Social Networks and Income Inequality

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Social Interactions **and** Inequality

- "The outstanding discovery of recent historical and anthropological research is that man's economy, as a rule, is submerged in his social relationships. He does not act so as to safeguard his individual interest in the possession of material goods; he acts so as to safeguard his social standing, his social claims, his social assets. He values material goods only in so far as they serve this end." (Polanyi, 1944)

- "Economics is all about how people make choices. Sociology is all about why they don't have any choices to make." (Duesenberry, 1960)

Where do Social Interactions Appear?

Phenomena

- Labor markets
  - Career Choices
  - Retirement
- Fertility
- Health
- Education Outcomes
- Violence

Mechanisms

- Peer effects
  - Stigma
- Role models
- Social Norms
- Social Learning
- Social Capital?

Research Methodologies

Ethnographies

Classics

- William F. Whyte, *Street Corner Society: The Social Stucture of an Italian Slum*, 1943.
- Berelson, Lazarsfeld, and McPhee, *Voting: A Study of Opinion Formation in a Presidential Campaign*, 1954.
- Oscar Lewis, *Five Families; Mexican Case Studies in the Culture of Poverty*, 1959.
- Elliot Liebow, *Talley's Corner: A study of Negro Streetcorner Men*, 1967.

Modern Ethnographies

- Katherine S. Newman, *No Shame in My Game: The Working Poor in the Inner City*, 1999.
- — *Chutes and Ladders: Navigating the Low-Wage Labor Market*, 2006.
- Kate Meagher, *Identity Economics: Social Networks and the Informal Economy in Africa*, 2010.

Research Methodologies

Experiments

- Robbers' Cave, M. Sherif *et* al. 1954.
- Conformity, S. Asch *et* al. 1951.
- 6 Degrees of Separation, S. Milgram, 1967.

Large-Scale “Natural” Experiments

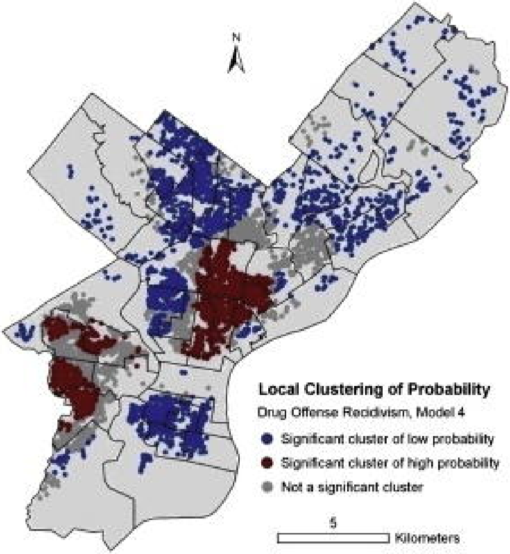
- Gautreaux Assisted Housing Program
- Moving to Opportunity

Questions

- What are appropriate tools for modelling social interactions?
- Describe the peer effects. What goes on at the micro level?

- What are the aggregate effects of interaction on social networks?

Crime



N
Local Clustering of Probability
Drug Offense Recidivism, Model 4
Significant cluster of low probability
Significant cluster of high probability
Not a significant cluster
5
Kilometers

Aggregate Analysis

Glaeser Sacerdote and Scheinkman 1996.

> *The most puzzling aspect of crime is not its overall level nor the relationships between it and either deterrence or economic opportunity. Rather, following Quetelet [1835], we believe that the most intriguing aspect of crime is its astoundingly high variance across time and space.*

> *Positive covariance across agents' decisions about crime is the only explanation for variance in crime rates higher than the variance predicted by difference in local conditions.*

A Model

- $2N + 1$ individuals live on the integer lattice at points $-N, \ldots, N$.
- Type 0s never commit a crime; Type 1's always do; Type 2's imitate the neighbor to the left.
- Type of individual i is p_i.

Model (of sorts)

- Expected distance between fixed agents determines group size — range of interaction effects.
- Social interactions magnify the effect of fixed agents.

$$E\{a_i\} = \frac{p_1}{p_0 + p_1} \equiv p, \qquad S_n = \sum_{|i| \le n} \frac{a_i - p}{2n + 1}.$$

$$\sqrt{2n+1} S_n \to N(0, \sigma^2), \qquad \sigma^2 = p(1-p)\frac{2-\pi}{\pi}$$

where

$$\pi = p_0 + p_1, \qquad f(\pi) = \frac{2-\pi}{\pi}.$$

Aggregate Statistics

TABLE IIA
ESTIMATES OF $f(\pi)$

Data series	Crimes per capita (p) times 1-crimes per capita ($1 - p$)	$\frac{\text{Sample variance}}{p(1-p)}$	Estimated λ^2	Estimated $f(\pi)$	Estimated $f(\pi)$ $\lambda^2 = .008$
Serious crime					
1985	0.073	1313.8	.013	754.6	604.7
N = 658			(.003)	(118.2)	
1970	0.042	1045.5	.004	475.1	284.3
N = 617			(.001)	(42.5)	
NYC	0.053	575.1			248.1
N = 70					
1986	0.078	1500.0	.0003	155.0	73.2
N = 631			(.0015)	(58.5)	

Empirical problems

- Unobserved correlated shocks
- Endogeneity of the network
- Distinguishing endogenous and contextual effects

Plan

- Network Science
- Labor Markets — Weak and Strong Ties
- Peer Effects and Complementarities — Games on Networks
- Social Capital
- Social Learning
- Diffusion
- Community Structure

Network Science

Graphs

A directed graph $\mathcal{G}$ is a pair (V, E) where V is a set of vertices, or nodes, and E is a set of Edges. An edge is an ordered pair (v, w), meaning that there is a connection **from** v **to** w. If $(w, v) \in E$ whenever $(v, w) \in E$, then $\mathcal{G}$ is an undirected graph.

The degree of a node in an undirected graph $\mathcal{G}$ is $\#\{w : (v, w) \in E\}$.

A path of $\mathcal{G}$ is an ordered list of nodes $(v_0, \ldots, v_N)$ such that $(v_{n-1}, v_n) \in E$ for all $1 \leq n \leq N$. A geodesic is a shortest-length path connecting v_0 and v_n.

Graphs

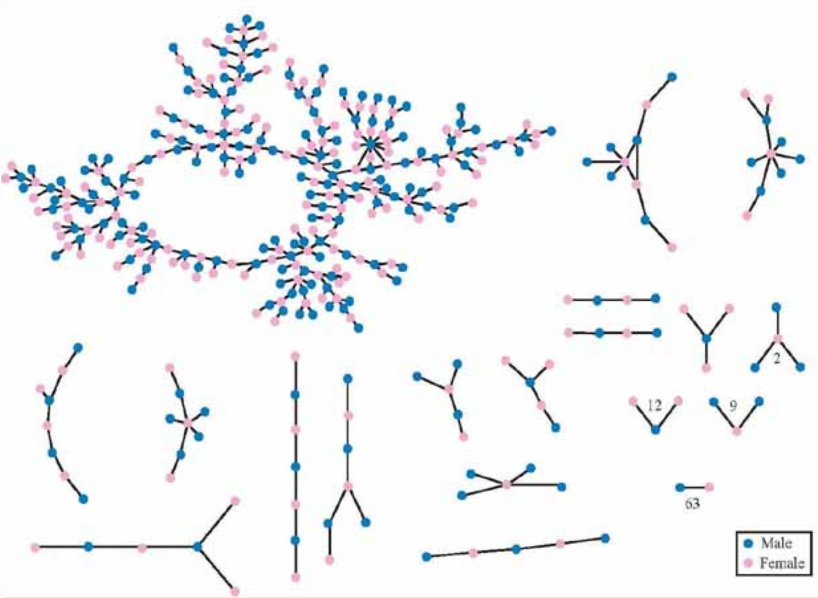
A subset of vertices is connected if there is a path between every two of them. A component of $\mathcal{G}$ is a set of vertices maximal with respect to connectedness. A clique is a component for which all possible edges are in E.

A graph $\mathcal{G}$ has a matrix representation. A adjacency matrix for a graph (V, E) is a $\#V \times \#V$ matrix A such that $a_{vw} = 1$ if $(v, w) \in E$, and 0 otherwise. A weighted adjacency matrix has non-zero numbers corresponding to edges in E.

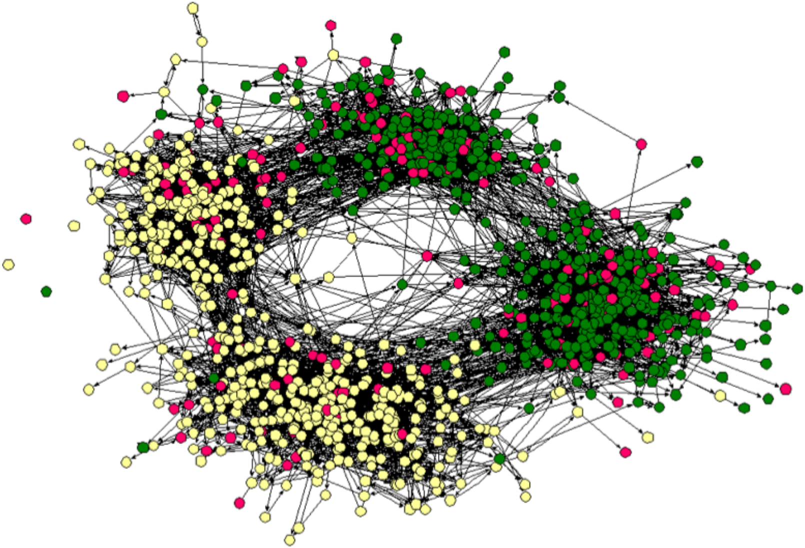
Common Network Measurements

- Graph diameter — maximal geodesic length.
- Mean geodesic length.
- Degree distribution.
- Clustering coefficient — the average (over individuals) of the number of length 2 paths containing i that are part of a triangle. (Measures degree of transitivity.)
- Component size distribution

Some Social Networks



Some Social Networks II



Some Social Networks

Network	Type	n	m	z	ℓ	α	$C^{(1)}$	$C^{(2)}$	r
film actors	undirected	449 913	25 516 482	113.43	3.48	2.3	0.20	0.78	0.208
company directors	undirected	7 673	55 392	14.44	4.60	–	0.59	0.88	0.276
math coauthorship	undirected	253 339	496 489	3.92	7.57	–	0.15	0.34	0.120
physics coauthorship	undirected	52 909	245 300	9.27	6.19	–	0.45	0.56	0.363
biology coauthorship	undirected	1 520 251	11 803 064	15.53	4.92	–	0.088	0.60	0.127
telephone call graph	undirected	47 000 000	80 000 000	3.16		2.1			
email messages	directed	59 912	86 300	1.44	4.95	1.5/2.0		0.16	
email address books	directed	16 881	57 029	3.38	5.22	–	0.17	0.13	0.092
student relationships	undirected	573	477	1.66	16.01	–	0.005	0.001	−0.029
sexual contacts	undirected	2 810				3.2			

n – # nodes, m – # edges, z – mean degree,
l – mean geodesic length, α – exponent of degree dist.,
$C^{(k)}$ - clustering coeff.s, r degree corr. coeff.

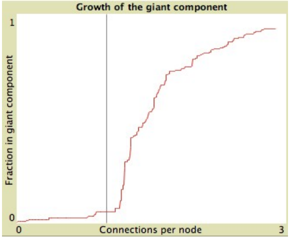
Comparison: Erdös-Rényi Random Graphs

Every possible (v, w) edge is assigned to E with probability p.

Poisson random graphs: A sequence of graphs $\mathcal{G}_n$ with $|V_n| = n$ such that $p \cdot (n-1) \to z$.

Large n facts:

- Phase transition at $z = 1$.
- Low-density: Exponential component size distribution with a finite limit mean.
- High-density: a giant connected component of size $O(n)$. Remainder size distribution exponential ....
- Clustering coefficient is $C^2 = O(n^{-1})$.

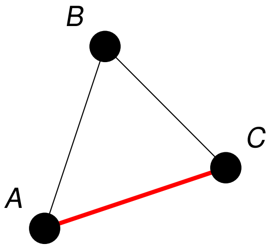


Simulation of Erdös-Rényi random sets on 300 nodes.

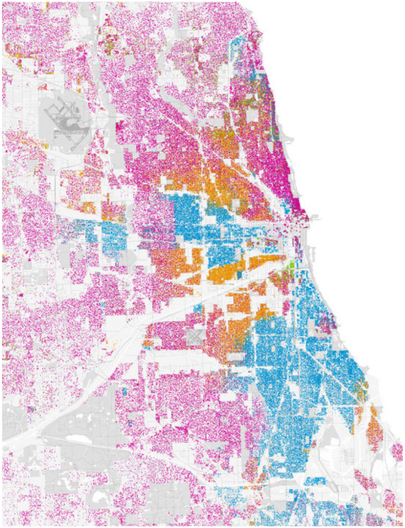
Transitivity

"If two people in a social network have a friend in common, then there is an increased likelihood that they will become friends themselves at some point in the future." Rappoport (1953)

- Clustering coefficient:
  Fraction of connected triples that are triangles.
- Why transitivity?



Homophily



"Similarity begets friendships."
Plato

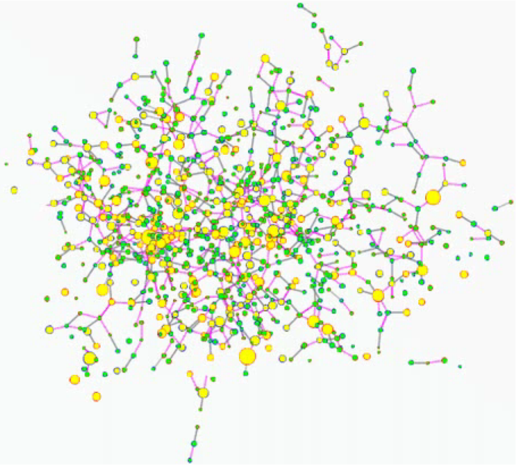
"All things akin and like are for the most part pleasant to each other, as man to man, horse to horse, youth to youth. This is the origin of the proverbs: The old have charms for the old, the young for the young, like to like, beast knows beast, ever jackdaw to jackdaw, and all similar sayings." Aristotle, *Nicomachean Ethics*

Sources of Homophily

- Status Homophily: We feel more comfortable when we interact with others who share a similar cultural background.
- Value Homophily: We often feel justified in our opinions when we are surrounded by others who share the same beliefs.
- Opportunity Homophily: We mostly meet people like us.

Sources of Homophily

- Fixed attributes
  - Selection
- Variable attributes
  - Social influence
- Identification



Measuring Homophily

Consider a network with N individuals: Fraction p are males, fraction $q = 1 - p$ are females.

- Assign nodes to gender randomly, each node male with probability p.
- What is the probability of a "cross-gender" edge?

Measuring Homophily

Consider a network with N individuals: Fraction p are males, fraction $q = 1 - p$ are females.

- Assign nodes to gender randomly, each node male with probability p.
- What is the probability of a "cross-gender" edge?
- A fraction of cross-gender edges less than $2pq$ is evidence for homophily.

Small Worlds

"Arbitrarily selected individuals (N=296) in Nebraska and Boston are asked to generate acquaintance chains to a target person in Massachusetts, employing "the small world method" (Milgram, 1967). Sixty-four chains reach the target person. Within this group the mean number of intermediaries between starters and targets is 5.2. Boston starting chains reach the target person with fewer intermediaries than those starting in Nebraska; subpopulations in the Nebraska group do not differ among themselves. The funneling of chains through sociometric "stars" is noted, with 48 per cent of the chains passing through three persons before reaching the target. Applications of the method to studies of large scale social structure are discussed."

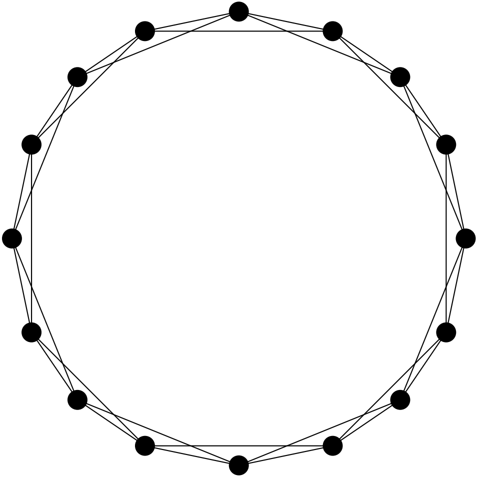
Travers and Milgram (1969)

Watts-Strogatz Model

Homophily

$+$

Weak Ties

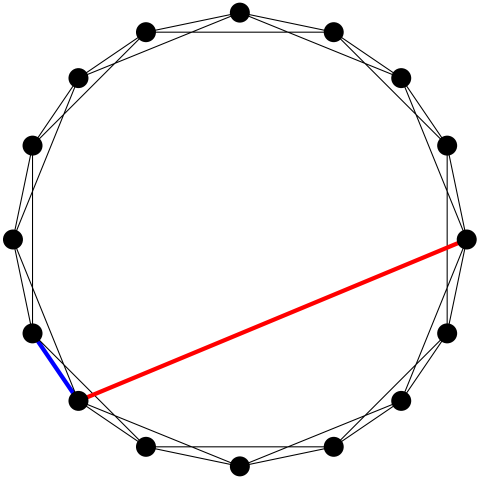


Watts-Strogatz Model

Homophily

+

Weak Ties

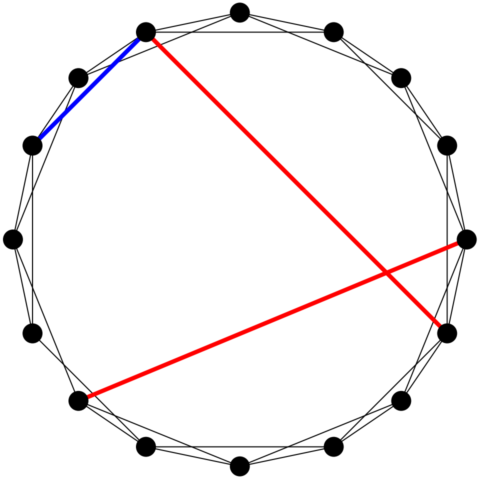


Watts-Strogatz Model

Homophily

+

Weak Ties



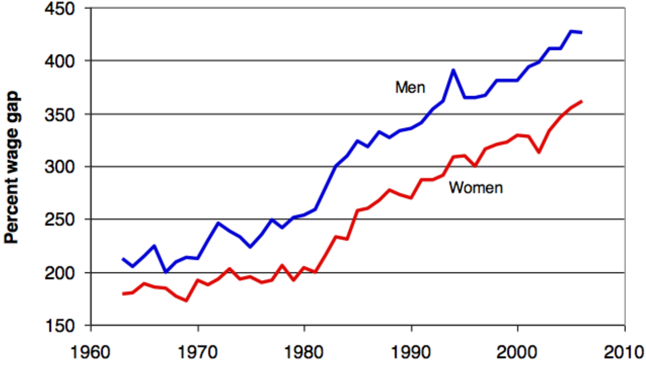
Is The World Small?

My Wife “ What a suprise meeting you here. The world is indeed small.”

Friend: “No, it’s very stratified.”

Labor Markets

Inequality in Labor Markets



Inequality in Labor Markets

Table 2: Changes in the 90-50 and 50-10 Wage Gaps

	Male			
	1980-90		1990-2000	
	90-50	***50-10***	***90-50***	***50-10***
Total change	.1035	.0886	.0660	-.0774
Due to wage dispersion within occupations	.0778	.0507	.0342	-.0537
Due to wage changes between occupations	.0588	.0257	.0184	-.0154
	Female			
	1980-90		1990-2000	
	90-50	***50-10***	***90-50***	***50-10***
Total change	.0592	.1526	.0368	.0153
Due to wage dispersion within occupations	.0292	.1074	.0343	.0044
Due to wage changes between occupations	.0417	.0660	.0103	-.0045

Job Search

TABLE 1—JOB-FINDING METHODS USED BY WORKERS

	Percentage of jobs found using each method					
Source/data	Friends/relatives	Gate application	Employment agency	Ads	Other	Sample size
Myers and Shultz (1951)/sample of displaced textile workers:						
First job	62	23	6	2	7	144
Mill job	56	37	3	2	2	144
Present job	36	14	4	0	46[a]	144
Rees and Shultz (1970)/Chicago labor-market study, 12 occupations:[b]						
Typist	37.3	5.5	34.7	16.4	6.1	343
Keypunch operator	35.3	10.7	13.2	21.4	19.4	280
Accountant	23.5	6.4	25.9	26.4	17.8	170
Tab operator	37.9	3.2	22.2	22.2	14.5	126
Material handler	73.8	6.9	8.1	3.8	7.4	286
Janitor	65.5	13.1	7.3	4.8	9.3	246
Janitress	63.6	7.5	5.2	11.2	12.5	80
Fork-lift operator	66.7	7.9	4.7	7.5	13.2	175
Punch-press operator	65.4	5.9	7.7	15.0	6.0	133
Truck driver	56.8	14.9	1.5	1.5	25.3	67
Maintenance electrician	57.4	17.1	3.2	11.7	10.6	129
Tool and die maker	53.6	18.2	1.5	17.3	9.4	127
Granovetter (1974)/sample of residents of Newton, MA:						
Professional	56.1	18.2	15.9[c]	—[c]	9.8	132
Technical	43.5	24.6	30.4	—	1.4	69
Managerial	65.4	14.8	13.6	—	6.2	81
Corcoran et al. (1980)/Panel Study of Income Dynamics, 11th wave:						
White males	52.0	—[d]	5.8	9.4	33.8[d]	1,499
White females	47.1	—	5.8	14.2	33.1	988
Black males	58.5	—	7.0	6.9	37.6	667
Black females	43.0	—	15.2	11.0	30.8	605

[a]Most of these workers were rehired at a previous mill job or hired at a new mill established in one of the vacated mills.

[b]In computing the percentages, workers rehired by previous employers and those not reporting the job-finding source are excluded from the denominator and subtracted from the sample size.

[c]Agencies and ads are combined under the heading "formal means."

[d]Gate applications are included under "other."

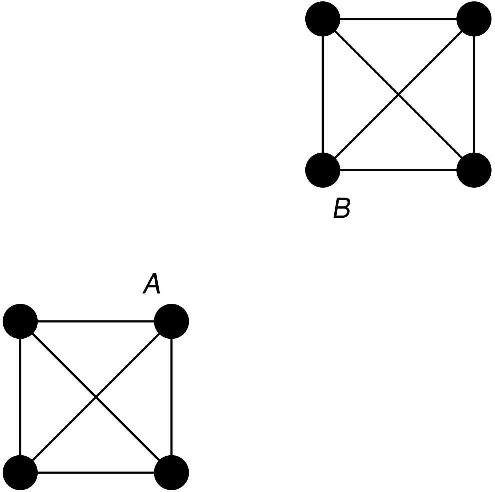
The Strentgh of Weak Ties

"...[T]he strength of a tie is a (probably linear) combination of the amount of time, the emotional intensity, the intimacy (mutual confiding), and the reciprocal services which characterize the tie. Each of these is somewhat independent of the other, though the set is obviously highly intracorrelated. Discussion of operational measures of and weights attaching to each of the four elements is postponed to future empirical studies. It is sufficient for the present purpose if most of us can agree, on a rough intuitive basis, whether a given tie is strong, weak, or absent."

Granovetter (1973)

Why do Weak Ties Matter?

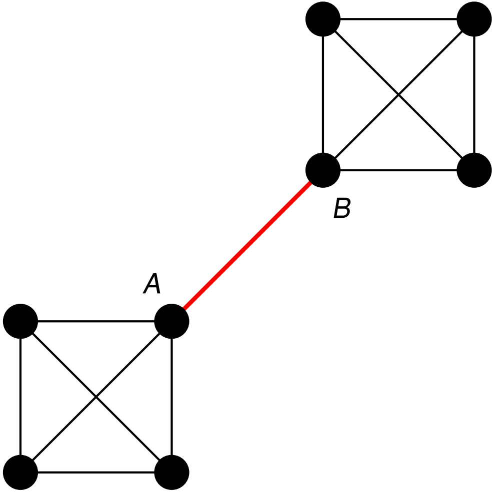
Two cliques.



Why do Weak Ties Matter?

Two cliques.

A–*B* is a **bridge**.

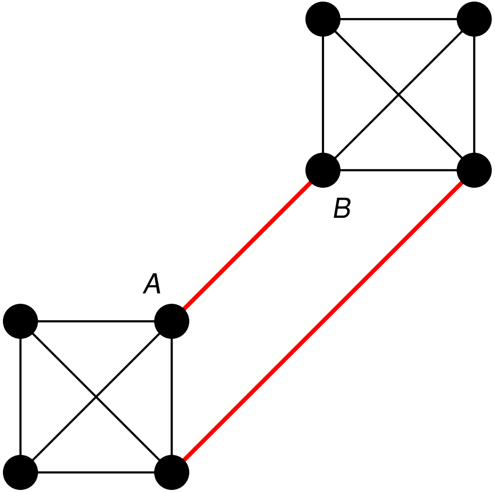


Why do Weak Ties Matter?

Two cliques.

A–*B* is a **bridge**.

Local bridge's endpoints have no common friends.



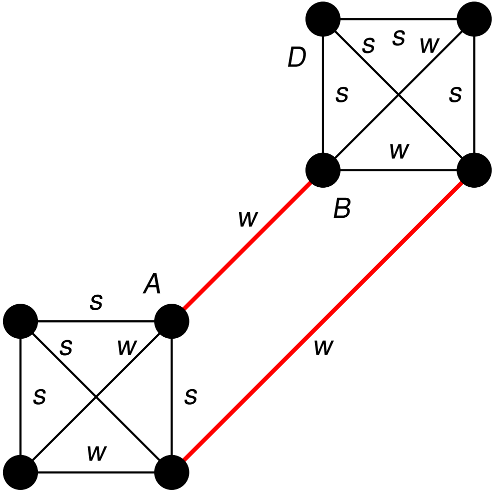
Why do Weak Ties Matter?

Two cliques.

A–*B* is a **bridge**.

Local bridge's endpoints have no common friends.

Triadic closure: A length-2 path containining only strong edges is a closed triad.



Ties and Inequality

Model

- Workers live for two periods, $\#W$ identical in both periods.
- Half of the workers are high-ability, produce 1.
- Half of the workers are low-ability, produce 0.
- Workers are observationally indistinguishable.

- Each firm employs 1 worker.
- $\pi =$ employee productivity – wage.
- Free entry, risk-neutral entrepreneurs.

- Equilibrium condition: Firms expected profit is 0. Wage offers are expected productivity.

- Each $t = 1$ worker knows at most 1 $t = 2$ worker.
- Each $t = 1$ worker has a *social tie* with pr $= \tau$.
- Conditional on having a tie, it is to the same type with probability $\alpha > 1/2$.
- Assignments of a $t = 1$ worker to a specific $t = 2$ worker is random.

- τ — “network density”
- α — “inbreeding bias”

Ties and Inequality III

Timing

- Firms hire period 1 workers through the anonymous market, clears at wage w_{m1}.
- Production occures. Each firm learns its worker's productivity.
- Firm f sets a referral offer, w_{rf}, for a second period worker.
- Social ties are assigned.
- $t = 1$ workers with ties relay w_{ri}.
- $t = 2$ workers decide either to accept an offer or enter the market.
- Period 2 market clears at wage w_{m2}.
- Production occurs

- Only firms with 1-workers will make referral offers.
- Referral wages offers are distributed on an interval $[w_{m2}, w_R]$.
- $0 < w_{m2} < 1/2$.
- $\pi_2 > 0$.
- $w_{m1} = E\Big\{\text{production value} + \text{referral value}\Big\} > 1/2$.

$$\alpha, \tau \uparrow \quad \Longrightarrow \quad \begin{cases} w_{m2} \downarrow \\ w_R \uparrow \\ \pi_2 \uparrow \\ w_{m1} \uparrow \end{cases}$$

- in the market-only model, $w_{m1} = w_{m2} = 1/2$.
- $t = 2$ 1-types are better off, $t = 2$ low types are worse off. Social structure magnifies income inequality in the second period.
- The total wage bill in the second period is less with social structure.

Network Structure and Inequality

- Dynamic Markov model
- Illustrate how network structure matters

- Discrete time.
- N individuals.
- Symmetric adjacency matrix A.
- A configuration of the model is a map $s : \{1, \ldots, N\} \to \{0, 1\}$.
  Interpretation: 0 is unemployed, 1 is employed.
- p is the probability that an individual learns about a job opening.

1. With probability $p + q \leq 1$, a *job event* happens. With probability qk/N one of the k employed individuals loses her job. With probability p a single randomly chosen individual learns about a job.

2. If she is unemployed, she takes the job.

3. If she is employed, she passes the offer on to an unemployed neighbor, chosen at random.

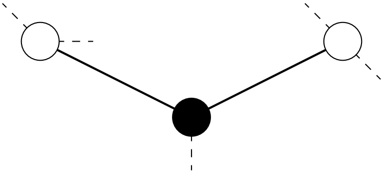
4. If all neighbors are employed, the referral dies.

In any period, the configuration can change in one of three ways:

- A 0 can change to a 1;
- A 1 can change to a 0;
- The configuration can remain unchanged.

$$\Pr\left\{s_{t+1}(i) = 1 \,\middle|\, s_t(i) = 0, s_t(-i)\right\} = \frac{p}{N}\left(1 + \sum_j a_{ij} s_t(j) \frac{1}{\sum_k a_{jk} s_t(k)}\right)$$

$$\Pr\left\{s_{t+1}(i) = 0 \,\middle|\, s_t(i) = 1, s_t(-i)\right\} = \frac{q}{N}$$



$$\mathrm{Cov}\Big(s_{t+1}(1), s_{t+1}(3)\,\Big|\, s_t = (0,1,0)\Big) =$$

$$E\Big\{s_{t+1}(1)\cdot s_{t+1}(3)\Big|(0,1,0)_t\Big\}$$

$$-E\Big\{s_{t+1}(1)\Big|(0,1,0)\Big\}\cdot E\Big\{s_{t+1}(3)\Big|(0,1,0)_t\Big\} =$$

$$\left(\frac{q}{N}+\frac{p}{N}+\frac{p}{N}+z\right)\cdot 0$$

$$-\left(\frac{q}{N}\cdot 0+\frac{p}{N}\cdot 1+\frac{p}{N}\cdot 0+z\cdot 0\right)^2 = -\frac{p^2}{N^2}$$

- Equilibrium is an invariant distribution of the Markov chain.
- The transition matrix is irreducible, so the invariant distribution μ is unique!
- $\mathrm{Cov}_{\mu}\big(s(i), s(j)\big) \geq 0$.
- $\mathrm{Cov}_{\mu}\big(s(i), s(j)\big) > 0$ if and only if i and j are in the same connected component.

Because of symmetry, this is a Markov process on the number of employed. m_{ij} is the probability that j workers will be employed tomorrow if i workers are employed today.

$$M = \begin{pmatrix} 1-p & p & 0 \\ \frac{q}{2} & 1-p-\frac{q}{2} & p \\ 0 & q & 1-q \end{pmatrix}$$

The invariant distribution is a probability distribution that solves

$$\rho M = \rho.$$

$$\rho(0) = \frac{q^2}{\Delta} \qquad \rho(1) = \frac{2pq}{\Delta} \qquad \rho(2) = \frac{2p^2}{\Delta}.$$

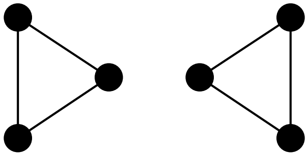
$$M = \begin{pmatrix} 1-p & p & 0 \\ \frac{q}{2} & 1-\frac{q}{2}-\frac{p}{2} & \frac{p}{2} \\ 0 & q & 1-q \end{pmatrix}$$

$$\rho(0) = \frac{q^2}{\Delta} \qquad \rho(1) = \frac{2pq}{\Delta} \qquad \rho(2) = \frac{p^2}{\Delta}.$$

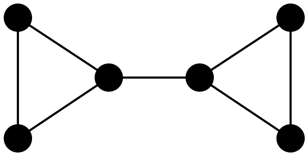
Suppose that $emp = k$ out of N individuals are employed after t events.

$$\Pr\{emp_{t+1} = k + 1 | emp_t = k\} = p,$$
$$\Pr\{emp_{t+1} = k - 1 | emp_t = k\} = \frac{kq}{N}.$$

$$\frac{\rho(k+1)}{\rho(k)} = \frac{N}{k}\frac{p}{q} \qquad \frac{\rho(k)}{\rho(0)} = \frac{N^k}{k!}\left(\frac{p}{q}\right)^k$$



Product distribution



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Peer Effects and Complementarities

Behaviors on Networks

Three Types of Network Effects

- Information and social learning.
- Network externalities.
- Social norms.

School Effort

Complementarities

A model of choice on a network as a normal-form game.

Students have a pre-existing social network, which is described by a weighted sociomatrix A. a_{ij} is the degree to which j influences i. W.l.o.g. row sums are 1.

Strategies are effort is $e \in \mathbf{R}$.

The payoff functions are

$$u_i(e_1, \ldots, e_N) = \theta_i e_i - \frac{1}{2} e_i^2 + \phi_i \sum_{j=1}^{N} a_{ij} e_i e_j$$

Story: Blue is the return to effort, and red is the disutility of providing effort. The strategic complementarity is a network externality.

School Effort

Social norms

A model of choice on a network as a normal-form game.

Students have a pre-existing social network, which is described by a weighted sociomatrix A. a_{ij} is the degree to which j influences i. W.l.o.g. row sums are 1.

Strategies are effort is $e \in \mathbf{R}$.

The payoff functions are

$$u_i(e_1, \ldots, e_N) = \theta_i e_i - \frac{1}{2} e_i^2 - \frac{\phi_i}{2} \left(e_i - \sum_{j=1}^{N} a_{ij} e_j \right)^2 .$$

Story: Blue is the private utility of providing effort, which has both costs and benefits, and red is the cost of diverging from the social norm. Students respond to peer pressure to conform.

Private Component

To complete the model, specify how individual characteristics matter.

$$\theta_i = \gamma x_i + \delta \sum_j c_{ij} x_j + z$$

Direct Effect

Contextual Effect

Equilibrium

first order conditions

Complementarities

$$\theta_i - e_i + \phi_i \sum_j a_{ij} \mathrm{E}\left\{e_j | x, z_i\right\} = 0$$

Social norms

$$\theta_i - (1 + \phi_i) e_i + \phi_i \sum_j a_{ij} \mathrm{E}\left\{e_j | x, z_i\right\} = 0$$

Equilibrium

Computation

$$e_i = \frac{1}{1+\phi_i}\theta_i + \frac{\phi_i}{1+\phi_i}\sum_j a_{ij}\,\mathrm{E}\left\{e_j|x, z_i\right\}.$$

A computation gives:

$$e = f(x, z) = \Phi(I - \Phi A)^{-1}(\gamma I + \delta C)x + g(x, z)$$

$$g_i(x, z_i) = \frac{1}{1+\phi_i}z_i + \frac{\phi_i}{1+\phi_i}\sum_j a_{ij}\,\mathrm{E}\left\{g_j(x, z_j)|x, z_i\right\}$$

If z is independent of x, then g_i depends only upon z_i.

$$g_i(z_i) = \frac{1}{1+\phi_i}z_i + \frac{\phi_i}{1+\phi_i}\sum_j a_{ij}\,\mathrm{E}\left\{g_j(z_j)|z_i\right\}$$

If the z_i are all pairwise independent, then g depends only upon x.

Econometric Model

Statistical model:

$$e = b + Bx + \epsilon,$$

Theoretical model:

$$e = \Phi(I - \Phi A)^{-1}(\gamma I + \delta C)x + g(z)$$

The “Reflection” Problem

For all $g \in G$ and all $i \in g$,

$$e_i = \alpha + \gamma x_i + \delta x_g + \phi \mu_i + \varepsilon_i \qquad \text{(Behavior)}$$

$$x_g = \frac{1}{N_g} x_i \qquad \text{(Behavior)}$$

$$\mu_i = \frac{1}{N_g} \sum_{j \in g} \mathrm{E}\left\{e_j\right\} \qquad \text{(Equilibrium)}$$

The reduced form is

$$e_i = \frac{\alpha}{1 - \phi} + \gamma x_i + \frac{\phi\gamma + \delta}{1 - \phi} x_g + \varepsilon_i$$

Identification

$$e = b + Bx + \epsilon.$$

- If there is enough variation in x, then B is identified.
- The problem is to infer γ, δ and Φ from B.

Identification

known A

Theorem: Suppose that for all i and j, x_i and ϵ_j are uncorrelated, and at least one of γ and δ is not 0. If A, C and AC are distinct, then linear independence of the matrices I, A, C and AC is necessary and sufficient for identification of γ, δ and Φ.

Corollary: Suppose there are distinct individuals i and j who are connected by a sequence of edges, some in the peer-effects network and some in the contextual-effects network, and who are not connected by a path in either the peer-effects network or the contextual-effects network alone. Then γ, δ and Φ are identified.

Identification

unknown A

Suppose $\Phi = \phi\mathbb{I}$, and $\phi > 0$.

Theorem: Assume that contextual-effects sociomatrix C is known *a priori*. Assume too that the peer-effects sociomatrix A is unknown but the peer-effects network is known *a priori*. Suppose that $N \geq 3$. Suppose that there are two distinct individuals i and j who are known to be unconnected in the peer-effects network. If $B^{-1}_{ij} \neq 0$, then the utility parameters γ, δ and ϕ are identified from the conditional mean of ω given x.

Theorem: For all C and "generic" A, γ, ϕ, δ and A are identified.

Non-Linear Aggregators

Bad apple The worst student does enormous harm.

Shining light A single student with sterling outcomes can inspire all others to raise their achievement.

Invidious comparison Outcomes are harmed by the presence of better achieving peers.

Boutique A student will have higher achievement whenever she is surrounded by peer with similar characteristics.

Social Capital

Networks and Social Capital

"the aggregate of the actual or potential resources which are linked to possession of a durable network of more or less institutionalized relationships of mutual acquaintance or recognition." (Bourdieux and Wacquant, 1992)

"the ability of actors to secure benefits by virtue of membership in social networks or other social structures." (Portes, 1998)

"features of social organization such as networks, norms, and social trust that facilitate coordination and cooperation for mutual benefit." (Putnam, 1995)

"Social capital is a capability that arises from the prevalence of trust in a society or in certain parts of it. It can be embodied in the smallest and most basic social group, the family, as well as the largest of all groups, the nation, and in all the other groups in between. Social capital differs from other forms of human capital insofar as it is usually created and transmitted through cultural mechanisms like religion, tradition, or historical habit." (Fukuyama, 1996)

"naturally occurring social relationships among persons which promote or assist the acquisition of skills and traits valued in the marketplace. . . " (Loury, 1992)

Networks and Social Capital

"…social capital may be defined operationally as *resources embedded in social networks and accessed and used by actors for actions.* Thus, the concept has two important components: (1) it represents resources embedded in social relations rather than individuals, and (2) access and use of such resources reside with actors." (Lin, 2001)

Information

- Search is a classic example according to Lin's (2001) definition.
- Search has nothing to do with values and social norms beyond the willingness to pass on a piece of information.

Trust

Three Stories about Trust:

Reciprocity: Reputation games, folk theorems, . . .

Social Learning: Generalized trust.

Behavioral Theories: Evolutionary Psychology, prosocial preferences, . . .

Inequality and Trust

- Evidence for a correlation between trust and income inequality
  - Rothstein and Uslaner (2005), Uslaner and Brown (2005).
- Trust is correlated with optimism about one's own life chances
  - Uslaner (2002)

Networks, Trust, and Development

- Informal social organization substitutes for markets and formal social institutions in underdeveloped economies.
- In the US, periods of high growth have also been periods of decline in social capital (Putnam, 2000)
- Possibly: Social capital is needed for economic development only up to some intermediate stage, where generalized trust in institutions takes the place of informal trust arrangements.

Does Social Capital Have an Economic Payoff?

Knaak and Keefer (1997). “Does social capital have a payoff".

$$g_i = \mathbf{X}_i\gamma + \mathbf{Z}_i\pi + \text{CIVIC}_i\alpha + \text{TRUST}_i\beta + \epsilon_i$$

g_i real per-capita growth rate.

$\mathbf{X}_i$ control variables — Solow.

$\mathbf{Z}_i$ control variables — “endogenous” growth models.

CIVIC_i index of the level of civic cooperation.

TRUST_i the percentage of survey respondents (after omitting those responding ‘don’t know’) who, when queried about the trustworthiness of others, replied that ‘most people can be trusted’.

A Model of Trust

- A population of N completely anonymous individuals.
- Individuals have no distinguishing features, and so no one can be identified by any other.
- Individuals are randomly paired at each discrete date t, with the opportunity to pursue a joint venture. Simultaneously with her partner, each individual has to choose whether to participate (P) in the joint venture, or to pursue an independent venture (I). The entirety of her wealth must be invested in one or the other option. The individual with wealth w receives a gross return $w\pi$ from her choice, where π is realized from the following payoff matrix:

		partner P	partner I
investor	P	$\tilde{R}$	$\tilde{r}$
	I	$\tilde{e}$	$\tilde{e}$

Gross Returns

A Model of Trust

- $E\tilde{R} > E\tilde{e} > E\tilde{r}$.
- Individuals reinvest a constant fraction β of their wealth.
- Strategies for i are functions which map the history of is experience in the game to actions in the current period.
- Equilibria: Always play P, always play I are two equilibria.

Learning

Each individual i has a prior belief ρ, about the probability of one's opponent choosing P. The prior distribution is beta with parameters $a^i, b^i > 0$. In more detail,

$$\begin{aligned}\rho^i(x) &= \beta(a_0^i, b_0^i)\\ &= \frac{\Gamma(a_0^i + b_0^i)}{\Gamma(a_0^i)\Gamma(b_0^i)} x^{a_0^i - 1}(1-x)^{b_0^i - 1}.\end{aligned}$$

Let ρ_t^i denote individual i's posterior beliefs after t rounds of matching. The posterior densities ρ_t^i and ρ_t^j will be conditioned on different data, since all information is private. The updating rule for the β distribution has

$$\rho_t^i(h_t) \equiv \beta(a_t^i, b_t^i) = \beta(a_0^i + n, b_0^i + t - n)$$

for histories containing n P's and therefore $t - n$ I's. The posterior mean of the β distribution is $a_t^i / (a_t^i + b_t^i)$.

Optimal Play

$$q^* = (e - r)/(R - r)$$

- Let m_t^i denote i's mean of ρ_t.
- An optimal strategy for individual i is: Choose P if $m_t > q^*$ and choose I otherwise.

Theorem 3: For all initial beliefs $(\rho_0^1, \ldots, \rho_0^N)$, almost surely either $\lim_t n_t^P = 0$ or $lim_t n_t^P = N$. The probabilities of both are positive. The limit wealth distributions in both cases is $\Pr\{\lim_t w_t > w\} \sim cw^k$, where k is k_P or k_I, and $k_P < k_I$.

Social Learning

Averaging the Opinions of Others

- DeGroot (1974)
- X is some event. $p_i(t)$ is the probability that i assigns to the occurance of X at time t.
- M is a stochastic matrix. m_{ij} is the weight i gives to j's opinion.
- $p(t) = Mp(t-1) = \cdots = M^t p(0)$.

Averaging the Opinions of Others

Example

$$M = \begin{pmatrix} 1/3 & 1/3 & 1/3 \\ 1/2 & 1/2 & 0 \\ 0 & 1/2 & 1/2 \end{pmatrix},$$

$$p(2) = M^2 p(0) = \begin{pmatrix} 5/18 & 8/18 & 5/18 \\ 5/12 & 5/12 & 2/12 \\ 1/4 & 1/2 & 1/4 \end{pmatrix} p(0),$$

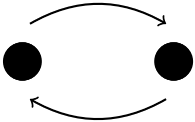
$$p(t) = M^t p(0) \to \begin{pmatrix} 3/9 & 4/9 & 2/9 \\ 3/9 & 4/9 & 2/9 \\ 3/9 & 4/9 & 2/9 \end{pmatrix} p(0).$$

$$p_i(\infty) = (1/9)\Big(3p_1(0) + 4p_2(0) + 2p_3(0)\Big).$$

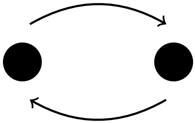
Averaging the Opinions of Others

Distinct Limits

$$M = \begin{pmatrix} 1/2 & 1/2 & 0 & 0 \\ 1/3 & 2/3 & 0 & 0 \\ 0 & 0 & 1/2 & 1/2 \\ 0 & 0 & 2/3 & 1/3 \end{pmatrix}$$



$$M^t \to \begin{pmatrix} 2/5 & 3/5 & 0 & 0 \\ 2/5 & 3/5 & 0 & 0 \\ 0 & 0 & 3/5 & 2/5 \\ 0 & 0 & 3/5 & 2/5 \end{pmatrix}$$



$$p_i(t) \to (1/5)\big(2p_1(0) + 3p_2(0)\big) \quad \text{for } i = 1, 2.$$

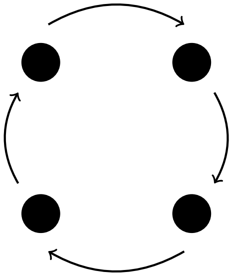
$$p_i(t) \to (1/5)\big(3p_3(0) + 2p_4(0)\big) \quad \text{for } i = 3, 4.$$

Averaging the Opinions of Others

No Limit

$$M = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

$$M^t = M^{(t-1) \bmod 3 + 1}$$



Theorem: If M is irreducible and aperiodic, then beliefs converge to a limit probability. $\lim_{t\to\infty} p(t) = \sum_i \pi_i p_i(0)$, where π is the left Perron eigenvector of M.

Connection to Markov processes.

Influential individuals are those who influence other influential individuals. We want to measure this by a scalar s_i for each individual i.

Definition: The Bonacich (eigenvector) centrality of individual j is the average of the social influences of those he inflluences, weighted by the amount he influences them (Bonacich, 1987).

Then s solves

$$s_j = \sum_i m_{ij} s_i,$$

Thus s is the left Perron eigenvector of M, and so $s = \pi$.

Limit Beliefs and the "Wisdom of Crowds"

- Suppose that $p_i(0) = p + \epsilon_i$. The ϵ_i are all independent, have mean 0, and variances are bounded.
- What is the relationship between $p_i(\infty)$ and p?
- A sequence of networks $(V_n, E_n)_{n=1}^{\infty}$, $|V_n| = n$, with centrality vectors s^n, and belief sequences $p^n(t)$.

Definition: The sequence learns if for all $\epsilon > 0$,
$\Pr\{|\lim_{n\to\infty} \lim_{t\to\infty} p^n(t) - p| > \epsilon\} = 0$.

Theorem: If there is a $B > 0$ such that for all i, $s_n^i \leq B/n$, then the sequence learns.

- What conditions on the networks guarantee this?

Bayesian Learning on Networks I

Multi-armed bandit problem

- An undirected network $\mathcal{G}$.
- Two actions, *A* and *B*. *A* pays off 1 for sure. *B* pays off 2 with probability p and 0 with probability $1 - p$.
- At times $t = \{1, 2, \ldots\}$, each individual makes a choice, to maximize $\mathrm{E}\left\{\sum_{\tau=t}^{\infty} \beta^{\tau} \pi_{i\tau} | h_t\right\}$, the expected present value of the discounted payoff stream given the information.
- $p \in \{p_1, \ldots, p_K\}$. W.l.o.g. $p_j \neq p_k$ and $p_k \neq 1/2$.
- Each individual has a full-support prior belief μ_i on the p_k.
- Individuals see the choices of his neighbors, and the payoffs.

Bayesian Learning on Networks II

Multi-armed bandit problem

- If the network contains only one member, this is the classic multi-armed bandit problem.
- How does the network change the classic results?
- What does one learn from the behavior of others?

Theorem: With probability one, there exists a time such that all individuals in a component play the same action from that time on.

- In one-individual problem, it is possible to lock into A when B is optimal. How does the likelihood of this change in a network?

Diffusion

Varieties of Action

- Graphical Games — Diffusion of action
  - Blume (1993, 1995) — Lattices
  - Morris (2000) — General graphs
  - Young and Kreindler (2011) — Learning is fast
- Social Learning — Diffusion of knowledge
  - Banerjee, QJE (1992)
  - Bikchandani, Hershleifer and Welch (1992)
  - Rumors

Coordination Games

	A	B
A	a,a	0,0
B	0,0	b,b

$a, b > 0$

Pure coordination game

Three equilibria:

$$\langle a, a\rangle, \quad \langle b, b\rangle, \quad \text{and} \quad \langle(\frac{b}{a+b}, \frac{a}{a+b}), (\frac{b}{a+b}, \frac{a}{a+b})\rangle$$

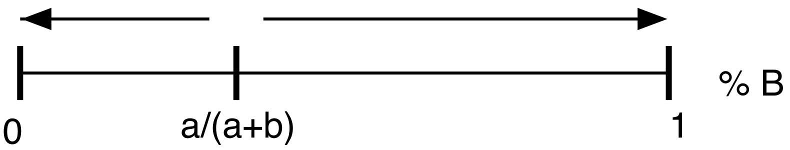
Coordination Games

	A	B
A	a,a	0,0
B	0,0	b,b

$a, b > 0$

Pure coordination game

Best response dynamics



Coordination Games

	A	B
A	a,a	d,c
B	c,d	b,b

$a > c,\ b > d$

General coordination game

Here the symmetric mixed equilibrium is at
$p^* = (b - d)/(a - c + b - d)$.

Suppose $b - d > a - c$. Then $p^* > 1/2$. At $(1/2, 1/2)$, A is the best response. This is not inconsistent with $b > a$.

- A is Pareto dominant if $a > b$.
- B is risk dominant if $b - d < a - c$.

Coordination Games — Stochastic Stability

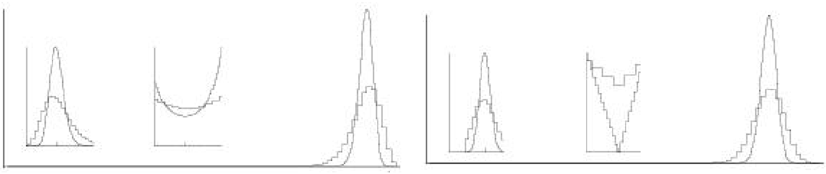
Continuous time stochastic process

- Each player has an alarm clock. When it goes off, she makes a new strategy choice. The interval between rings has an exponential distribution.
- Strategy revision:
  - Each individual best-responds with prob. $1 - \epsilon$, Kandori, Mailath and Robb (1993); Young (1993)

    or
  - The log-odds of choosing A over B is proportional to the payoff difference — logit choice, Blume (1993, 1995).

The Stochastic Process

This is a Markov process on the state space $[0, \ldots, N]$, where the state is the number of players choosing B.

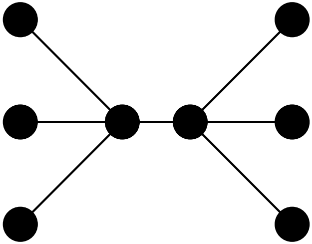


Logit Choice   Mistakes

In both cases, as $\text{Prob}\{\text{best response} \uparrow 1\}$, $\text{Prob}\{N\} \uparrow 1$.

Coordination on Networks

- Is the answer the same on every graph?



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Coordination on Networks

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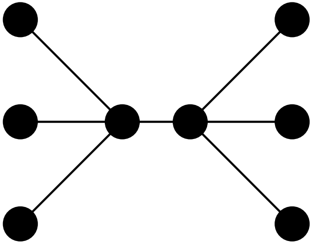
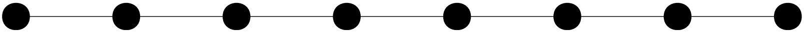


Mistake: $0 : 0.5\,N : 0.5$. Logit: $N : 1$.

Diffusion of Coordination — Line

When the Poisson alarm clock rings, the player best responds to his neighbors. $p^* < 1/2$. Questions:

- Are islands of risk dominance stable?
- Can risk dominance spread?



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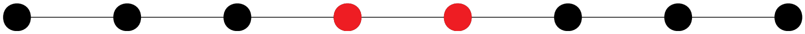

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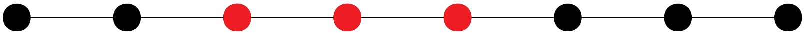

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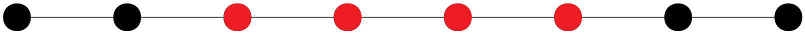

Diffusion of Coordination — Lattices

When the Poisson alarm clock rings, the player best responds to his neighbors. $p^* < 1/4$. Questions:

- Are islands of risk dominance stable?
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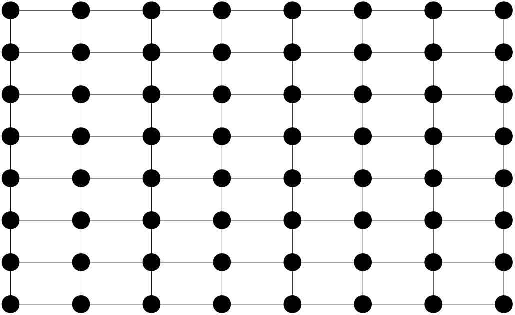

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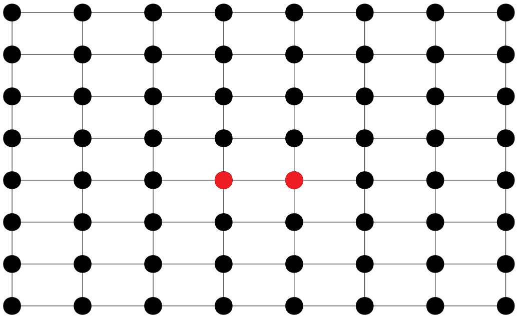

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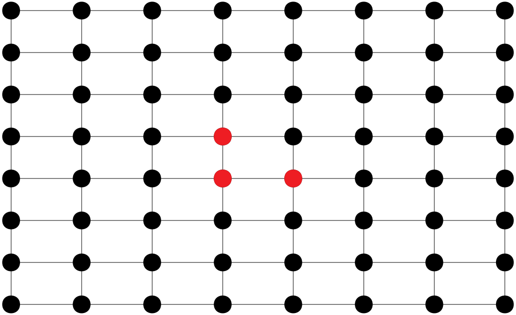

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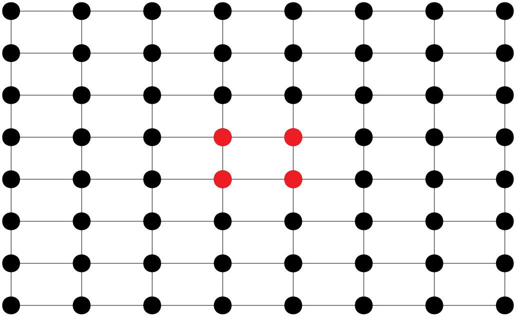
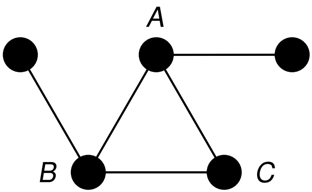


Diffusion of coordination — General Graphs

- A cluster of density p is a set of vertices C such that for each $v \in C$, at least fraction p of v's neighbors are in C.

The set $C = \{A, B, C\}$ is a cluster of density $2/3$.



General Graphs

Two observations:

- Every Graph will have a cascade threshold.
- Diffusion can only move forward.

General Graphs: Clusters Stop Cascades

Consider a set S of initial adopters in a network with vertices T, and suppose that remaining nodes have threshold q.

Claim: If S^c contains a cluster with density greater than $1 - q$, then S will not cause a complete cascade.

Proof: Suppose the innovation spreads from S to T and then gets stuck. No vertex in T^c wants to switch, so less than a fraction q of its neighbors are in T, more than fraction $1 - q$ are out. That is T^c has density greater than $1 - q$.

General Graphs: Clusters Stop Cascades

Claim: If a set $S \subset V$ of initial adopters of an innovation with threshold q fails to start a cascade, then there is a cluster $C \in V/S$ of density greater than $1 - q$.

Proof: Suppose the innovation spreads from S to T and then gets stuck. No vertex in T^c wants to switch, so less than a fraction q of its neighbors are in T, more than fraction $1 - q$ are out. That is T^c has density greater than $1 - q$.

Networks and Optimality

- Networks make it easier for cascades to take place.
  - In the fully connected graph, a cascade from a small group never takes place. With stochastic adjustment in the mistakes model, the probability of transiting from all A to all B is $O(\epsilon^{pN})$, where p is the indifference threshold. On a network, the probability of transiting from all A to all B is on the order of ϵ^{K}, where K is the size of a group needed to start a cascade, and thiss is independent of N.

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- This is not always optimal!
  - Risk dominance and Pareto dominance can be different. This can be understood as a robustness question. If the population has correlated on the efficient action, how easy is it to undo? Hard if the efficient action is risk dominant. Easy on networks, but harder in nearly completely connected networks.

Community Structure

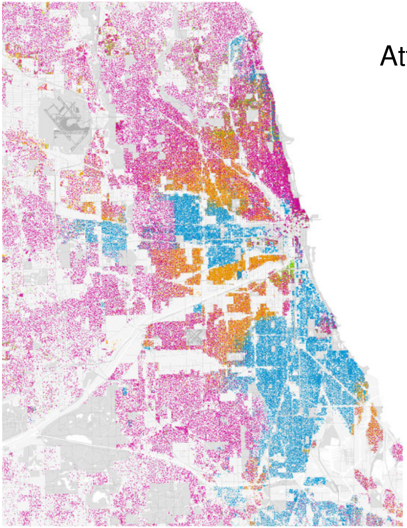
Under Construction

Two Problems

Imagine a social network, such as a friendship network in a school or network of information sharing in a village. Suppose the network participants represent several ethnic groups, races or tribes.

- How “integrated” is the network with respect to predefined communities?
- What are the implicit “comunities” of highly mutually interactive neighbors?
- How do these community structures map onto each other?

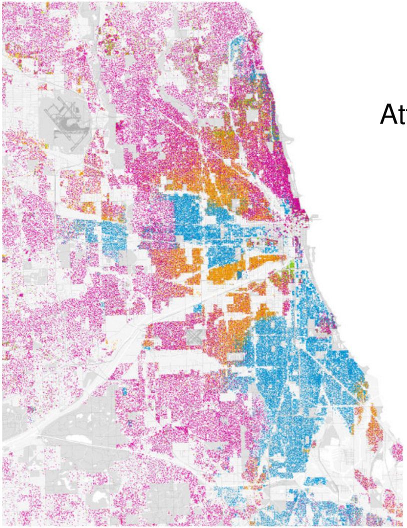
Measuring Segregation



Attributes of physical segregation.

- Evenness — Differential distribution of two groups across the network.
- Exposure — The degree to which different groups are in contact.
- Concentration — Relative concentration of physical space occupied by different groups.

Measuring Segregation



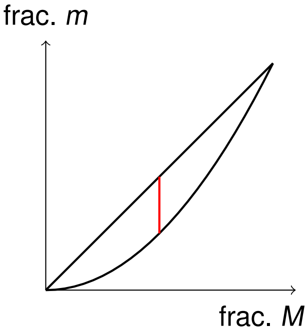
Attributes of physical segregation.

- Centraliztion — Extent to which a group is near the center.
- Clustering — Degree to which group members are connected to others in the group.

Dissimilarity Index

A city is divided into N areas. Area i has minority population m_i and majority population M_i. Total populations are m and M, respectively.

$$\text{dissimilarity index} = \frac{1}{2}\sum_{i=1}^{N}\left|\frac{m_i}{m} - \frac{M_i}{M}\right|.$$



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