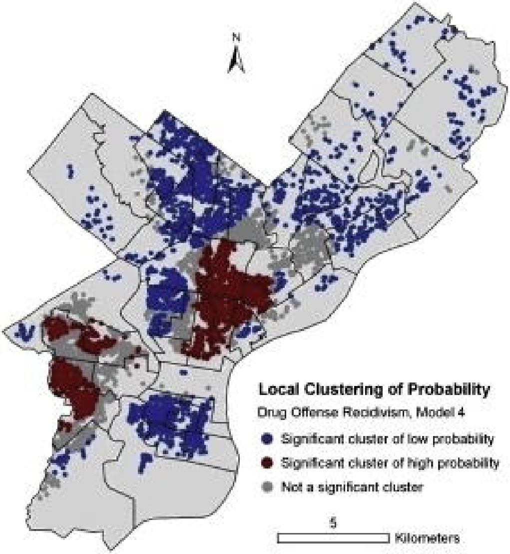
Mennis and Harris (2001)

Although other research has investigated deviant peer contagion, and still other research has examined offense specialization among delinquent youths, we have found that deviant peer contagion influences juvenile recidivism, and that contagion is likely to be associated with repeat offending. These findings suggest that juveniles are drawn to specific types of offending by the spatially-bounded concentration of repeat offending among their peers. Research on causes of delinquency within neighborhoods, then, may produce more useful causal models than studies that ignore spatial concentrations of offense patterns.

Table 4
Logistic regression of person offense recidivism ($N = 7166$).

Explanatory variables	Model 1	Model 2	Model 3
Individual			
Age	0.91*** (13.32) CI 0.87–0.96	0.92*** (10.13) CI 0.88–0.97	0.92*** (9.94) CI 0.88–0.97
White	0.85 (1.55) CI 0.66–1.10	0.86 (1.41) CI 0.67–1.11	0.83 (1.98) CI 0.64–1.08
Hispanic	0.64*** (11.26) CI 0.49–0.83	0.65*** (10.01) CI 0.50–0.85	0.73* (5.39) CI -0.56–0.95
Public assistance	0.99 (0.01) CI 0.84–1.17	1.00 (0.00) CI 0.84–1.18	1.01 (0.01) CI 0.01
Parental crime	1.35*** (9.25) CI 1.11–1.65	1.35*** (8.85) CI 1.11–1.64	1.35*** (8.84) CI 1.11–1.64
Number of prior arrests	1.12*** (15.87) CI 1.06–1.19	1.13*** (16.69) CI 1.06–1.19	1.12*** (15.74) CI 1.06–1.19
Prior institutional living arrangement	1.15 (2.44) CI 0.97–1.36	1.14 (2.35) CI 0.96–1.36	1.13 (1.92) CI 0.95–1.34
Instant offense type			
Person offense		1.23* (6.39) CI 1.05–1.44	1.22* (5.58) CI 1.03–1.43
Contagion effects			
Area person recidivism rate (×10)			2.77*** (84.05) CI 2.23–3.45
Constant	0.40* (5.91)	0.31*** (8.99)	
AUC	0.58***	0.59***	0.63***

A gray box indicates a variable that was excluded from that model run. Cell values indicate odds ratios. Wald statistic is shown in parentheses. "C.I." indicates confidence interval at 95% confidence. *$p < 0.05$, **$p < 0.01$, ***$p < 0.005$.


N
Local Clustering of Probability
Drug Offense Recidivism, Model 4
Significant cluster of low probability
Significant cluster of high probability
Not a significant cluster
5
Kilometers

Aggregate Analysis

Glaeser Sacerdote and Scheinkman (1996). “Crime and Social Interaction.”

> *The most puzzling aspect of crime is not its overall level nor the relationships between it and either deterrence or economic opportunity. Rather, following Quetelet [1835], we believe that the most intriguing aspect of crime is its astoundingly high variance across time and space.*
>
> *Positive covariance across agents’ decisions about crime is the only explanation for variance in crime rates higher than the variance predicted by difference in local conditions.*

A Model (of sorts)

- $2N + 1$ individuals live on the integer lattice at points $-N, \ldots, N$.
- Type 0s never commit a crime; Type 1's always do; Type 2's imitate the neighbor to the right.
- Type of individual i is p_i.

A Model (of sorts)

- Expected distance between fixed agents determines group size — range of interaction effects.
- Social interactions magnify the effect of fixed agents.

$$E\{a_i\} = \frac{p_1}{p_0 + p_1} \equiv p, \qquad S_n = \sum_{|i| \leq n} \frac{a_i - p}{2n+1}.$$

$$\sqrt{2n+1} S_n \to N(0, \sigma^2), \qquad \sigma^2 = p(1-p)\frac{2-\pi}{\pi}$$

where

$$\pi = p_0 + p_1, \qquad f(\pi) = \frac{2-\pi}{\pi}.$$

Aggregate Statistics

TABLE IIA
ESTIMATES OF $f(\pi)$

Data series	Crimes per capita (p) times 1-crimes per capita ($1-p$)	Sample variance / $p(1-p)$	Estimated λ^2	Estimated $f(\pi)$	Estimated $f(\pi)$ $\lambda^2 = .008$
Serious crime					
1985	0.073	1313.8	.013	754.6	604.7
N = 658			(.003)	(118.2)	
1970	0.042	1045.5	.004	475.1	284.3
N = 617			(.001)	(42.5)	
NYC	0.053	575.1			248.1
N = 70					
1986	0.078	1500.0	.0003	155.0	73.2
N = 631			(.0015)	(58.5)	

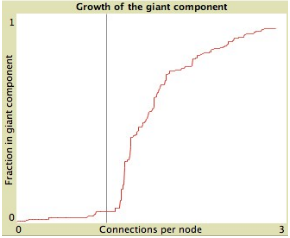
Erdös-Rényi Random Graphs

Undirected graph. Every pair of vertices is chosen as an edge independently with probability p.

Poisson random graphs: A sequence of graphs $\mathcal{G}_n$ with $|V_n| = n$ such that $p \cdot (n-1) \to z$.

Large n facts:

- Phase transition at $z = 1$.
- Low-density: Exponential component size distribution with a finite limit mean.
- High-density: a giant connected component of size $O(n)$. Remainder size distribution exponential . . . .
- Clustering coefficient is $C^2 = O(n^{-1})$.
- Poisson degree distribution with mean z.



Simulation of Erdös-Rényi random sets on 300 nodes.

Preferential Attachment

- A source of power laws.
- Introduced by Eggenberger and Polya (1923).
- Popularized by Zipf (1949) (city size) and Simon (1955) (wealth).

Preferential Attachment

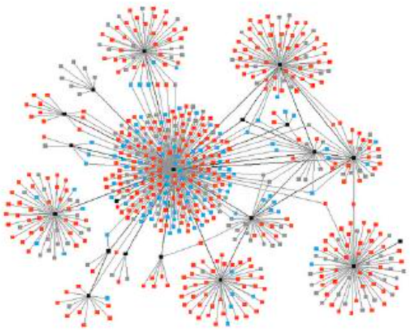
A directed graph.

- A vertex set V of size N.
- For nodes $i > 1$, with probability p i links to a randomly chosen node $j < i$.
- With probability $(1 - p)$ i links to the immediate ancestor of j.

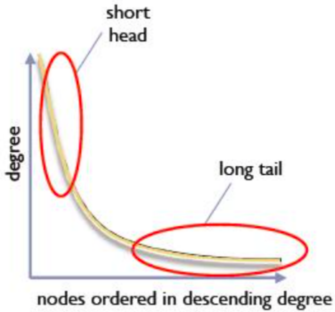
The graph is surely connected.

For large n the fraction of nodes with in-degree k is $1/k^r$ where r depends on p. The fraction P_r of vertices with r edges converges as N gets large, and $P_r = \Theta(r^{-\frac{2-p}{1-p}})$. See Kumar et al. (2000).

Preferential Attachment

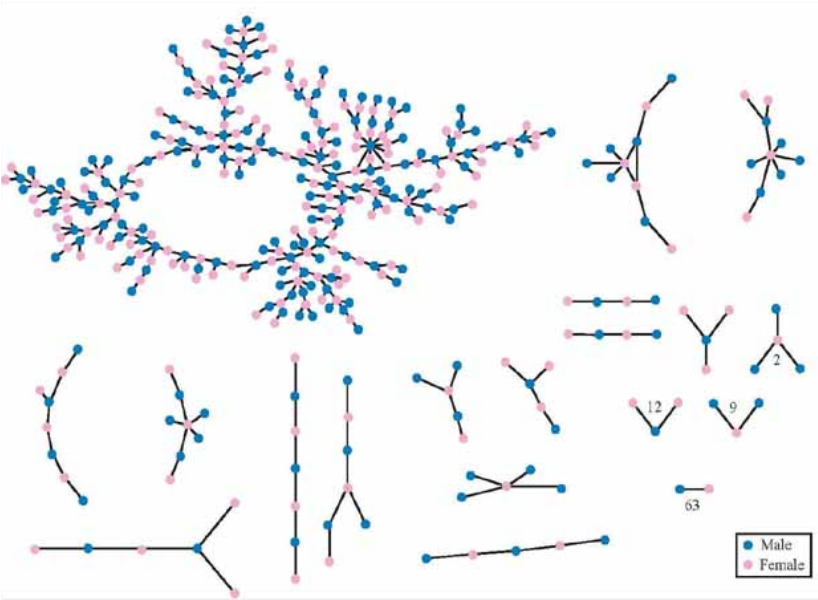


Example of network with preferential attachment

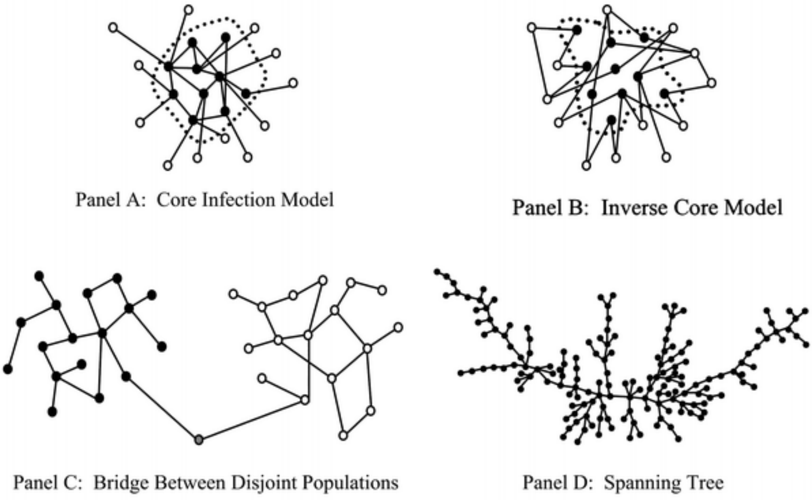


Sketch of long-tailed degree distribution

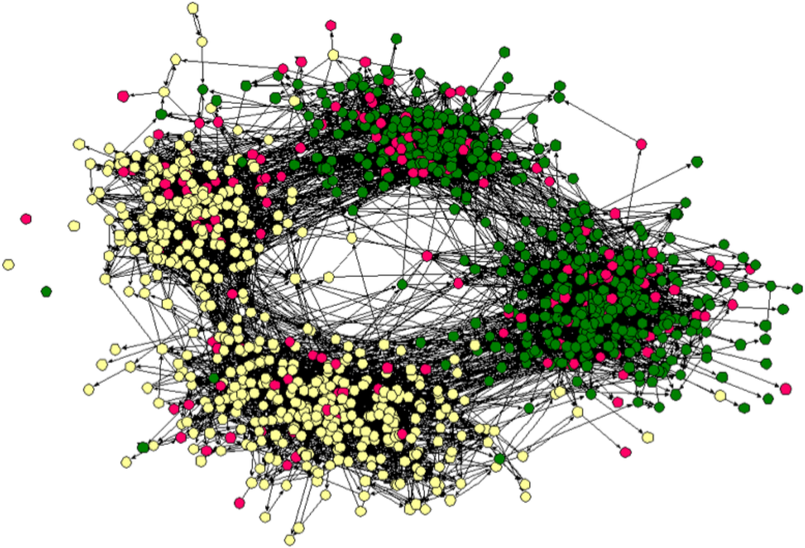
Some Social Networks



Some Social Networks II



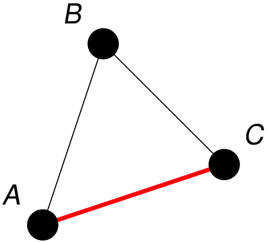
Some Social Networks III



Transitivity

"If two people in a social network have a friend in common, then there is an increased likelihood that they will become friends themselves at some point in the future." Rappoport (1953)

- Clustering coefficient: Fraction of connected triples that are triangles.
- Why transitivity?



Centrality

Types of Centrality Measures:

Degree Centrality How many vertices can a vertex reach directly?

Betweeness Centrality How likely is this vertex to be on the geodesic between two randomly chosen vertices?

Closeness Centrality How fast can this vertex reach all vertices in the network.

Eigenvector Centrality How much does this vertex influence other important vertices?

Which nodes are important?

Let A be the adjacency matrix for a directed graph. $A_{ij} = 1$ if j influences i. e is the vector of 1's.

- Degree Centraility: How many nodes can a node directly influence?

‘

$$c^d = eA \qquad c_j = \sum_i A_{ij}$$

- Katz (1953) Centrality: How many nodes can a node reach?

$$c_j^k(\alpha) = \sum_i \left(\sum_{k>0} \alpha^k A^k \right)_{ij}$$

$$c(\alpha) = (I - \alpha A)^{-1} e - e.$$

A_{ij}^k is the number of paths of length k from i to j. The parameter α discounts longer paths. α must be less than the largest eigenvalue of A.

- Eigenvector Centrality: The centrality of j is proportional to the sum of the centralities of the nodes she influences.

$$c_j^e = \mu \sum_i c_i a_{ij}$$

where $\mu > 0$. If the network is strongly connected, then (Perron Frobenius Theorem) there is a unique scalar μ and a one-dimensional set of vectors $c \gg 0$ that solve this. μ is the inverse of the Perron eigenvalue, and c is in the corresponding left eigenspace. (Bonacich, 1987; Bonacich and Lloyd, 2001).

Suppose A is indecomposable and aperiodic. Let $\lambda \geq 1$ denote the Perron eigenvalue of A. Then $B = \lambda^{-1}A$ has Perron eigenvalue 1. Let v denote a (strictly positive) Perron right eigenvector of A, and V the diagonal matrix whose ith diagonal element is v_i. Then $M = V^{-1}BV$ is a Markov matrix, with a unique invariant vector of l_1-norm 1. Let π be that vector.

$$\begin{aligned} \pi &= \pi M \\ &= \pi V^{-1}BV \\ \pi V^{-1} &= \pi V^{-1}B \end{aligned}$$

so πV^{-1} is left-invariant under A and thus a centrality vector.

$C^k(\alpha)$ is the diagonal matrix with diagonal elements $c_i^k(\alpha)$, $\alpha < 1/\lambda$.

$$I + C^k(\beta/\lambda) = \sum_{k\geq 0} \beta^k \lambda^{-k} A^k = \sum_{k\geq 0} \beta^k B^k$$
$$= \sum_{k\geq 0} \beta^k (VMV^{-1})^k = V \left(\sum_{k\geq 0} \beta^k M^k \right) V^{-1}$$
$$(1-\beta)I + (1-\beta)V^{-1}C^k(\beta/\lambda)V = (1-\beta)\sum_{k\geq 0} \beta^k M^k \xrightarrow{\beta\to 1} \Pi$$
$$\lim_{\beta\to 1}(1-\beta)C^k(\beta/\lambda) = VC^e$$

as $\beta \to 1$, that is, $\alpha \to 1/\lambda$.

To round this out, $\lim_{\alpha\to 0} \alpha^{-1} c^k(\alpha) = c^d$.

Two sources of centrality:

- Who you are connected to.
- What you 'bring to the table'.

$$
\begin{aligned}
c^\alpha(d) &= \alpha c^\alpha A + d \\
&= d(I - \alpha A)^{-1} \\
&= d(I + \alpha A + \alpha^2 A^2 + \cdots)
\end{aligned}
$$

α-centrality takes $d = e$:

$$c^\alpha = c^\alpha(e).$$

A quadratic game in which each player is influenced by the average play of his neighbors.

$$u_i(x_i, x_{-i}) = h_i x_i - \frac{x_i^2}{2} - \frac{\beta}{2}(x_i - \bar{x}_i)^2, \qquad \bar{x}_i = \sum_j a_{ij} x_j.$$

The equilibrium is unique:

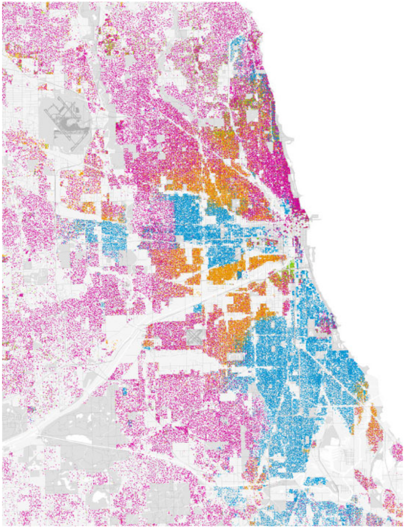
$$x = (1 - \phi)(I - \phi A)^{-1} h, \qquad \phi = \beta / 1 + \beta.$$

Average play in the population is

$$\begin{aligned} \frac{1}{n} e \cdot x &= \frac{1 - \phi}{n} e \left(I - \phi A\right)^{-1} h \\ &= \frac{1}{n}(1 - \phi) c^{\phi} h. \end{aligned}$$

Individual *i*'s influence on the average choice of the population is proportional to c^{ϕ}.

Homophily



“Similarity begets friendships.”
Plato

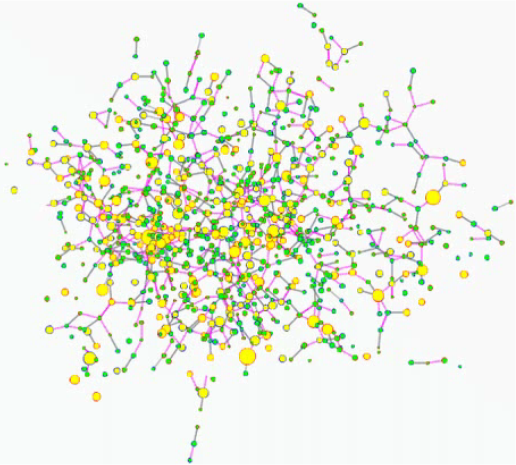
“All things akin and like are for the most part pleasant to each other, as man to man, horse to horse, youth to youth. This is the origin of the proverbs: The old have charms for the old, the young for the young, like to like, beast knows beast, ever jackdaw to jackdaw, and all similar sayings.” Aristotle, *Nicomachean Ethics*

Sources of Homophily

- Status Homophily: We feel more comfortable when we interact with others who share a similar cultural background.
- Value Homophily: We often feel justified in our opinions when we are surrounded by others who share the same beliefs.
- Opportunity Homophily: We mostly meet people like us.

Sources of Homophily

- Fixed attributes
  - Selection
- Variable attributes
  - Social influence
- Identification



Measuring Homophily

Consider a network with N individuals: Fraction p are males, fraction $q = 1 - p$ are females.

- Assign nodes to gender randomly, each node male with probability p.
- What is the probability of a "cross-gender" edge?

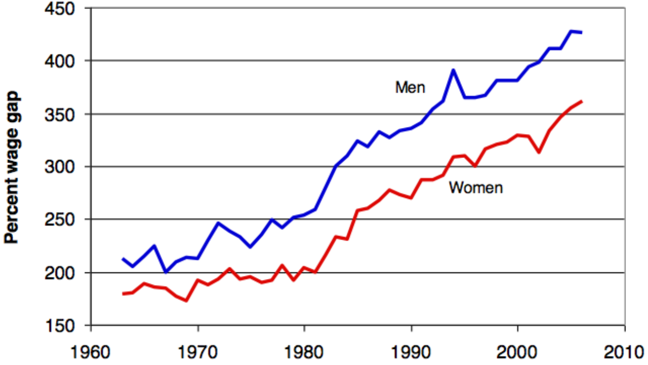
Measuring Homophily

Consider a network with N individuals: Fraction p are males, fraction $q = 1 - p$ are females.

- Assign nodes to gender randomly, each node male with probability p.
- What is the probability of a "cross-gender" edge?
- A fraction of cross-gender edges less than $2pq$ is evidence for homophily.

Labor Markets

Inequality in Labor Markets



Inequality in Labor Markets

Table 2: Changes in the 90-50 and 50-10 Wage Gaps

	Male			
	1980-90		1990-2000	
	90-50	***50-10***	***90-50***	***50-10***
Total change	.1035	.0886	.0660	-.0774
Due to wage dispersion within occupations	.0778	.0507	.0342	-.0537
Due to wage changes between occupations	.0588	.0257	.0184	-.0154
	Female			
	1980-90		1990-2000	
	90-50	***50-10***	***90-50***	***50-10***
Total change	.0592	.1526	.0368	.0153
Due to wage dispersion within occupations	.0292	.1074	.0343	.0044
Due to wage changes between occupations	.0417	.0660	.0103	-.0045

Job Search

TABLE 1—JOB-FINDING METHODS USED BY WORKERS

Source/data	Friends/relatives	Gate application	Employment agency	Ads	Other	Sample size
	Percentage of jobs found using each method					
Myers and Shultz (1951)/sample of displaced textile workers:						
First job	62	23	6	2	7	144
Mill job	56	37	3	2	2	144
Present job	36	14	4	0	46[a]	144
Rees and Shultz (1970)/Chicago labor-market study, 12 occupations:[b]						
Typist	37.3	5.5	34.7	16.4	6.1	343
Keypunch operator	35.3	10.7	13.2	21.4	19.4	280
Accountant	23.5	6.4	25.9	26.4	17.8	170
Tab operator	37.9	3.2	22.2	22.2	14.5	126
Material handler	73.8	6.9	8.1	3.8	7.4	286
Janitor	65.5	13.1	7.3	4.8	9.3	246
Janitress	63.6	7.5	5.2	11.2	12.5	80
Fork-lift operator	66.7	7.9	4.7	7.5	13.2	175
Punch-press operator	65.4	5.9	7.7	15.0	6.0	133
Truck driver	56.8	14.9	1.5	1.5	25.3	67
Maintenance electrician	57.4	17.1	3.2	11.7	10.6	129
Tool and die maker	53.6	18.2	1.5	17.3	9.4	127
Granovetter (1974)/sample of residents of Newton, MA:						
Professional	56.1	18.2	15.9[c]	—[c]	9.8	132
Technical	43.5	24.6	30.4	—	1.4	69
Managerial	65.4	14.8	13.6	—	6.2	81
Corcoran et al. (1980)/Panel Study of Income Dynamics, 11th wave:						
White males	52.0	—[d]	5.8	9.4	33.8[d]	1,499
White females	47.1	—	5.8	14.2	33.1	988
Black males	58.5	—	7.0	6.9	37.6	667
Black females	43.0	—	15.2	11.0	30.8	605

[a]Most of these workers were rehired at a previous mill job or hired at a new mill established in one of the vacated mills.
[b]In computing the percentages, workers rehired by previous employers and those not reporting the job-finding source are excluded from the denominator and subtracted from the sample size.
[c]Agencies and ads are combined under the heading "formal means."
[d]Gate applications are included under "other."

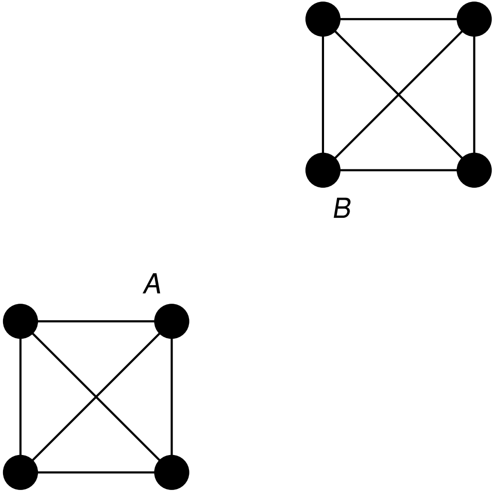
The Strentgh of Weak Ties

"…[T]he strength of a tie is a (probably linear) combination of the amount of time, the emotional intensity, the intimacy (mutual confiding), and the reciprocal services which characterize the tie. Each of these is somewhat independent of the other, though the set is obviously highly intracorrelated. Discussion of operational measures of and weights attaching to each of the four elements is postponed to future empirical studies. It is sufficient for the present purpose if most of us can agree, on a rough intuitive basis, whether a given tie is strong, weak, or absent."

Granovetter (1973, p. 1361)

Why do Weak Ties Matter?

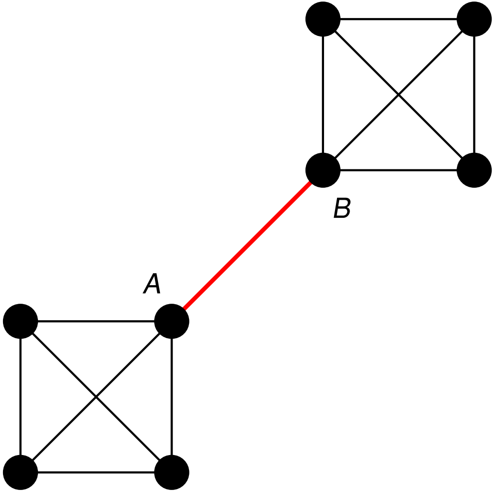
Two cliques.



Why do Weak Ties Matter?

Two cliques.

A–*B* is a **bridge**.

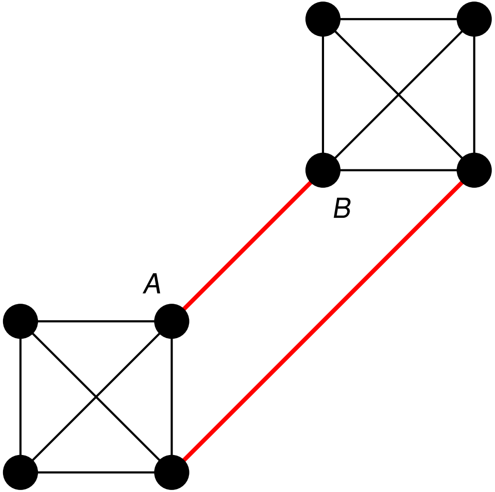


Why do Weak Ties Matter? I

Two cliques.

A–*B* is a **bridge**.

Local bridge's endpoints have no common friends.



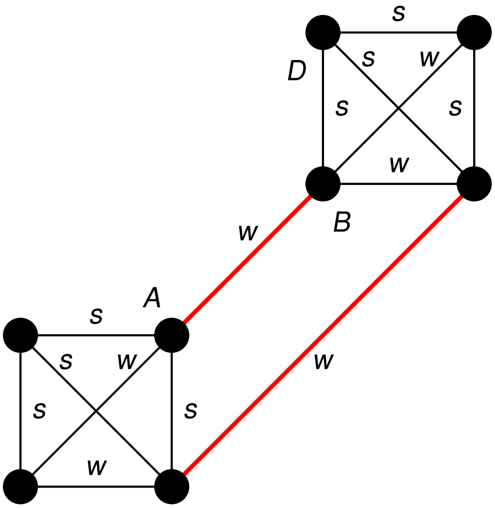
Why do Weak Ties Matter?

Two cliques.

A–*B* is a **bridge**.

Local bridge's endpoints have no common friends.

Triadic closure: A length-2 path containining only strong edges is a closed triad.



Ties and Inequality

Montgomery (1991)

- Workers live for two periods, $\#W$ identical in both periods.
- Half of the workers are high-ability, produce 1.
- Half of the workers are low-ability, produce 0.
- Workers are observationally indistinguishable.

- Each firm employs 1 worker.
- $\pi =$ employee productivity – wage.
- Free entry, risk-neutral entrepreneurs.

- Equilibrium condition: Firms expected profit is 0. Wage offers are expected productivity.

Ties and Inequality II

Social Structure

- Each $t = 1$ worker knows at most 1 $t = 2$ worker.
- Each $t = 1$ worker has a *social tie* with pr $= \tau$.
- Conditional on having a tie, it is to the same type with probability $\alpha > 1/2$.
- Assignments of a $t = 1$ worker to a specific $t = 2$ worker is random.

- τ — “network density”
- α — “inbreeding bias”

Ties and Inequality III

Timing

- Firms hire period 1 workers through the anonymous market, clears at wage w_{m1}.
- Production occures. Each firm learns its worker's productivity.
- Firm f sets a referral offer, w_{rf}, for a second period worker.
- Social ties are assigned.
- $t = 1$ workers with ties relay w_{ri}.
- $t = 2$ workers decide either to accept an offer or enter the market.
- Period 2 market clears at wage w_{m2}.
- Production occurs

- Only firms with 1-workers will make referral offers.
- Referral wages offers are distributed on an interval $[w_{m2}, w_R]$.
- $0 < w_{m2} < 1/2$.
- $\pi_2 > 0$.
- $w_{m1} = E\Big\{\text{production value} + \text{referral value}\Big\} > 1/2$.

V

$$\alpha, \tau \uparrow \quad \Longrightarrow \quad \begin{cases} w_{m2} \downarrow \\ w_R \uparrow \\ \pi_2 \uparrow \\ w_{m1} \uparrow \end{cases}$$

- in the market-only model, $w_{m1} = w_{m2} = 1/2$.
- $t = 2$ 1-types are better off, $t = 2$ low types are worse off. Social structure magnifies income inequality in the second period.
- The total wage bill in the second period is less with social structure.

Weak Ties in China

Tian, Felicia and Nan Lin. 2016. “Weak ties, strong ties, and job mobility in urban China: 1978–2008”. *Social Networks* 44, 117–129.

. . . Using pooled data from three cross-sectional surveys in urban China, the results show a steady increase in the use of weak ties and an increasing and persistent use of strong ties in finding jobs between 1978 and 2008. The results also show no systematic difference between the use of weak ties for finding jobs in the market sector versus the state sector. However, they show faster growth in the use of strong ties for finding jobs in the state sector, compared to the market sector.

Peer Effects and Complementarities

Behaviors on Networks

Three Types of Network Effects

- Information and social learning.
- Network externalities.
- Social norms.

A Common Regression

$$\omega_i = \pi_0 + x_i\pi_1 + \bar{x}_g\pi_2 + y_g\pi_3 + \varepsilon_i$$

Where

- ω_i is a choice variable for an individual,
- x_i is a vector of individual correlates,
- $\bar{x}_g$ is a vector of group averages of individual correlates,
- y_g is a vector of other group effects, and
- ε_i is an unobserved individual effect.

For all $g \in G$ and all $i \in g$,

$$\omega_i = \alpha + \beta x_i + \delta x_g + \gamma \mu_i + \varepsilon_i \qquad \text{(Behavior)}$$

$$x_g = \frac{1}{N_g} x_i \qquad \text{(Behavior)}$$

$$\mu_i = \frac{1}{N_g} \sum_{j \in g} \mathrm{E}\left\{\omega_j\right\} \qquad \text{(Equilibrium)}$$

The reduced form is

$$\omega_i = \frac{\alpha}{1 - \gamma} + \beta x_i + \frac{\gamma\beta + \delta}{1 - \gamma} x_g + \varepsilon_i$$

General Linear Network Model

$$\omega_i = \beta' x_i + \delta' \sum_j c_{ij} x_j + \gamma' \sum_j a_{ij} \operatorname{E}\left\{\omega_j | x\right\} + \eta_i$$

This is the general linear model

$$\Gamma\omega + \Delta x = \eta.$$

Question:

- How do we interpret the parameters?
- What kind of restrictions on the coefficients are reasonable, and do they lead to identification.

These questions require a theoretical foundation.

Incomplete-Information Game

- I individuals; each i described by a type vector $(x_i, z_i) \in \mathbf{R}^2$.
  x_i is publicly observable, z_i is private.
- There is a Harsanyi prior ρ on the space of types $\mathbf{R}^{2I}$.
- Actions are $\omega_i \in \mathbf{R}$.
- Payoff functions:

$$U_i(\omega_i, \omega_{-i}; x, z_i) = \theta_i \omega_i - \frac{1}{2}\omega_i^2 - \frac{\phi}{2}\left(\omega_i - \sum_j a_{ij}\, \omega_j\right)^2$$

- a_{ij} — peer effect of j on i.

Private Component

To complete the model, specify how individual characteristics matter.

$$\theta_i = \gamma x_i + \delta \sum_j c_{ij} x_j + z$$

Direct Effect

Contextual Effect

c_{ij} — contextual/direct effect of j on i.

Equilibrium

$$(1+\phi)\left(I - \frac{\phi}{1+\phi}A\right)\omega - (\gamma I + \delta C)x = \eta$$

$$\Gamma\omega + \Delta x = \eta.$$

Constraints imposed by the theory:

$$a_{ii} = c_{ii} = 0, \quad \sum_j a_{ij} = \sum_j c_{ij} = 1.$$

$$\Gamma_{ii} = 1 + \phi, \quad \sum_{j \neq i} \Gamma_{ij} = -\phi, \quad \Delta_{ii} = -(\gamma + \delta), \quad \sum_{j \neq i} \Delta_{ij} = \delta.$$

Even more constraints if you insist on $A = C$.

When is the first equation identified?

- Order condition: $\#\{j \nsim_C 1\} + \#\{j \nsim_A 1\} \geq N - 1$.
- For each (γ, δ) pair there is a generic set of C-matrices such that the rank condition is satisfied.
- If two individuals' exclusions satisfy the order condition, there is a generic set of C-matrices such that the rank condition is satisfied for all γ and δ.

Non-Linear Aggregators

Bad apple The worst student does enormous harm.

Shining light A single student with sterling outcomes can inspire all others to raise their achievement.

Invidious comparison Outcomes are harmed by the presence of better achieving peers.

Boutique A student will have higher achievement whenever she is surrounded by peer with similar characteristics.

Matching and Network Formation

- Market Design

- Matching problems are models of network formation

    - Bipartite matching with transferable utility
    - Bipartite matching without exchange
    - Generalization to networks

Stable Matches

Given are two sets of objects X and Y. e.g. workers and firms. Both sides have preferences over whom they are matched with, but with no externalities, that is, given that a is matched with x, he does not care if b is matched with y and z. The literature divides over the information parties have when they choose partners, and whether compensating transfers can be made. The organizing principle is that of a stable match.

Assume w.l.o.g. $|X| \leq |Y|$.

Definition: A match is one-to-one map from X to Y. A match is stable if there are no pairs $x \leftrightarrow y$ and $x' \leftrightarrow y'$ such that $y' \succ_x y$ and $x \succ_{y'} x'$.

Find the optimal match by maximizing total surplus:

$$
\begin{aligned}
v(L \cup F) = \max_x \quad & \sum_{l,f} v_{lf} x_{lf} \\
\text{s.t.} \quad & \sum_f x_{lf} \leq 1 \quad \text{for all } l, \\
& \sum_l x_{lf} \leq 1 \quad \text{for all } f, \\
& x \geq 0
\end{aligned}
$$

The vertices for this problem are integer solutions, that is, non-fractional matches. A solution to the primal is an optimal matching.

Set of laborers L and firms F. v_{lf} is the value or surplus generated by matching worker l and firm f.

The surplus of a match is split between the firm and worker. Suppose $i \leftrightarrow f$ and $j \leftrightarrow g$. Payments to each are w_i and w_j, and π_i and π_j.

Since this is a division of the surplus,

$$w_i + \pi_f = v_{if} \quad \text{and} \quad w_j + \pi_g = v_{jg}.$$

If $w_i + \pi_g < v_{ig}$, then there is a split of the surplus v_{ig} such that i and g would both prefer to match with each other than with their current partners. The match is not stable. Stability requires

$$w_i + \pi_g \geq v_{ig} \quad \text{and} \quad w_j + \pi_f \geq v_{jf}.$$

Matching with Transferable Utility

The dual has variables for each individual and firm.

$$\begin{aligned} \min_{w,\pi} \quad & \sum_{l,f} w_l + \pi_f \\ \text{s.t.} \quad & \pi_f + w_l \geq v_{lf} \quad \text{for all pairs } l, f, \\ & \pi \geq 0,\ w \geq 0. \end{aligned}$$

Solutions to the dual satisfy the stability condition.

Complementary slackness says that matched laborer-firm pairs split the surplus, $\pi_f + w_l = v_{lf}$.

Characterizing Matches

Theorem: A matching is stable if and onl if it is optimal.

Lemma: Each laborer with a positive payoff in any stable outcome is matched in every stable matching.

Proof: Complementary slackness.

Lemma: If laborer l is matched to firm f at stable matching x, and there is another stable matching x' which l likes more, then f likes it less.

Proof: Formalize this as follows: If x is a stable matching and $\langle w', \pi' \rangle$ is another stable payoff, then $w' > w$ implies $\pi > \pi'$. This follows from complementary slackness, since $w_l + \pi_f = v_{lf} = w'_l + \pi'_f$.

Suppose X and Y are each partially-ordered sets, and $v : X \times Y \to \mathbf{R}$ is a function.

Definition: $v : X \times Y \to \mathbf{R}$ has increasing differences iff $x' > x$ and $y' > y$ implies that

$$v(x', y') + v(x, y) \geq v(x', y) + v(x, y').$$

An important special case is where X and Y are intervals of $\mathbf{R}$, each with the usual order, and v is C^2.

$$v(x', y') - v(x, y') \geq v(x', y) - v(x, y).$$

Then

$$D_x v(x, y') \geq D_x v(x, y)$$

From this it follows that $D_{xy} v(x, y) \geq 0$.

Generalizations

- Matching without exchange. Gale and Shapley (1962).
- The roommate problem.
- Generalization of non-transferable matching to networks. Jackson and Wolinsky (1996).

Network Formation with Contagious Risk

Blume et al. (2013)

A set V of N agents form no more than Δ bilateral relationships with each other, thereby constructing a graph $G = (V, E)$. Each agent receives payoff $a > 0$ from each of her links.

Then, cascades occur. Each node fails independently with probably q. Each failed node transmits failure to her neighbors with independent probability p, and so on. The edges that transmit, and the nodes they connect are the live-edge subgraph.

A falied agent loses all benefits and pays a cost b.

$$\pi_i = ad_i(1 - \phi_i) - b\phi_i$$

where d_i is the degree of agent i and ϕ_i is the probability i fails.

Network Formation with Contagious Risk

Rawlsian welfare — minimum welfare among all agents.

Definition: A graph is stable if:

- no node can strictly increase its payoff by deleting all its incident links (hence removing itself from the network), and
- there is no pair of unconnected nodes (i, j) such that adding an (i, j) edge to G would make them both better off.

Assumptions

- $a > pqb$.
- $a < pb$.
- $a < qb$.

We want the bounds to hold very loosely. “Separation parameter” δ:

Assumption $\mathcal{P}(\delta)$: There is a small constant δ such that

$$\delta^{-1} pqb < a < \delta \min\{pb, qb\}.$$

- Results provide asymptotically tight characterizations of the welfare obtained by both socially optimal and stable graphs.
- If each node forms more than $1/p$ links, the live-edge subgraph has a giant connected component.
- "..., we find roughly that social optimality occurs just beyond the edge of a phase transition that controls how failures propagate, while stable graphs lie slightly further still past this phase transition, at a point where most of the welfare has already been wiped out."

Social Capital

Networks and Social Capital

"the aggregate of the actual or potential resources which are linked to possession of a durable network of more or less institutionalized relationships of mutual acquaintance or recognition." (Bourdieux and Wacquant, 1992)

"the ability of actors to secure benefits by virtue of membership in social networks or other social structures." (Portes, 1998)

"features of social organization such as networks, norms, and social trust that facilitate coordination and cooperation for mutual benefit." (Putnam, 1995)

"Social capital is a capability that arises from the prevalence of trust in a society or in certain parts of it. It can be embodied in the smallest and most basic social group, the family, as well as the largest of all groups, the nation, and in all the other groups in between. Social capital differs from other forms of human capital insofar as it is usually created and transmitted through cultural mechanisms like religion, tradition, or historical habit." (Fukuyama, 1996)

"naturally occurring social relationships among persons which promote or assist the acquisition of skills and traits valued in the marketplace. . . " (Loury, 1992)

Networks and Social Capital

"…social capital may be defined operationally as *resources embedded in social networks and accessed and used by actors for actions.* Thus, the concept has two important components: (1) it represents resources embedded in social relations rather than individuals, and (2) access and use of such resources reside with actors."

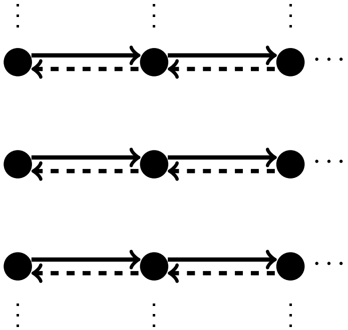
(Lin, 2001)

Information

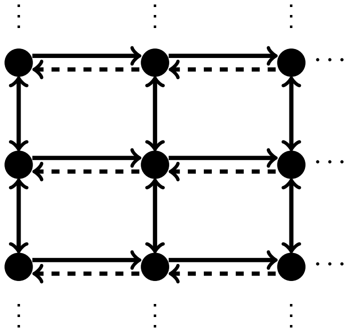
- Search is a classic example according to Lin's (2001) definition.
- Search has nothing to do with values and social norms beyond the willingness to pass on a piece of information.

Intergenerational Transfers

Loury (1981)



Only Intergenerational Transfers



Intergenerational Transfers with Re-distribution

x	output
α	ability, realized in adults.
e	investment
c	consumption
y	income
$h(\alpha, e)$	production function
$U(c, V)$	parent's utility

$c + e = y$ parental budget constraint

Assumptions:

A.1. U is strictly monotone, strictly concave, C^2, Inada condition at the origin. $\gamma < U_v < 1 - \gamma$ for some $0 < \gamma < 1$.

A.2 h is strictly increasing, strictly concave in e, C^1, $h(0,0) = 0$ and $h(0,e) < e$. $h_\alpha \geq \beta > 0$. For some $\hat{e} > 0$, $h_e \leq \rho < 1$ for all $e > \hat{e}$ and α.

A.3. $0 \leq \alpha \leq 1$, distributed i.i.d. μ. μ has a continuous and strictly positive density on $[0,1]$.

Parent's utility of income y is described by a Bellman equation:

$$V^*(y) = \max_{0 \leq c \leq y} \mathrm{E}\left\{U\Big(c, V^*\big(h(\tilde{\alpha}, y - c)\big)\Big)\right\}.$$

- The Bellman equation has a unique solution, and there is a $\bar{y}$ such that $y \leq \bar{y}$ for all α.

The solution defines a Markov process of income.

$$
\begin{tikzcd}
y \arrow[r] & e, \alpha \arrow[d, "h"] & \nu \arrow[l, dashed] \\
& y \arrow[r] & \cdots
\end{tikzcd}
$$

- If education is a normal good, then the Markov process is ergodic, and the invariant distribution μ has support on $[0, \hat{y}]$, where $\hat{y}$ solves $h\big(1, e^*(y)\big) = y$.

An *education-specific tax policy* taxes each individual as a function of their education and their income. It is *redistributive* if the aggregate tax collection is 0 for every education level e.

Tax policy τ_1 is more egalitarian than tax policy τ_2 iff the distribution of income under τ_2 is riskier than that of τ_1 conditional on the education level e.

- If τ_1 and τ_2 are redistributive educational tax policies, and τ_1 is more egalitarian than τ_2, then for all income levels y, $V^*_{\tau_1}(y) > V^*_{\tau_2}(y)$.
- A result about universal public education.
- A result on the relationhip between ability and earnings.

Trust

Three Stories about Trust:

Reciprocity: Reputation games, folk theorems, . . .

Social Learning: Generalized trust.

Behavioral Theories: Evolutionary Psychology, prosocial preferences, . . .

Inequality and Trust

- Evidence for a correlation between trust and income inequality
  - Rothstein and Uslaner (2005), Uslaner and Brown (2005).
- Trust is correlated with optimism about one's own life chances
  - Uslaner (2002)

Networks, Trust, and Development

- Informal social organization substitutes for markets and formal social institutions in underdeveloped economies.
- In the US, periods of high growth have also been periods of decline in social capital (Putnam, 2000)
- Possibly: Social capital is needed for economic development only up to some intermediate stage, where generalized trust in institutions takes the place of informal trust arrangements.

Does Social Capital Have an Economic Payoff?

Knaak and Keefer (1997). “Does social capital have a payoff".

$$g_i = \mathbf{X}_i\gamma + \mathbf{Z}_i\pi + \mathrm{CIVIC}_i\alpha + \mathrm{TRUST}_i\beta + \epsilon_i$$

- g_i real per-capita growth rate.
- $\mathbf{X}_i$ control variables — Solow.
- $\mathbf{Z}_i$ control variables — “endogenous” growth models.
- CIVIC_i index of the level of civic cooperation.
- TRUST_i the percentage of survey respondents (after omitting those responding ‘don’t know’) who, when queried about the trustworthiness of others, replied that ‘most people can be trusted’.

A Model of Trust

- A population of N completely anonymous individuals.
- Individuals have no distinguishing features, and so no one can be identified by any other.
- Individuals are randomly paired at each discrete date t, with the opportunity to pursue a joint venture. Simultaneously with her partner, each individual has to choose whether to participate (P) in the joint venture, or to pursue an independent venture (I). The entirety of her wealth must be invested in one or the other option. The individual with wealth w receives a gross return $w\pi$ from her choice, where π is realized from the following payoff matrix:

$$
\begin{array}{cc|c|c|}
 & & \multicolumn{2}{c}{\text{partner}} \\
 & & P & I \\
\cline{3-4}
\text{investor} & P & \tilde{R} & \tilde{r} \\
\cline{3-4}
 & I & \tilde{e} & \tilde{e} \\
\cline{3-4}
\end{array}
$$

Gross Returns

A Model of Trust

- $E\tilde{R} > E\tilde{e} > E\tilde{r}$.
- Individuals reinvest a constant fraction β of their wealth.
- Strategies for *i* are functions which map the history of *i*s experience in the game to actions in the current period.
- Equilibria: Always play *P*, always play *I* are two equilibria.

Learning

Each individual i has a prior belief ρ, about the probability of one's opponent choosing P. The prior distribution is beta with parameters $a^i, b^i > 0$. In more detail,

$$\begin{aligned}\rho^i(x) &= \beta(a_0^i, b_0^i) \\ &= \frac{\Gamma(a_0^i + b_0^i)}{\Gamma(a_0^i)\Gamma(b_0^i)} x^{a_0^i - 1}(1-x)^{b_0^i - 1}.\end{aligned}$$

Let ρ_t^i denote individual i's posterior beliefs after t rounds of matching. The posterior densities ρ_t^i and ρ_t^j will be conditioned on different data, since all information is private. The updating rule for the β distribution has

$$\rho_t^i(h_t) \equiv \beta(a_t^i, b_t^i) = \beta(a_0^i + n, b_0^i + t - n)$$

for histories containing n P's and therefore $t - n$ I's. The posterior mean of the β distribution is $a_t^i / (a_t^i + b_t^i)$.

Optimal Play

$$q^* = (e - r)/(R - r)$$

- Let m_t^i denote i's mean of ρ_t.
- An optimal strategy for individual i is: Choose P if $m_t > q^*$ and choose I otherwise.

Theorem 3: For all initial beliefs $(\rho_0^1, \ldots, \rho_0^N)$, almost surely either $\lim_t n_t^P = 0$ or $lim_t n_t^P = N$. The probabilities of both are positive. The limit wealth distributions in both cases is $\Pr\{\lim_t w_t > w\} \sim cw^k$, where k is k_P or k_I, and $k_P < k_I$.

Social Learning

Averaging the Opinions of Others

- DeGroot (1974)
- X is some event. $p_i(t)$ is the probability that i assigns to the occurance of X at time t.
- A is a stochastic matrix. a_{ij} is the weight i gives to j's opinion.
- $p(t) = Ap(t-1) = \cdots = A^t p(0)$.

Averaging the Opinions of Others

Example

$$A = \begin{pmatrix} 1/3 & 1/3 & 1/3 \\ 1/2 & 1/2 & 0 \\ 0 & 1/2 & 1/2 \end{pmatrix},$$

$$p(2) = A^2 p(0) = \begin{pmatrix} 5/18 & 8/18 & 5/18 \\ 5/12 & 5/12 & 2/12 \\ 1/4 & 1/2 & 1/4 \end{pmatrix} p(0),$$

$$p(t) = A^t p(0) \rightarrow \begin{pmatrix} 3/9 & 4/9 & 2/9 \\ 3/9 & 4/9 & 2/9 \\ 3/9 & 4/9 & 2/9 \end{pmatrix} p(0).$$

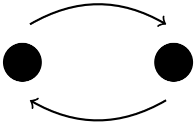
For all i,

$$p_i(\infty) = \frac{3}{9} p_1(0) + \frac{4}{9} p_2(0) + \frac{2}{9} p_3(0).$$

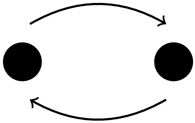
Averaging the Opinions of Others

Distinct Limits

$$A = \begin{pmatrix} 1/2 & 1/2 & 0 & 0 \\ 1/3 & 2/3 & 0 & 0 \\ 0 & 0 & 1/2 & 1/2 \\ 0 & 0 & 2/3 & 1/3 \end{pmatrix}$$



$$A^t \to \begin{pmatrix} 2/5 & 3/5 & 0 & 0 \\ 2/5 & 3/5 & 0 & 0 \\ 0 & 0 & 3/5 & 2/5 \\ 0 & 0 & 3/5 & 2/5 \end{pmatrix}$$



$$p_i(t) \to \frac{2}{5}p_1(0) + \frac{3}{5}p_2(0) \quad \text{for } i = 1, 2.$$

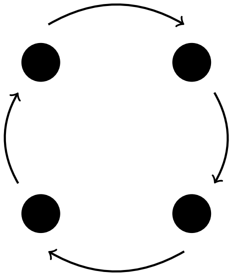
$$p_i(t) \to \frac{3}{5}p_3(0) + \frac{2}{5}p_4(0) \quad \text{for } i = 3, 4.$$

Averaging the Opinions of Others

No Limit

$$A = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

$$A^t = A^{(t-1) \bmod 3 + 1}$$



Averaging the Opinions of Others

Convergence

Theorem: If A is irreducible and aperiodic, then beliefs converge to a limit probability. For all j, $\lim_{t \to \infty} p_j(t) = \sum_i c_i^e p_i(0)$, where c^e is the unit-normalized eigenvalue centrality of A.

Speed of Convergence

How long does it take for an individual's belief to get near to the limit belief?

$$\left|p_i(t) - p_i(\infty)\right| = \left|\sum_j \left(A_{ij}^t - \sum_j c_j^e\right) p_j(0)\right|$$

For each j $0 < p_j(0) < 1$,

$$\sup_{p(0)} \left|\sum_j \left(A_{ij}^t - \sum_j c_j^e\right) p_j(0)\right| =$$

$$\max\left\{\sum_{j:A_{ij}^t \geq c_j^e} \left(A_{ij}^t - \sum_j c_j^e\right), -\sum_{j:A_{ij}^t \leq c_j^e} \left(A_{ij}^t - \sum_j c_j^e\right)\right\}$$

$$= \left\|A_{ij}^t - c^e\right\|_{TV}$$

Speed of Convergence

For x and y in the non-negative unit simplex,

$$\|x - y\|_{TV} = \sup_A |\sum_{i \in A} (x_i - y_i)|.$$

We want to max this over individuals, so

$$d(t) = sup_i \left\| A_{ij}^t - c^e \right\|_{TV}.$$

Define

$$t(\epsilon) = \min\{t : d(t) < \epsilon\}$$

$$t^* = t(1/4).$$

A Lower Bound for t^*

$$Q(i,j) = c_i^e A_{ij}, \qquad Q(A,B) = \sum_{i \in A\, j \in B} Q(i,j).$$

$Q(A,B)$ is the amount of influence B inherits from A.

$$\Phi(S) = \frac{Q(S, S^c)}{\sum_{i \in S} c_i^e}, \qquad \Phi^* = \inf_{S : \sum_{i \in S} c_i^e \leq 1/2} \Phi(S).$$

$\Phi(S)$ is the share of S's influence that is inherited by S^c.

Theorem: $$t^* \geq \frac{1}{4\Phi^*}.$$

Limit Beliefs and the "Wisdom of Crowds"

- Suppose that $p_i(0) = p + \epsilon_i$. The ϵ_i are all independent, have mean 0, and variances are bounded.
- What is the relationship between $p_i(\infty)$ and p?
- A sequence of networks $(V_n, E_n)_{n=1}^{\infty}$, $|V_n| = n$, with centrality vectors s^n, and belief sequences $p^n(t)$.

Definition: The sequence learns if for all $\epsilon > 0$,
$\Pr\{|\lim_{n\to\infty}\lim_{t\to\infty} p^n(t) - p| > \epsilon\} = 0$.

Theorem: If there is a $B > 0$ such that for all i, each individual's normailzed centrality is less than B/n, then the sequence learns.

- What conditions on the networks guarantee this?

Bayesian Learning on Networks

Multi-armed bandit problem

- An undirected network $\mathcal{G}$.
- Two actions, A and B. A pays off 1 for sure. B pays off 2 with probability p and 0 with probability $1 - p$.
- At times $t = \{1, 2, \ldots\}$, each individual makes a choice, to maximize $\mathrm{E}\left\{\sum_{\tau=t}^{\infty} \beta^{\tau} \pi_{i\tau} | h_t\right\}$, the expected present value of the discounted payoff stream given the information.
- $p \in \{p_1, \ldots, p_K\}$. W.l.o.g. $p_j \neq p_k$ and $p_k \neq 1/2$.
- Each individual has a full-support prior belief μ_i on the p_k.
- Individuals see the choices of his neighbors, and the payoffs.

- If the network contains only one member, this is the classic multi-armed bandit problem.
- How does the network change the classic results?
- What does one learn from the behavior of others?

Theorem: With probability one, there exists a time such that all individuals in a component play the same action from that time on.

- In one-individual problem, it is possible to lock into A when B is optimal. How does the likelihood of this change in a network?

Bayesian Learning on Networks

Common Knowledge

$(\Omega, \mathcal{F}, p)$ A probability space.

X A finite set of actions.

Y_i A finite set of signals observed by i. $y_i : \Omega \to Y_k$ is $\mathcal{F}$-measurable.

$\sigma(f)$ If f is a measurable mapping of Ω into any measure space, σf is the σ-algebra generated by f. Define $\sigma(y_k) = \mathcal{Y}_k$.

Definition: A decision function maps states Ω to actions X. A decision rule maps σ-fields on Ω to decision rules, that is, $d(\mathcal{G}) : \Omega \to X$. For any σ-field $\mathcal{G}$, $d(\mathcal{G})$ is $\mathcal{G}$-measurable. That is, $\sigma d(\mathcal{G}) \subset \mathcal{G}$.

Bayesian Learning on Networks

Common Knowledge

- Updating of beliefs:

$$\mathcal{F}_k(t+1) = \mathcal{F}_k(t) \vee \bigvee_{j \neq k} \sigma d\left(\mathcal{F}_j(t)\right),$$

$$\mathcal{F}_k(0) = \mathcal{Y}_k.$$

Key Property: If $\sigma d(\mathcal{G}) \subset \mathcal{H} \subset \mathcal{G}$, then $d(\mathcal{G}) = d(\mathcal{H})$.

Theorem: Suppose d has the key property. Then there are σ-algebras $\mathcal{F}_k \subset \bigvee_k Y_k$ and $T \geq 0$ such that $\mathcal{F}_k(t) = \mathcal{F}_k$ for all $t \geq T$, and for all k and j,

$$d(\mathcal{F}_k) = d(\mathcal{F}_j) = d\left(\bigwedge_i \mathcal{F}_i\right).$$

If the decision functions for all individuals are common knowledge, then they agree.

Bayesian Learning on Networks

Common Knowledge

Now given is a connected undirected network (V, E).

- Individuals i and k communicate directly if there is an edge connecting them.
- Individuals i and k communicate indirectly if there is a path connecting them.

Key Network Property: For any sequence of individuals $k = 1, 2, \ldots, n$, if $\sigma d(\mathcal{F}_k) \subset \mathcal{F}_{k+1}$ and $\sigma d(\mathcal{F}_n) \subset \mathcal{F}_1$, then $d(\mathcal{F}_k) = d(\mathcal{F}_1)$ for all k.

Bayesian Learning on Networks

Updating of beliefs:

$$\mathcal{F}_k(t+1) = \mathcal{F}_k(t) \vee \bigvee_{j \sim k} \sigma d\left(\mathcal{F}_j(t)\right),$$

$$\mathcal{F}_k(0) = \mathcal{Y}_k.$$

Theorem: Suppose d has the key network property. Then there are σ-algebras $\mathcal{F}_k \subset \bigvee_k Y_k$ and $T \geq 0$ such that $\mathcal{F}_k(t) = \mathcal{F}_k$ for all $t \geq T$, and for all k and j,

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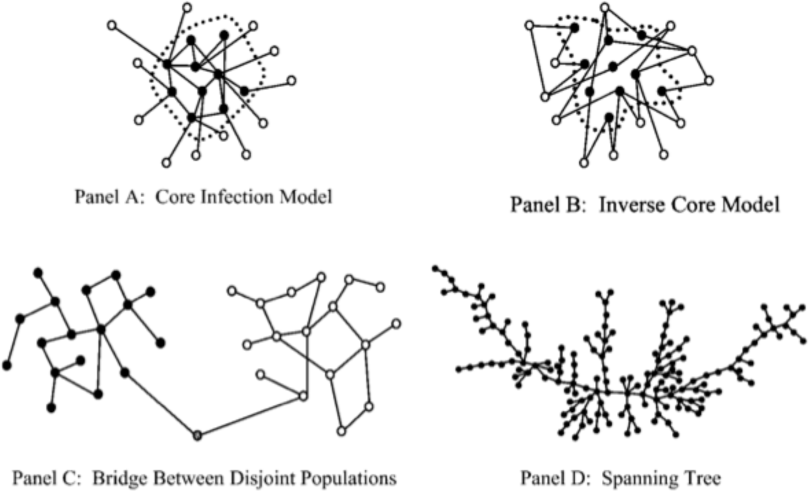
Diffusion

Network Effects and Diffusion



Varieties of Action

- Graphical Games — Diffusion of action
  - Blume (1993, 1995) — Lattices
  - Morris (2000) — General graphs
  - Young and Kreindler (2011) — Learning is fast
- Social Learning — Diffusion of knowledge
  - Banerjee, QJE (1992)
  - Bikchandani, Hershleifer and Welch (1992)
  - Rumors

Coordination Games

	A	B
A	a,a	0,0
B	0,0	b,b

$a, b > 0$

Pure coordination game

Three equilibria:

$$\langle a, a \rangle, \quad \langle b, b \rangle, \quad \text{and} \quad \langle (\frac{b}{a+b}, \frac{a}{a+b}), (\frac{b}{a+b}, \frac{a}{a+b}) \rangle$$

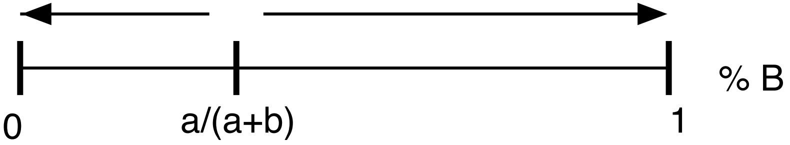
Coordination Games

	A	*B*
A	a,a	0,0
B	0,0	b,b

$a, b > 0$

Pure coordination game

Best response dynamics



Coordination Games

	A	B
A	a,a	d,c
B	c,d	b,b

$a > c$, $b > d$

General coordination game

Here the symmetric mixed equilibrium is at
$p^* = (b - d)/(a - c + b - d)$.

Suppose $b - d > a - c$. Then $p^* > 1/2$. At $(1/2, 1/2)$, A is the best response. This is not inconsistent with $b > a$.

- A is Pareto dominant if $a > b$.
- B is risk dominant if $b - d < a - c$.

Coordination Games — Stochastic Stability

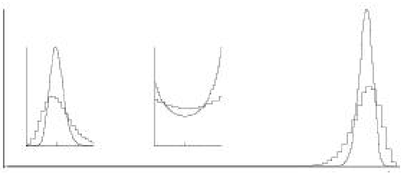
Continuous time stochastic process

- Each player has an alarm clock. When it goes off, she makes a new strategy choice. The interval between rings has an exponential distribution.
- Strategy revision:
  - Each individual best-responds with prob. $1 - \epsilon$, Kandori, Mailath and Robb (1993); Young (1993)

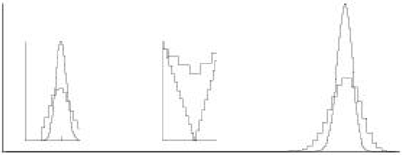
    or
  - The log-odds of choosing A over B is proportional to the payoff difference — logit choice, Blume (1993, 1995).

The Stochastic Process

This is a Markov process on the state space $[0, \ldots, N]$, where the state is the number of players choosing B.



Logit Choice

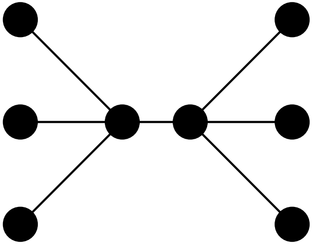


Mistakes

In both cases, as $\mathrm{Prob}\{\text{best response} \uparrow 1\}$, $\mathrm{Prob}\{N\} \uparrow 1$.

Coordination on Networks

- Is the answer the same on every graph?



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Coordination on Networks

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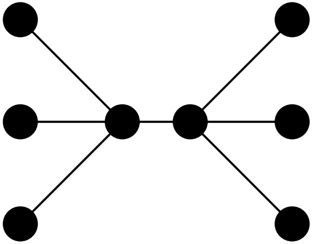


Mistake: $0 : 0.5\,N : 0.5$. Logit: $N : 1$.

General Analysis

- In general, the strategy revision process is an ergodic Markov process.
- There is no general characterization of the invariant distribution.
- The answer is well-understood for potential games and logit updating.

A General Diffusion Model

- Best response strategy revision. If fraction q or more of your neighbors choose A, then you choose A.
- Two obvious equilibria: All A and All B.
- How easy is it to "tip" from one to the other? What about intermediate equilibria?

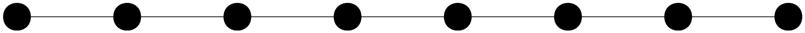
A General Diffusion Model

- Imagine that everyone initially uses B.
- Now a small group adopts A.
- When does it spread, when does it stop?
- The answer should depend on the network structure, who are the initial adopters, and the threshold p^*.

Diffusion of Coordination — Line

When the Poisson alarm clock rings, the player best responds to his neighbors. $p^* < 1/2$. Questions:

- Are islands of risk dominance stable?
- Can risk dominance spread?



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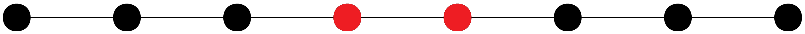

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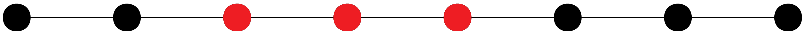

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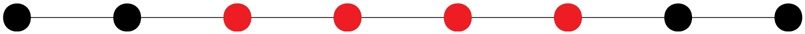
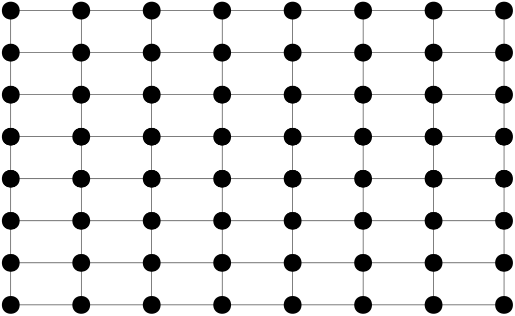


Diffusion of Coordination — Lattices

When the Poisson alarm clock rings, the player best responds to his neighbors. $p^* < 1/4$. Questions:

- Are islands of risk dominance stable?
- Can risk dominance spread?



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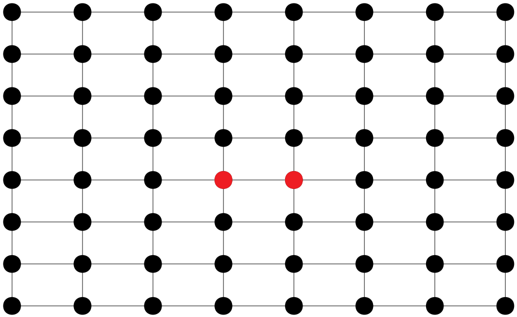

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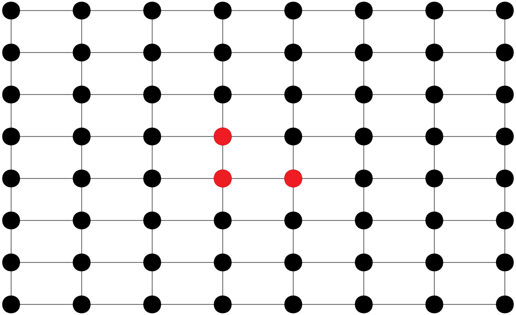

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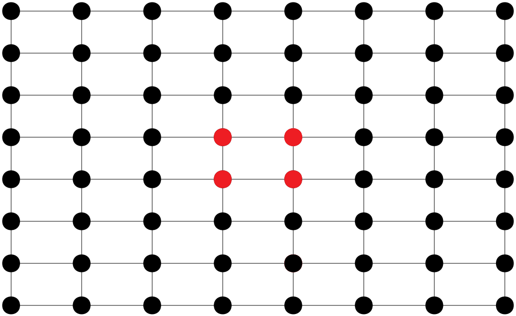
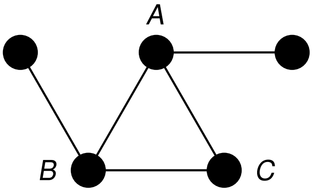


Diffusion of coordination — General Graphs

- A cluster of density p is a set of vertices C such that for each $v \in C$, at least fraction p of v's neighbors are in C.

The set $C = \{A, B, C\}$ is a cluster of density $2/3$.



General Graphs

Two observations:

- Every graph will have a cascade threshold.
- If the initial adoptees are a cluster of density at least p^*, then diffusion can only move forward.

General Graphs: Clusters Stop Cascades

Consider a set S of initial adopters in a network with vertices T, and suppose that remaining nodes have threshold q.

Claim: If S^c contains a cluster with density greater than $1 - q$, then S will not cause a complete cascade.

Proof: If there is a set $T \subset S^c$ with density greater than $1 - q$, then even if S / T chooses A, every member of T has fraction more than $1 - q$ choosing B, and therefore less than fraction q are choosing A. Therefore no member of T will switch.

General Graphs: Clusters Stop Cascades

Claim: If a set $S \subset V$ of initial adopters of an innovation with threshold q fails to start a cascade, then there is a cluster $C \in V / S$ of density greater than $1 - q$.

Proof: Suppose the innovation spreads from S to T and then gets stuck. No vertex in T^c wants to switch, so less than a fraction q of its neighbors are in T, more than fraction $1 - q$ are out. That is T^c has density greater than $1 - q$.

Networks and Optimality

- Networks make it easier for cascades to take place.
  - In the fully connected graph, a cascade from a small group never takes place. With stochastic adjustment in the mistakes model, the probability of transiting from all A to all B is $O(\epsilon^{qN})$, where q is the indifference threshold. On a network, the probability of transiting from all A to all B is on the order of ϵ^{K}, where K is the size of a group needed to start a cascade, and this is independent of N.

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- This is not always optimal!
  - Risk dominance and Pareto dominance can be different. This can be understood as a robustness question. If the population has correlated on the efficient action, how easy is it to undo? Hard if the efficient action is risk dominant. If the efficient action is not risk-dominant, it is easier to undo on sparse networks than on nearly completely connected networks.

Community Structure

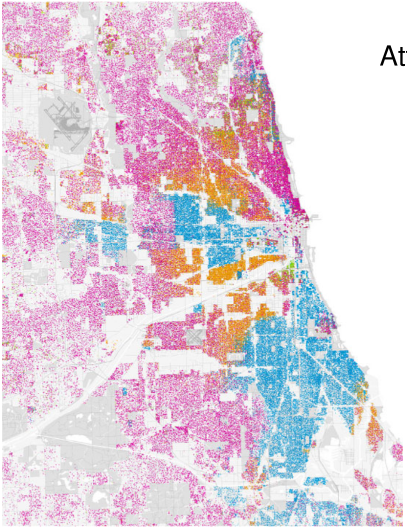
Under Construction

Two Problems

Imagine a social network, such as a friendship network in a school or network of information sharing in a village. Suppose the network participants represent several ethnic groups, races or tribes.

- How “integrated” is the network with respect to predefined communities?
- What are the implicit “comunities” of highly mutually interactive neighbors?
- How do these community structures map onto each other?

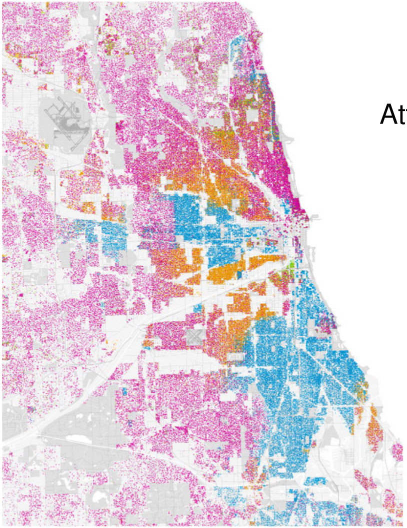
Measuring Segregation



Attributes of physical segregation.

- Evenness — Differential distribution of two groups across the network.
- Exposure — The degree to which different groups are in contact.
- Concentration — Relative concentration of physical space occupied by different groups.

Measuring Segregation



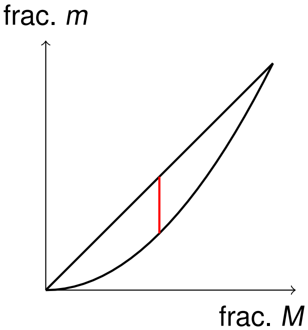
Attributes of physical segregation.

- Centraliztion — Extent to which a group is near the center.
- Clustering — Degree to which group members are connected to others in the group.

Dissimilarity Index

A city is divided into N areas. Area i has minority population m_i and majority population M_i. Total populations are m and M, respectively.

$$\text{dissimilarity index} = \frac{1}{2}\sum_{i=1}^{N}\left|\frac{m_i}{m} - \frac{M_i}{M}\right|.$$



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