Online Appendix for "Child skill production: Accounting for parental and market-based time and goods investments"

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A Analytical Issues

A.1 Separating the household's problem into an intratemporal and intertemporal problem

Full problem

The household's problem for periods $t = 1, ..., T$, is given by:

$$\begin{aligned} & V_t(\theta, H_m, H_f, A_t, y_t, \Pi_t, \Psi_t) \\ & = \max_{l_{m,t}, \tau_{m,t}, l_{f,t}, \tau_{f,t}, g_t, Y_{c,t}, A_{t+1}} u(c_t) + v_m(l_{m,t}) + v_f(l_{f,t}) + \beta V_{t+1}(\theta, H_m, H_f, A_{t+1}, y_{t+1}, \Pi_{t+1}, \Psi_{t+1}) \end{aligned}$$

subject to non-negative inputs $(\tau_{m,t}, \tau_{f,t}, g_t, Y_{c,t})$, $l_{j,t} \geq 0$ and $l_{j,t} + \tau_{j,t} \leq 1$ for $j = m, f$, child human capital production equation (1),

$$\begin{aligned} c_t + p_t g_t + W_{m,t}\tau_{m,t} + W_{f,t}\tau_{f,t} + P_{c,t}Y_{c,t} + A_{t+1} &= (1+r)A_t + y_t + W_{m,t}(1 - l_{m,t}) + W_{f,t}(1 - l_{f,t}), \\ A_{t+1} &\geq A_{min,t}, \\ V_{T+1}(\theta, H_m, H_f, A_{T+1}, y_{T+1}, \Pi_{T+1}, \Psi_{T+1}) &= \tilde{U}(H_m, H_f, A_{T+1}) + \tilde{V}(\Psi_{T+1}). \end{aligned}$$

We assume $u'(\cdot) > 0$, $u''(\cdot) < 0$, $v_j'(\cdot) > 0$, and $v_j''(\cdot) \leq 0$, $j = m, f$. We also assume standard Inada conditions for preferences over consumption and leisure.

Suppose both parents work in the market, $l_{j,t} + \tau_{j,t} < 1$. Let λ_t be the Lagrange multiplier on the period t budget constraint and ξ_t be the Lagrange multiplier on the period t borrowing constraint. The first order conditions for c_t, $\tau_{j,t}$, g_t, $Y_{c,t}$, $l_{j,t}$, A_{t+1}, are:

$$\lambda_t = u'(c_t), \tag{45}$$

$$\lambda_t W_{j,t} = \beta \frac{\partial V_{t+1}}{\partial \Psi_{t+1}} \frac{\partial \mathcal{H}_t}{\partial f_t} \frac{\partial f_t}{\partial \tau_{j,t}}, \tag{46}$$

$$\lambda_t p_t = \beta \frac{\partial V_{t+1}}{\partial \Psi_{t+1}} \frac{\partial \mathcal{H}_t}{\partial f_t} \frac{\partial f_t}{\partial g_t}, \tag{47}$$

$$\lambda_t P_{c,t} = \beta \frac{\partial V_{t+1}}{\partial \Psi_{t+1}} \frac{\partial \mathcal{H}_t}{\partial f_t} \frac{\partial f_t}{\partial Y_{c,t}}, \tag{48}$$

$$v_j'(l_{j,t}) = \lambda_t W_{j,t}, \tag{49}$$

$$\lambda_t + \xi_t = \lambda_{t+1}\beta(1+r). \tag{50}$$

We also have:

$$\lambda_t(c_t + p_t g_t + P_{c,t}Y_{c,t} + A_{t+1} - (1+r)A_t - y_t - W_{m,t}(1 - l_{m,t} - \tau_{m,t}) - W_{f,t}(1 - l_{f,t} - \tau_{f,t})) = 0, \tag{51}$$

$$\xi_t(A_{t+1} - A_{min,t}) = 0. \tag{52}$$

Note that if a parent does not work, the cost of child time investment is measured by the value of lost leisure, and $v_j'(l_{j,t}) = \beta \frac{\partial V_{t+1}}{\partial \Psi_{t+1}} \frac{\partial \mathcal{H}_t}{\partial f_t} \frac{\partial f_t}{\partial \tau_{j,t}}$.

Intratemporal problem

The intratemporal problem is to minimize expenditures, given X_t:

$$\min_{g_t,\tau_{m,t},\tau_{f,t},Y_{c,t}} p_t g_t + P_{c,t}Y_{c,t} + W_{m,t}\tau_{m,t} + W_{f,t}\tau_{f,t}$$

subject to non-negative inputs $(\tau_{m,t}, \tau_{f,t}, g_t, Y_{c,t})$, $\tau_{m,t} \leq 1$, $\tau_{f,t} \leq 1$, and $X_t = f_t(\tau_{m,t}, \tau_{f,t}, g_t, Y_{c,t}; H_m, H_f)$. Let $\bar{p}_t$ be the Lagrange multiplier on this constraint. The first order conditions for g_t, $\tau_{j,t}$, and $Y_{c,t}$, are:

$$\begin{aligned} p_t &= \bar{p}_t \frac{\partial f_t}{\partial g_t}, \\ W_{j,t} &= \bar{p}_t \frac{\partial f_t}{\partial \tau_{j,t}}, \\ P_{c,t} &= \bar{p}_t \frac{\partial f_t}{\partial Y_{c,t}}. \end{aligned}$$

Substitute these first order conditions into the minimand:

$$E_t = \bar{p}_t \left[g_t \frac{\partial f_t}{\partial g_t} + Y_{c,t}\frac{\partial f_t}{\partial Y_{c,t}} + \tau_{m,t}\frac{\partial f_t}{\partial \tau_{m,t}} + +\tau_{f,t}\frac{\partial f_t}{\partial \tau_{f,t}} \right].$$

Because $f_t(\tau_{m,t}, \tau_{f,t}, g_t, Y_{c,t})$ is homogenous of degree 1 (Constant Returns to Scale), we have:

$$X_t = f_t(\tau_{m,t}, \tau_{f,t}, g_t, Y_{c,t}) = \frac{\partial f_t}{\partial g_t} g_t + \frac{\partial f_t}{\partial \tau_{m,t}}\tau_{m,t} + \frac{\partial f_t}{\partial \tau_{f,t}}\tau_{f,t} + \frac{\partial f_t}{\partial Y_{c,t}} Y_{c,t},$$

and, $E_t = \bar{p}_t X_t$.

Intertemporal problem

Suppose in every period, $t = 1, ..., T$, along with leisure and assets, the household chooses an amount of child investment X_t, given a per period composite price $\bar{p}_t$. This problem can be written as follows:

$$V_t(\theta, H_m, H_f, A_t, y_t, \Pi_t, \Psi_t) = \max_{l_{m,t}, l_{f,t}, X_t, A_{t+1}} u(c_t) + v(l_{m,t}) + v(l_{f,t}) + \beta \left[V_{t+1}(\theta, H_m, H_f, A_{t+1}, y_{t+1}, \Pi_{t+1}, \Psi_{t+1}) \right]$$

subject to $0 \leq l_{m,t}, l_{f,t} \leq 1$, $X_t \geq 0$,

$$\begin{aligned} c_t + \bar{p}_t(\Pi_t, H_m, H_f)X_t + A_{t+1} &= (1+r)A_t + y_t + W_{m,t}(1 - l_{m,t}) + W_{f,t}(1 - l_{f,t}), \\ \Psi_{t+1} &= \mathcal{H}_t\left(X_t, \theta, \Psi_t\right), \\ A_{t+1} &\geq A_{min,t}, \\ V_{T+1}(\theta, H_m, H_f, A_{T+1}, y_{T+1}, \Pi_{T+1}, \Psi_{T+1}) &= \tilde{U}(H_m, H_f, A_{T+1}) + \tilde{V}(\Psi_{T+1}). \end{aligned}$$

The first order conditions for c_t, $l_{j,t}$, X_t, A_{t+1}, are:

$$\lambda_t = u'(c_t), \tag{53}$$

$$v_j'(l_{j,t}) = \lambda_t W_{j,t}, \tag{54}$$

$$\lambda_t \bar{p}_t = \beta \frac{\partial V_{t+1}}{\partial \Psi_{t+1}} \frac{\partial \mathcal{H}_t}{\partial f_t}, \tag{55}$$

$$\lambda_t + \xi_t = \lambda_{t+1}\beta(1+r). \tag{56}$$

We also have:

$$\lambda_t(c_t + \bar{p}_t(\Pi_t, H_m, H_f)X_t + A_{t+1} - (1+r)A_t - y_t - W_{m,t}(1-l_{m,t}) - W_{f,t}(1-l_{f,t})) = 0, \quad (57)$$

$$\xi_t(A_{t+1} - A_{min,t}) = 0. \quad (58)$$

Comparing first order conditions, we see the separated problem has first order Conditions (53), (54), (56), and (58) corresponding to the full problem Conditions (45), (49), (50), and (52). If we substitute $\bar{p}_t X_t = p_t g_t + P_{c,t}Y_{c,t} + W_{m,t}\tau_{m,t} + W_{f,t}\tau_{f,t}$, into Condition (57), we have Condition (51).

Take Condition (55) and multiply through by $X_t = f_t$:

$$\lambda_t \bar{p}_t X_t = \beta \frac{\partial V_{t+1}}{\partial \Psi_{t+1}} \frac{\partial \mathcal{H}_t}{\partial f_t} f_t.$$

Substitute in for $\bar{p}_t X_t = p_t g_t + P_{c,t}Y_{c,t} + W_{m,t}\tau_{m,t} + W_{f,t}\tau_{f,t}$:

$$\lambda_t[p_t g_t + P_{c,t}Y_{c,t} + W_{m,t}\tau_{m,t} + W_{f,t}\tau_{f,t}] = \beta \frac{\partial V_{t+1}}{\partial \Psi_{t+1}} \frac{\partial \mathcal{H}_t}{\partial f_t} f_t.$$

Because $f_t(\tau_{m,t}, \tau_{f,t}, g_t, Y_{c,t})$ is homogenous of degree 1, we have:

$$f_t(\tau_{m,t}, \tau_{f,t}, g_t, Y_{c,t}) = \frac{\partial f_t}{\partial g_t} g_t + \frac{\partial f_t}{\partial \tau_{m,t}} \tau_{m,t} + \frac{\partial f_t}{\partial \tau_{f,t}} \tau_{f,t} + \frac{\partial f_t}{\partial Y_{c,t}} Y_{c,t}.$$

Condition (55) becomes:

$$g_t \left[\beta \frac{\partial V_{t+1}}{\partial \Psi_{t+1}} \frac{\partial \mathcal{H}_t}{\partial f_t} \frac{\partial f_t}{\partial g_t} - \lambda_t p_t\right] + \tau_{m,t} \left[\beta \frac{\partial V_{t+1}}{\partial \Psi_{t+1}} \frac{\partial \mathcal{H}_t}{\partial f_t} \frac{\partial f_t}{\partial \tau_{m,t}} - \lambda_t W_{m,t}\right] +$$

$$\tau_{f,t} \left[\beta \frac{\partial V_{t+1}}{\partial \Psi_{t+1}} \frac{\partial \mathcal{H}_t}{\partial f_t} \frac{\partial f_t}{\partial \tau_{f,t}} - \lambda_t W_{f,t}\right] + Y_{c,t} \left[\beta \frac{\partial V_{t+1}}{\partial \Psi_{t+1}} \frac{\partial \mathcal{H}_t}{\partial f_t} \frac{\partial f_t}{\partial Y_{c,t}} - \lambda_t P_{c,t}\right] = 0,$$

and implies Conditions (46) ($j = m, f$), (47), and (48).

A.2 Expenditure shares

Throughout this subsection of the Appendix, define $D \equiv p + P_c\Phi_c + wH_m\Phi_m$.

Proof of Proposition 1

We can differentiate shares with respect to P_c:

$$\frac{\partial S_g}{\partial P_c} = \frac{\gamma p \Phi_c}{(1-\gamma)D^2}, \qquad \frac{\partial S_\tau}{\partial P_c} = \frac{\gamma w H_m \Phi_m \Phi_c}{(1-\gamma)D^2}, \qquad \frac{\partial S_{Yc}}{\partial P_c} = \frac{-\gamma[pg + wH_m\tau]\Phi_c}{(1-\gamma)D^2}.$$

The stated results in Proposition 1 are immediate from these derivatives.

Proof of Proposition 2

We can differentiate expenditure shares with respect to p:

$$\begin{aligned}
\frac{\partial S_g}{\partial p} &= \frac{-\left\{\rho(1-\gamma)[P_c\Phi_c a_m\Phi_m^\rho + wH_m\Phi_m(a_m\Phi_m^\rho + a_g)] + \gamma(1-\rho)P_c\Phi_c a_g\right\}}{(1-\gamma)(1-\rho)[a_m\Phi_m^\rho + a_g]D^2} \\
\frac{\partial S_\tau}{\partial p} &= \frac{wH_m\Phi_m\left\{p\rho(1-\gamma)[a_m\Phi_m^\rho + a_g] + P_c\Phi_c(\rho-\gamma)a_g\right\}}{p(1-\rho)(1-\gamma)[a_m\Phi_m^\rho + a_g]D^2} \\
\frac{\partial S_{Yc}}{\partial p} &= \frac{\gamma P_c\Phi_c a_g\left\{p + wH_m\Phi_m\right\}}{p(1-\gamma)[a_m\Phi_m^\rho + a_g]D^2},
\end{aligned}$$

and with respect to P_c:

$$\begin{aligned}
\frac{\partial S_g}{\partial w} &= \frac{p\left\{P_c\Phi_c(\rho-\gamma)a_m\Phi_m^\rho + \rho wH_m\Phi_m(1-\gamma)[a_m\Phi_m^\rho + a_g]\right\}}{D^2 w(1-\gamma)(1-\rho)[a_m\Phi_m^\rho + a_g]} \\
\frac{\partial S_\tau}{\partial w} &= -\frac{H_m\Phi_m\left\{p\rho(1-\gamma)[a_m\Phi_m^\rho + a_g] + P_c\Phi_c[\gamma(1-\rho)a_m\Phi_m^\rho + \rho(1-\gamma)a_g]\right\}}{(1-\rho)(1-\gamma)[a_m\Phi_m^\rho + a_g]D^2} \\
\frac{\partial S_{Yc}}{\partial w} &= \frac{\gamma P_c p\Phi_c a_m\Phi_m^\rho}{a_g w(1-\gamma)D^2}.
\end{aligned}$$

The stated results in Proposition 2 are immediate from these derivatives.

Proof of Proposition 3

Differentiating D with respect to H_m yields:

$$\frac{\partial D}{\partial H_m} = \frac{P_c\Phi_c[a_m\Phi_m^\rho((\gamma-\rho)(1-\bar{\varphi}_m) + \rho(\gamma-1)\bar{\varphi}_g) + a_g(\rho-1)\gamma\bar{\varphi}_g] + wH_m\Phi_m\rho(\gamma-1)(1-\bar{\varphi}_m+\bar{\varphi}_g)[a_m\Phi_m^\rho + a_g]}{(1-\gamma)(1-\rho)H_m[a_m\Phi_m^\rho + a_g]}.$$

Using this, we have

$$\begin{aligned}
\frac{\partial S_g}{\partial H_m} &= \frac{-p\frac{\partial D}{\partial H_m}}{D^2} \\
\frac{\partial S_\tau}{\partial H_m} &= \frac{w\Phi_m p\rho(\gamma-1)(1-\bar{\varphi}_m+\bar{\varphi}_g)[a_m\Phi_m^\rho + a_g]}{(1-\gamma)(1-\rho)[a_m\Phi_m^\rho + a_g]D^2} \\
&\quad + \frac{w\Phi_m P_c\Phi_c\left(\gamma(\rho-1)(1-\bar{\varphi}_m)a_m\Phi_m^\rho + (\bar{\varphi}_g(\gamma-\rho) + \rho(\gamma-1)(1-\bar{\varphi}_m))a_g\right)}{(1-\gamma)(1-\rho)[a_m\Phi_m^\rho + a_g]D^2} \\
\frac{\partial S_{Yc}}{\partial H_m} &= \frac{\gamma P_c\Phi_c[pa_m\Phi_m^\rho(1-\bar{\varphi}_m-\bar{\varphi}_g) - pa_g\bar{\varphi}_g + wH_m\Phi_m a_m\Phi_m^\rho(1-\bar{\varphi}_m)]}{(1-\gamma)H_m[a_m\Phi_m^\rho + a_g]D^2}.
\end{aligned}$$

The stated results in Proposition 3 are immediate from these derivatives.

A.3 Intertemporal problem

We note that Proposition 4 is immediate based on the text preceding the result.

Proof of Proposition 5

Use the implicit function theorem and differentiate Equation (23) with respect to prices, non-labor income, and maternal human capital to determine how consumption adjusts. Let π generically reflect these parameters, so:

$$\frac{\partial c}{\partial \pi} = \frac{\sum_{j=0}^{T-t}(1+r)^{-j}\left[\left(1-\psi_m^{1/\nu}c^{\sigma/\nu}\left(1-\frac{1}{\nu}\right)W_{m,t+j}^{-1/\nu}\right)\frac{\partial W_{m,t+j}}{\partial \pi}+\left(1-\psi_f^{1/\nu}c^{\sigma/\nu}\left(1-\frac{1}{\nu}\right)W_{f,t+j}^{-1/\nu}\right)\frac{\partial W_{f,t+j}}{\partial \pi}\right]}{\Upsilon_{T-t}+\sum_{j=0}^{T-t}(1+r)^{-j}\psi_m^{1/\nu}W_{m,t+j}^{(\nu-1)/\nu}\left(\frac{\sigma}{\nu}\right)c^{(\sigma-\nu)/\nu}+\bar{K}_t\sigma c^{\sigma-1}-(1+r)^{-(T-t)}\frac{\sigma c^{-\sigma-1}}{\beta\Delta'(\Delta^{-1}(\beta^{-1}c^{-\sigma}))}}.$$

$$+\frac{(1+r)^{-(T-t)}\left[D_m\frac{\partial H_m}{\partial \pi}+D_f\frac{\partial H_f}{\partial \pi}\right]+\sum_{j=0}^{T-t}(1+r)^{-j}\frac{\partial y_{t+j}}{\partial \pi}}{\Upsilon_{T-t}+\sum_{j=0}^{T-t}(1+r)^{-j}\psi_m^{1/\nu}W_{m,t+j}^{(\nu-1)/\nu}\left(\frac{\sigma}{\nu}\right)c^{(\sigma-\nu)/\nu}+\bar{K}_t\sigma c^{\sigma-1}-(1+r)^{-(T-t)}\frac{\sigma c^{-\sigma-1}}{\beta\Delta'(\Delta^{-1}(\beta^{-1}c^{-\sigma}))}}.$$

The denominator is strictly positive, because $\sigma > 0$, $\nu > 0$, $\bar{K}_t > 0$, and $\Delta'(\cdot) < 0$. Furthermore, the first order condition for leisure implies $l_{j,t} = \psi_j^{1/\nu}W_{j,t}^{-1/\nu}c^{\sigma/\nu} < 1$, so the numerator terms $\left(1-\psi_j^{1/\nu}c^{\sigma/\nu}\left(1-\frac{1}{\nu}\right)W_{j,t+j}^{-1/\nu}\right)$ are strictly positive.

Thus, consumption is strictly increasing in current and future non-labor income, current and future skill prices, and parental human capital, while it is independent of (current and future) prices for home investment goods and child care services.

Equation (19) implies that $\partial E_t/\partial\pi = K_t\sigma c^{\sigma-1}(\partial c_t/\partial\pi)$ (for π reflecting non-labor income, prices, and parental human capital), which implies the results of Proposition 5.

A.4 Levels

In this subsection of the appendix, we discuss comparative statics results for input levels. The solution for goods investment when families are borrowing constrained is

$$g_t = \left(\frac{(1+r)A_t + y_t - A_{min,t} + W_{m,t}}{p_t + P_{c,t}\Phi_{c,t} + W_{m,t}\Phi_{m,t}}\right)\left(\frac{K_t}{1+\psi_m+K_t}\right).$$

When unconstrained, the solution is

$$g_t = \left(\frac{(1+r)A_t + \sum_{j=0}^{T-t}(1+r)^{-j}\left[W_{m,t+j}+y_{t+j}\right]+(1+r)^{t-T}D_mH_m}{p_t + P_{c,t}\Phi_{c,t} + W_{m,t}\Phi_{m,t}}\right)\left(\frac{K_t}{(1+\psi_m)\Upsilon_{T-t}+(1+r)^{t-T}\beta D_0+\bar{K}_t}\right).$$

As noted in the text, in both cases $\tau_{m,t} = \Phi_{m,t}g_t$ and $Y_{c,t} = \Phi_{c,t}g_t$.

For our comparative statics analysis below, it is useful to write the problem in a general way such that our results apply equally to both the constrained and unconstrained cases. To that end, we can write g_t in the following general form:

$$g_t = \tilde{K}_t\left(\frac{\tilde{\Omega}_t + \bar{W}_tH_m}{p_t + P_{c,t}\Phi_{c,t} + w_{m,t}H_m\Phi_{m,t}}\right), \tag{59}$$

where we continue to define $D_t \equiv p_t + P_{c,t}\Phi_{c,t} + W_{m,t}\Phi_{m,t}$ (a function of all input prices and H_m). The constant $\tilde{K}_t > 0$ depends on whether constraints are binding or not:

$$\tilde{K}_t = \begin{cases} \frac{K_t}{1+\psi_m+K_t} & \text{if borrowing constrained} \\ \frac{K_t}{(1+\psi_m)\Upsilon_{T-t}+(1+r)^{t-T}\beta D_0+\bar{K}_t} & \text{if always unconstrained.} \end{cases}$$

The terms collected into $\tilde{\Omega}_t$ and $\bar{W}_t$ will depend on the particular proposition and constrained vs. unconstrained case as discussed below.

Proof of Proposition 6

Here, we consider the effects of changes in $w_{m,t}$ on g_t, $\tau_{m,t}$, and $Y_{c,t}$. We define the $\tilde{\Omega}_t$ and $\bar{W}_t$ terms in Equation (59) as follows:

$$\tilde{\Omega}_t = \begin{cases} (1+r)A_t + y_t - A_{min,t} & \text{if borrowing constrained} \\ (1+r)A_t + \sum_{j=0}^{T-t}(1+r)^{-j}y_{t+j} + (1+r)^{t-T}D_m H_m + \sum_{j=1}^{T-t}(1+r)^{-j}W_{m,t+j} & \text{if always unconstrained.} \end{cases}$$

and $\bar{W}_t = w_{m,t} > 0$ in both the constrained and always unconstrained cases. Here, $\tilde{\Omega}_t$ reflects all currently available resources not earned from current work and is independent of the prices we consider varying here. As discussed in the text, we assume conditions that ensure $\tilde{\Omega}_t \geq 0$. Here, the conditions are extremely weak in that they only require that the vale of current debt not exceed the present discounted value of all future income (from all sources, including returns on human capital beyond year T).

We now differentiate g_t in Equation (59) with respect to $w_{m,t}$:

$$\frac{\partial g_t}{\partial w_{m,t}} = \tilde{K}_t \left(\frac{D_t H_m - (\tilde{\Omega}_t + w_{m,t}H_m)D_t'}{D_t^2} \right),$$

where D_t' is the derivative of D_t with respect to $w_{m,t}$. Because $D_t H_m > 0$ and $\tilde{\Omega}_t + w_{m,t}H_m \geq 0$, the numerator is strictly positive if $D_t' \leq 0$. Notice

$$D_t' = \frac{(\gamma-\rho)P_{c,t}\Phi_{c,t}a_m\Phi_{m,t}^{\rho}}{w_{m,t}(1-\gamma)(1-\rho)\left[a_m\Phi_{m,t}^{\rho} + a_g\right]} - \frac{\rho H_m \Phi_{m,t}}{1-\rho},$$

which is weakly negative if $\rho \geq \max\{0,\gamma\}$. Therefore, $\frac{\partial g_t}{\partial w_{m,t}} > 0$ if $\rho \geq \max\{0,\gamma\}$, as stated in Proposition 6.

Next, consider the effects of $w_{m,t}$ on $\tau_{m,t}$:

$$\begin{aligned} \frac{\partial \tau_{m,t}}{\partial w_{m,t}} &= \frac{\partial \Phi_{m,t}}{\partial w_{m,t}} g_t + \frac{\partial g_t}{\partial w_{m,t}} \Phi_{m,t} \\ &= \frac{\Phi_{m,t}\tilde{K}_t}{(1-\rho)w_{m,t}D_t^2} \left\{ \tilde{\Omega}_t[w_{m,t}(\rho-1)D_t' - D_t] + w_{m,t}H_m[\rho(D_t'w_{m,t} - D_t) - D_t'w_{m,t}] \right\}. \end{aligned}$$

We sign $[w_{m,t}(\rho-1)D_t' - D_t]$ and $[\rho(D_t'w_{m,t} - D_t) - D_t'w_{m,t}]$ separately. First,

$$w_{m,t}(\rho-1)D_t' - D_t =$$

$$\frac{p_t(1-\gamma)\left[a_m\Phi^\rho_{m,t}+a_g\right]+P_{c,t}\Phi_{c,t}[(1-\rho)a_m\Phi^\rho_{m,t}+(1-\gamma)a_g]+w_{m,t}H_m\Phi_{m,t}(1-\rho)(1-\gamma)\left[a_m\Phi^\rho_{m,t}+a_g\right]}{(\gamma-1)\left[a_m\Phi^\rho_{m,t}+a_g\right]}<0.$$

Because $\tilde{\Omega}_t \geq 0$, we have $\tilde{\Omega}_t[w_{m,t}(\rho-1)D'_t - D_t] \leq 0$. Next,

$$\rho(D'_t w_{m,t}-D_t)-D'_t w_{m,t}=\frac{\rho p_t(1-\gamma)\left[a_m\Phi^\rho_{m,t}+a_g\right]+P_{c,t}\Phi_{c,t}[\gamma(1-\rho)a_m\Phi^\rho_{m,t}+\rho(1-\gamma)a_g]}{(\gamma-1)\left[a_m\Phi^\rho_{m,t}+a_g\right]},$$

which is strictly negative if $\min\{\gamma,\rho\}>0$. Therefore, $\frac{\partial \tau_t}{\partial w_{m,t}}<0$ if $\min\{\gamma,\rho\}>0$ as stated in Proposition 6.

Finally, consider the effects of $w_{m,t}$ on $Y_{c,t}$:

$$\frac{\partial Y_{c,t}}{\partial w_{m,t}}=\frac{\Phi_{c,t}\tilde{K}_t\left\{\tilde{\Omega}_t\Theta_{1,t}+w_{m,t}H_m\Theta_{2,t}\right\}}{w_{m,t}(1-\gamma)(1-\rho)\left[a_m\Phi^\rho_{m,t}+a_g\right]D_t^2},$$

where

$$\begin{aligned}\Theta_{1,t} &= \gamma(1-\rho)a_m\Phi^\rho_{m,t}[p_t+w_{m,t}H_m w_{m,t}\Phi_{m,t}]\\ \Theta_{2,t} &= (1-\rho)\left\{a_m\Phi^\rho_{m,t}[p_t+(1-\gamma)P_{c,t}\Phi_{c,t}+w_{m,t}H_m\Phi_{m,t}]+(1-\gamma)a_g[p_t+P_{c,t}\Phi_{c,t}+w_{m,t}H_m\Phi_{m,t}]\right\}>0.\end{aligned}$$

Clearly, $\frac{\partial Y_{c,t}}{\partial w_{m,t}}>0$ when $\gamma \geq 0$ as stated in Proposition 6. Also note that if $\tilde{\Omega}_t=0$ (e.g. no non-labor income and no borrowing/saving), then $\frac{\partial Y_{c,t}}{\partial w_{m,t}}>0$ holds regardless of γ.

Permanent changes in $w_{m,t}$

When there are permanent changes in maternal wages, the impacts are equivalent to only a current change in $w_{m,t}$ when the family is constrained. This is not the case when families are always unconstrained; however, the qualitative results are the same.

In considering the effects of permanent changes in wages for always unconstrained families, define $w_{m,t}=\tilde{w}_{mt}\bar{w}_m$ where $\bar{w}_m$ reflects the permanent component of wages. We now define $\tilde{\Omega}_t$ so that it no longer includes future labor earnings:

$$\tilde{\Omega}_t=(1+r)A_t+\sum_{j=0}^{T-t}(1+r)^{-j}y_{t+j}+(1+r)^{t-T}D_mH_m \geq 0,$$

where the conditions on debt that ensure $\tilde{\Omega}_t \geq 0$ are now stronger than before. (For married couples, $\tilde{\Omega}_t$ would also include the discounted present value of all spousal wages.) All maternal earnings are now included in $\bar{W}_{m,t}=\sum_{j=0}^{T-t}(1+r)^{-j}w_{m,t+j}>0$. Based on these definitions and Equation (59), the same approach as above shows that all qualitative properties in Proposition 6 apply to permanent changes in wages, $\bar{w}_m$.

Proofs of Propositions 7 and 8

In Propositions 7 and 8, we study the effects of H_m on input choices. Here, we continue to use the same family resource decomposition as above for constrained families: $\tilde{\Omega}_t=(1+r)A_t+y_t-A_{min,t}\geq 0$

and $\bar{W}_{m,t} = w_{m,t}$. For always unconstrained families, we decompose resources into those related and unrelated to mother's human capital as follows:

$$\begin{aligned}\tilde{\Omega}_t &= (1+r)A_t + \sum_{j=0}^{T-t}(1+r)^{-j}y_{t+j} \geq 0 \\ \bar{W}_{m,t} &= (1+r)^{t-T}D_m + \sum_{j=0}^{T-t}(1+r)^{-j}w_{m,t+j} > 0,\end{aligned}$$

where $\tilde{\Omega}_t \geq 0$ now requires our strongest condition on the value of debt (i.e., it cannot exceed the discounted value of all non-labor income). Again, for married couples, $\tilde{\Omega}_t$ would also include the discounted present value of all spousal wages, substantially weakening the condition on debt. The expression $\bar{W}_{m,t}$ corresponds to returns to human capital relevant for the investment decision at time t. For constrained families, it only includes current labor returns, while for unconstrained families, it contains current and all future returns (including the continuation value that depends on maternal human capital).

We denote the derivative of D_t with respect to maternal human capital by $D'_t = P_{c,t}\frac{\partial \Phi_{c,t}}{\partial H_m} + w_{m,t}\Phi_{m,t} + w_{m,t}H_m\frac{\partial \Phi_{m,t}}{\partial H_m}$. Consider the effects of changes in H_m on g_t by differentiating Equation (59):

$$\frac{\partial g_t}{\partial H_m} = \tilde{K}_t\left(\frac{D_t\bar{W}_{m,t} - (\tilde{\Omega}_t + \bar{W}_{m,t}H_m)D'_t}{D_t^2}\right),$$

which is positive if $D'_t \leq 0$. Notice

$$D'_t = \frac{P_{c,t}\Phi_{c,t}\left\{a_m\Phi_{m,t}^{\rho}\left[(\gamma-\rho)(1-\bar{\varphi}_m) + \rho(\gamma-1)\bar{\varphi}_g\right] + \bar{\varphi}_g(\rho-1)\gamma a_g\right\} + w_{m,t}H_m\Phi_{m,t}\rho(\gamma-1)(\bar{\varphi}_g + 1 - \bar{\varphi}_m)[a_m\Phi_{m,t}^{\rho} + a_g]}{H_m(1-\rho)(1-\gamma)[a_m\Phi_{m,t}^{\rho} + a_g]}$$

We see that $D'_t \leq 0$, and therefore $\frac{\partial g_t}{\partial H_m} > 0$ if $(\rho-\gamma)(1-\bar{\varphi}_m) + \rho(1-\gamma)\bar{\varphi}_g \geq 0$, $\gamma\bar{\varphi}_g \geq 0$, and $\rho(\bar{\varphi}_g + 1 - \bar{\varphi}_m) \geq 0$.

When $\bar{\varphi}_g = 0$, we have $(\rho-\gamma)(1-\bar{\varphi}_m) \geq 0$ and $\rho(1-\bar{\varphi}_m) \geq 0$ (Proposition 7). And, when $\bar{\varphi}_g > 0$ and $\bar{\varphi}_m = 1$, we have $\rho \geq 0$ and $\gamma \geq 0$ (Proposition 8).

Next, consider maternal time investment:

$$\begin{aligned}\frac{\partial \tau_{m,t}}{\partial H_m} &= \Phi_{m,t}\frac{\partial g_t}{\partial H_m} + \frac{\partial \Phi_{m,t}}{\partial H_m}g_t \\ &= \frac{\Phi_{m,t}\tilde{K}_t}{D_t^2 H_m(1-\rho)}\left[\bar{W}_{m,t}H_m\big(\rho(\bar{\varphi}_m - 1 - \bar{\varphi}_g)D_t + (\rho-1)D'_tH_m\big) + \tilde{\Omega}_t\big((\rho(\bar{\varphi}_m - \bar{\varphi}_g) - 1)D_t + (\rho-1)D'_tH_m\big)\right].\end{aligned}$$

We have two parts of this expression to sign. First:

$$\bar{W}_{m,t}H_m\left\{\rho(\bar{\varphi}_m - 1 - \bar{\varphi}_g)D_t + (\rho-1)D'_tH_m\right\} = \left[\frac{1}{(1-\gamma)[a_m\Phi_{m,t}^{\rho} + a_g]}\right]\Big\{p_t\rho(\bar{\varphi}_m - \bar{\varphi}_g - 1)(1-\gamma)[a_m\Phi_{m,t}^{\rho} + a_g] +$$

$$P_{c,t}\Phi_{c,t}\left[a_m\Phi_{m,t}^{\rho}\gamma(1-\rho)(\bar{\varphi}_m - 1) + a_g[(\gamma-\rho)\bar{\varphi}_g + \rho(1-\gamma)(\bar{\varphi}_m - 1)]\right]\Big\},$$

which is positive when: $\rho(\bar{\varphi}_m - \bar{\varphi}_g - 1) \geq 0$, $\gamma(\bar{\varphi}_m - 1) \geq 0$, and $(\gamma-\rho)\bar{\varphi}_g + \rho(1-\gamma)(\bar{\varphi}_m - 1) \geq 0$. It is negative when: $\rho(\bar{\varphi}_g + 1 - \bar{\varphi}_m) \geq 0$, $\gamma(1-\bar{\varphi}_m) \geq 0$, and $(\rho-\gamma)\bar{\varphi}_g + \rho(1-\gamma)(1-\bar{\varphi}_m) \geq 0$.

Second:

$$\tilde{\Omega}_t \left\{(\rho(\bar{\varphi}_m - \bar{\varphi}_g) - 1)D_t + (\rho - 1)D_t' H_m\right\} = \left[\frac{1}{(1-\gamma)[a_m \Phi_{m,t}^{\rho} + a_g]}\right] \Big\{ w_{m,t} H_m \Phi_{m,t}(\rho-1)(1-\gamma)[a_m \Phi_{m,t}^{\rho} + a_g] +$$

$$p(\rho(\bar{\varphi}_m - \bar{\varphi}_g) - 1)(1-\gamma)[a_m \Phi_{m,t}^{\rho} + a_g] + P_{c,t}\Phi_{c,t}\left[a_m \Phi_{m,t}^{\rho}(1-\rho)(\gamma\bar{\varphi}_m - 1) + a_g[(\gamma-\rho)\bar{\varphi}_g + (1-\gamma)(\rho\bar{\varphi}_m - 1)]\right]\Big\}.$$

Because the first part of the expression in braces $w_{m,t} H_m \Phi_{m,t}(\rho-1)(1-\gamma)[a_m \Phi_{m,t}^{\rho} + a_g] < 0$, there is always a negative force (independent of parameters) impacting the effect of mother's human capital on time investment when $\tilde{\Omega}_t > 0$. We can only give cases where the derivative is (strictly) decreasing in mother's human capital. The entire expression related to $\tilde{\Omega}_t$ is negative when: $(1-\gamma)(1-\rho\bar{\varphi}_m) + \bar{\varphi}_g(\rho - \gamma) \geq 0$, $1 - \gamma\bar{\varphi}_m \geq 0$, and $1 + \rho(\bar{\varphi}_g - \bar{\varphi}_m) \geq 0$.

Altogether, conditions that imply a strictly negative (when $\tilde{\Omega}_t > 0$) impact of maternal human capital on time investment are as follows:

1. $\rho + \rho(\bar{\varphi}_g - \bar{\varphi}_m) \geq 0$,
2. $\gamma - \gamma\bar{\varphi}_m \geq 0$,
3. $(1-\gamma)\rho(1-\bar{\varphi}_m) + \bar{\varphi}_g(\rho - \gamma) \geq 0$,
4. $(1-\gamma)(1-\rho\bar{\varphi}_m) + \bar{\varphi}_g(\rho - \gamma) \geq 0$,
5. $1 - \gamma\bar{\varphi}_m \geq 0$,
6. $1 + \rho(\bar{\varphi}_g - \bar{\varphi}_m) \geq 0$.

Note that condition 1 implies condition 6, condition 2 implies condition 5, and condition 3 implies condition 4. We are left with conditions 1-3. When $\bar{\varphi}_g = 0$, we have $\rho(1-\bar{\varphi}_m) \geq 0$ and $\gamma(1-\bar{\varphi}_m) \geq 0$ (Proposition 7). And, when $\bar{\varphi}_g > 0$ and $\bar{\varphi}_m = 1$, we have $\rho \geq 0$ and $\rho \geq \gamma$ (Proposition 8).

A.5 Effects of a Small Price Change

Here, we derive expressions for the price elasticity of total investment X_t under no borrowing/saving as given by Equation (21). In this case, total investment depends only on input prices Π_t through the composite price of investment $\overline{p}_t(\Pi_t)$.

First, notice that the composite price can be written as the minimum unit cost of production:

$$\overline{p}_t(\Pi_t) = \min_{\tau_{m,t},\tau_{f,t},g_t,Y_{c,t}} \Big\{ W_{m,t}\tau_{m,t} + W_{f,t}\tau_{f,t} + p_t g_t + P_{c,t}Y_{c,t} | f_t(\tau_{m,t}, \tau_{f,t}, g_t, Y_{c,t}) \geq 1 \Big\}.$$

Let $(\underline{\tau}_{m,t}(\Pi_t), \underline{\tau}_{f,t}(\Pi_t), \underline{g}_t(\Pi_t), \underline{Y}_{c,t}(\Pi_t))$ be the solution to this problem. Then, by the envelope theorem, we have

$$\frac{\partial \overline{p}_t(\Pi_t)}{\partial p_t} = \underline{g}_t(\Pi_t).$$

Therefore, the elasticity of X_t with respect to p_t is

$$\frac{\partial \ln X_t}{\partial \ln p_t} = -\frac{\partial \ln \overline{p}_t(\Pi_t)}{\partial \ln p_t} = -\frac{p_t \underline{g}_t(\Pi_t)}{\overline{p}_t(\Pi_t)} = -S_{g,t}(\Pi_t).$$

Similarly, the elasticity of X_t with respect to $P_{c,t}$ is

$$\frac{\partial \ln X_t}{\partial \ln P_{c,t}} = -\frac{\partial \ln \bar{p}_t(\Pi_t)}{\partial \ln P_{c,t}} = -\frac{P_{c,t}\underline{Y}_{ct}(\Pi_t)}{\bar{p}_t(\Pi_t)} = -S_{Y_c,t}(\Pi_t).$$

Elasticities with respect to parental wages (for $y_t = A_t = A_{min,t} = 0$) are given by

$$\frac{\partial \ln X_t}{\partial \ln W_{j,t}} = \frac{W_{j,t}}{W_{m,t}+W_{f,t}} - \frac{\partial \ln \bar{p}_t(\Pi_t)}{\partial \ln W_{j,t}} = \frac{W_{j,t}}{W_{m,t}+W_{f,t}} - \frac{W_{j,t}\,\underline{\tau}_{j,t}(\Pi_t)}{\bar{p}_t(\Pi_t)} = \frac{W_{j,t}}{W_{m,t}+W_{f,t}} - S_{\tau_j,t}(\Pi_t), \quad \text{for } j \in \{m, f\}.$$

These results imply that the elasticity of total investment with respect to price depends only on the expenditure shares and wages as long as the price change is small.

B Additional Data Sources

B.1 Child Care Prices

Child care costs for 4-year old family care (and center-based care), P_c, are obtained from annual reports on the cost of child care in the U.S. compiled by Child Care Aware of America (2009–2019).[59] These costs represent the average annual price charged by full-time family care/center providers in each state covering 2006 to 2018. Several values from annual reports were dropped if they were imputed based on previous survey years or were taken from different sources or subsets of locations.

In order to obtain child care cost measures going back to 1997, we use our data (from 2006–2018) to regress state-year child care costs on state fixed effects, a linear time trend, and average state-year hourly wages for child care workers.[60] Average wages for child care workers are estimated from the 1992–2019 monthly *Current Population Surveys (CPS)*.[61] We then use the estimated coefficients, including the state fixed effects, to impute child care costs back to 1997 (or for any missing observations) using *CPS* average wages for child care workers for each state and year.

Finally, to put child care prices in roughly hourly terms, consistent with our parental wage measures, we divide our child care cost measures by 33 × 52, reflecting an average of 33 hours per week spent in family- or center-based child care among young children of employed mothers (Laughlin 2013).

B.2 Household Input Prices

We obtain state-year measures of household-based goods input prices, p, from a combination of goods and services price series from the *Regional Price Parities by State (RPP)* from the U.S. Bureau of Economic Analysis (BEA) and the *Consumer Price Index (CPI-U)* from the U.S. Bureau of Labor

[59] We are grateful to Kristina Haynie of Child Care Aware of America for providing us with a digital compendium of child care prices from all annual reports. Each year, states report the annual prices that child care providers charge for their services. These reports are provided by Child Care Resource and Referral (CCR&R) agencies in each state. Family care is provided in a home setting for a smaller group of children (usually under 12 children). Center-based child care is provided for a larger group of children in a facility that is outside of a private home.

[60] For the 4-year old family care costs, the estimated coefficient on the linear time trend is 158.99, while the coefficient on average wages for child care workers is 15.47. The state-fixed effects explain most of the variation, and the R^2 statistic for this regression is 0.86.

[61] We restrict our *CPS* sample to workers who are at least 18 years old, report either weekly earnings or an hourly wage, and report an occupation of either child care worker or preschool or kindergarten teacher (2010 occupation classification codes 4600 or 2300). Among workers reporting weekly earnings, an hourly wage is calculated from weekly earnings divided by usual hours worked per week. CPS weights are used to calculate state-year average wages.

Statistics (BLS). The *RPP*'s measure price level differences relative to the U.S. average by state and are available from 2008 to 2017. These indices are divided into several categories: All items; Goods; Services: Rent; and, Other Services.

To create the goods price series by state, we take the U.S. average of the *CPI* for "Commodities" and multiply it by each state's "Goods" *RPP*. This produces price measures by state for 2008–2017. To project back to 1997, we take the regional *CPI* for "Commodities" and use the year-over-year change of this index for each state within its Census region (Northeast, Midwest, South and West), working back from 2008 values. To create the services price series, we follow the same steps, using the "Services: Other" component from the *RPP*'s and the "Services less rent of shelter" index from the CPI. All these prices are year averages using a base year of 2000.

Finally, we use as our household goods input price, p, a weighted average of these goods and services price series, with a weight of 0.3 on services, reflecting the greater share of goods in the bundle of child investment inputs. For example, we use the 2003–18 *Consumer Expenditure Survey (CEX)* to create a comprehensive measure of potential household investments in children that includes expenditures on "goods" and "services" as described in Appendix B.3 and Appendix Table B-1. Based on this comprehensive measure of household investment inputs, we find that families with 1–2 children, both ages 0–12, spend an average of 35% of all household investment dollars on services. Taking a more limited household investment measure closer to that used in our *PSID-CDS* analysis suggests that families spend, on average, 20% on service-related child investments.

B.3 Consumer Expenditure Survey

The *Consumer Expenditure Survey (CEX)* is a rotating panel gathered by the U.S. Bureau of Labor Statistics. It collects detailed information on consumption, income and household's characteristics, and is representative of the U.S. population. The unit of measurement for the survey is given by Consumer Units. These units are broadly defined as members of a household that are related, or two or more persons living together that use their incomes to make joint expenditures decisions. Each unit is interviewed for up to four times during a 12-month period and is asked to report their expenditures on a detailed set of categories for the preceding three months. After completing the four interviews, each consumer unit is replaced.

For each parent, the *CEX* includes information on gender, age, education (less than high school, high school graduate, some college, and college graduates or above), and marital status (married, unmarried partner, or single parent families). In addition, we are able to determine the number of children in the household and the age of each child.

The sample we use runs from 2003 to 2018. We exclude consumer units that do not complete all four interviews and those whose key characteristics are inconsistent over time (i.e., changes in age or race of the reference person, or if the family size changes by more than two members), indicating a likely change in families in the unit. We limit our sample to families with parents ages 18–65, mothers who were ages 16–45 when their youngest child was born, and with only 1–2 children (all age 12 or younger).

We use the Universal Classification Codes (UCCs) for expenditure categories to create our household-level investment measures. Our preferred investment measure is composed of two broad categories: investment in goods and in services. Investment in goods includes expenditures on books (for school or other, magazines, etc.), toys, games, musical instruments, and other learning equipment such as computers and accessories for nonbusiness use. The services measure includes admission fees for recreational activities, fees for recreational lessons and tutoring services. We sum the quarterly expenditures reported by each household (across categories and their four interviews) to obtain annual investment measures, then divide by 52 to create weekly measures. The *CEX* also provides information on child care expenditures, which we also aggregate to the annual level before dividing by 52.

Table B-1 provides a more detailed look at the expenditure categories that compose our household investment measure along with average weekly expenditures within UCC categories.[62] We also report household investment expenditure categories consistent with those collected by the *PSID-CDS*. Altogether, the *PSID-CDS* categories aggregate to a weekly expenditure amount of $585.25, roughly 60% of the spending we include from the *CEX*.

[62]We aggregate a few categories, because some categories split over time.

Table B-1: Household Investment Expenditure Categories and Average Weekly Expenditures in the CEX

UCC	Description	PSID CDS	Average Expenditure (2002 dollars)
	Goods:		**561.75**
590220	-Books through book clubs	X	4.41
590230	-Books not through book clubs	X	43.00
590310	-Magazine or newspaper subscription		17.06
590410	-Magazine or newspaper, single copy		6.38
610110	-Toys, games, arts, crafts, tricycles, and battery powered riders	X	203.71
610120	-Playground equipment	X	10.89
610130	-Musical instruments, supplies, and accessories		26.02
660210	-School books, supplies, equipment for elementary, high school	X	24.36
660310	-Encyclopedia and other sets of reference books	X	0.31
660900, 660901	-School books, supplies, equipment for day care, nursery, preschool.	X	2.63
660902	-School books, supplies, and equipment for other schools	X	1.71
660410	-School books, supplies, equipment for vocational and technical schools	X	0.51
670902	-Other school expenses including rentals	X	47.61
690111	-Computers and computer hardware for nonbusiness use		134.65
690112, 690119, 690120	-Computer software and accessories for non-business use		22.48
690117	-Portable memory		2.88
690118	-Digital book readers	X	10.72
690230	-Business equipment for home use		2.43
	Services:		**421.09**
620211, 620212, 620213, 620214, 620215, 620216	-Admission fees for entertainment activities, including movie, theater, concert, opera or other musical series (single admissions and season tickets)		179.22
620310	-Fees for recreational lessons or other instructions	X	223.87
620904	-Rental and repair of musical instruments, supplies, and accessories		2.56
670903	-Test preparation, tutoring services	X	11.53
690113	-Repair of computer systems for nonbusiness use		3.92
	Total Investment		**982.85**

B.4 American Time Use Survey

The American Time Use Survey (ATUS) is a comprehensive survey of time use in the U.S. and has been administered annually since 2003. The ATUS sample is drawn from the Current Population Surveys (CPS), covering the population of non-institutionalized civilians at least 15 years old. Typical sample sizes have been about 26,000 respondents since 2004 with surveys administered evenly throughout the year. We use sample weights designed to adjust for stratified sampling, non-response, and to get a representative measure for each day of the year.

The survey asks individuals detailed information about all of their activities over the previous day, including who they were with at the time. The survey also collects information about the respondent and household. It can be linked with the CPS data. Our analysis combines data from the 2003–2018 surveys, limiting our sample to parents ages 18–65, in families with mothers ages 16–45 at youngest child's birth, and with only 1–2 children (all age 12 or younger). Because the survey only collects information on the respondent's time allocation, we never observe time spent by both parents in a household.

Our measure of time investment sums all of the time parents report spending with children in each of the following activities (categories based on the 2003 ATUS Activity Lexicon):

<u>(03.01) Caring For and Helping Household Children</u>: (03.01.02) Reading to/with household children; (03.01.03) Playing with household children, not sports; (03.01.04) Arts and crafts with household children; (03.01.05) Playing sports with household children; (03.01.06) Talking with/listening to household children; (03.01.07) Helping/teaching household children (not related to education); (03.01.08) Organization and planning for household children; (03.01.09) Looking after household children (as a primary activity; (03.01.10) Attending household children's events.

<u>(03.02) Activities Related to Household Children's Education</u>: (03.02.01) Homework (household children); (03.02.02) Meetings and School Conferences (household children); (03.02.03) Home schooling of household children.

<u>(03.03) Activities Related to Household Children's Health</u>: (03.03.01) Providing medical care to household children; (03.03.02) Obtaining medical care for household children.

<u>(12.03) Relaxing and Leisure</u>: (12.03.07) Playing games; (12.03.09) Arts and crafts as a hobby.

<u>(12.04) Arts and Entertainment (other than sports)</u>: (12.04.01) Attending performing arts; (12.04.02) Attending museum; (12.04.03) Attending movies/film.

<u>(13.01) Participating in Sports, Exercise, and Recreation</u>: all subcategories.

C Details on Counterfactual Analysis

Our counterfactual analysis assumes that parents have log preferences for consumption and leisure and are borrowing constrained. As shown in Section 3.2.1, these assumptions permit a closed form solution for total investment, Equation (21). We further assume that parents have no non-labor income and cannot borrow or save ($y_t = A_t = A_{min,t} = 0$). Their subjective discounter factor is $\beta = 1/1.02$ and they value their children's achievement at age 13 ($T = 13$). Finally, individuals are endowed with 100 hours per week (5,200 hours per year), which they can use for market work, leisure, or time investment in children.

These assumptions, along with estimated technology parameters and calibrated preference parameters, allow us to simulate investment and achievement for each child in 2002 PSID.

C.1 Calibration of Preference Parameters

The utility weights of the Cobb-Douglas utility function (α, ψ_m, and ψ_f) determine how households allocate their resources between consumption, leisure, and child investment in each period. For example, Equation (21) shows that two-parent households spend a fraction $K_t/(1 + \psi_m + \psi_f + K_t)$ of their full income on total investment in children. Therefore, given prices and technology parameters, the preference parameters can be identified from the levels of parental time spent on market work and child investment. We choose the preference parameters so that the model replicates weekly time use patterns from the 2002 PSID.

Table C-1: Calibration Targets: Weekly Hours of Time Investment and Work

	Mother's Education	
	Non-College	College
A. Single Mothers		
Mother's Time Investment	17.70	22.11
Mother's Hours Worked	35.99	37.62
B. Two-Parent Households		
Mother's Time Investment	18.29	18.75
Mother's Hours Worked	41.13	39.42
Father's Hours Worked	41.56	43.88

Tables C-1 and C-2 show calibration targets and calibrated parameters, separately by marital status and mother's education (non-college vs. college). The calibrated parameters imply that college-educated mothers have a stronger preference for their child's skills (α) compared to non-college-educated mothers. College educated single mothers have a lower value of leisure than their non-college counterparts, while the opposite is true for married mothers. College educated fathers have a lower value of leisure than non-college fathers.

Table C-2: Calibrated Preference Parameters

	Mother's Education	
	Non-College	College
A. Single Mothers		
α	8.00	9.12
ψ_m	1.52	1.28
B. Two-Parent Households		
α	4.98	5.31
ψ_m	0.37	0.45
ψ_f	0.60	0.44

C.2 Monetary Measure of Distortions

We measure the efficiency loss due to price distortions in monetary units.[63] For expositional purposes, we only discuss single mother households.

First, notice that the present discounted utility of single mothers can be written as a constant term plus

$$\sum_{t=1}^{T} \beta^{t-1} \mathcal{U}_t(c_t, l_{m,t}, X_t),$$

where

$$\mathcal{U}_t(c_t, l_{m,t}, X_t) \equiv \ln c_t + \psi_m \ln l_{m,t} + K_t \ln X_t.$$

Because of the no saving/borrowing assumption, the utility maximization problem in each period can be solved separately. The indirect utility function in period t is

$$\mathcal{V}_t(\Pi_t, W_{m,t}) \equiv \max_{c_t, l_{m,t}, X_t} \Big\{ \mathcal{U}_t(c_t, l_{m,t}, X_t) | c_t + W_{m,t} l_{m,t} + \overline{p}_t(\Pi_t) X_t \leq W_{m,t} \Big\}.$$

Let $\hat{c}_t(\Pi_t, W_{m,t})$, $\hat{l}_{mt}(\Pi_t, W_{m,t})$, and $\hat{X}_t(\Pi_t, W_{m,t})$ be the Marshallian demand functions that solve this problem.

Let $\Pi_t^* \equiv (W_{m,t}^*, p_t^*, P_{c,t}^*)$ be the "undistorted" prices that reflect social marginal costs of producing inputs. For given prices Π_t, we define the distortion in the level of consumption, leisure, and total investment as follows:

$$\Big\{ \hat{c}_t(\Pi_t, W_{m,t}) + W_{m,t}^* \hat{l}_{mt}(\Pi_t, W_{m,t}) + \overline{p}_t(\Pi_t^*) \hat{X}_t(\Pi_t, W_{m,t}) \Big\} - \mathcal{E}_t\big(\Pi_t^*, \mathcal{V}_t(\Pi_t, W_{m,t})\big), \tag{60}$$

where $\mathcal{E}_t(\Pi_t, \tilde{u})$ is the expenditure function in period t:

$$\mathcal{E}_t(\Pi_t, \tilde{u}) \equiv \min_{c_t, l_{m,t}, X_t} \Big\{ c_t + W_{m,t} l_{m,t} + \overline{p}_t(\Pi_t) X_t | \mathcal{U}_t(c_t, l_{m,t}, X_t) \geq \tilde{u} \Big\}.$$

[63] This is based on Park (2019), who considers a more general case where prices can depend on quantities.

The term in braces in Equation (60) is total household expenditure evaluated at the undistorted prices Π_t^*. When households face distorted prices $\Pi_t \neq \Pi_t^*$, this expenditure is not necessarily minimized. Therefore, there is a way to deliver the same level of utility $\mathcal{V}_t(\Pi_t, W_{m,t})$ at a lower expenditure, $\mathcal{E}_t\big(\Pi_t^*, \mathcal{V}_t(\Pi_t, W_{m,t})\big)$. The difference between these two expenditures represents efficiency loss due to the deviation of the prices from Π_t^*; it is always non-negative.

Similarly, we define the distortion in relative investment inputs conditional on total investment level as follows:

$$\Big\{W_{m,t}^* \underline{\tau}_{mt}(\Pi_t) + p_t^* \underline{g}_t(\Pi_t) + P_{c,t}^* \underline{Y}_{ct}(\Pi_t) - \overline{p}_t(\Pi_t^*)\Big\}\, \hat{X}_t(\Pi_t, W_{m,t}), \tag{61}$$

where $(\underline{\tau}_{mt}(\Pi_t), \underline{g}_t(\Pi_t), \underline{Y}_{ct}(\Pi_t))$ is the solution to the unit cost minimization problem as defined in Appendix A.5. Notice that this is also always non-negative due to the definition of the composite price.

The total distortion is the sum of (60) and (61):

$$\Big\{\hat{c}_t(\Pi_t, W_{m,t}) + W_{m,t}^* \hat{l}_{mt}(\Pi_t, W_{m,t}) + \big[W_{m,t}^* \underline{\tau}_{mt}(\Pi_t) + p_t^* \underline{g}_t(\Pi_t) + P_{c,t}^* \underline{Y}_{ct}(\Pi_t)\big] \hat{X}_t(\Pi_t, W_{m,t})\Big\} - \mathcal{E}_t\big(\Pi_t^*, \mathcal{V}_t(\Pi_t, W_{m,t})\big).$$

Using the budget constraint $\hat{c}_t(\Pi_t, W_{m,t}) + W_{m,t}\hat{l}_{mt}(\Pi_t, W_{m,t}) + \overline{p}_t(\Pi_t)\hat{X}_t(\Pi_t, W_{m,t}) = W_{m,t}$ and the identity $W_{m,t}^* = \mathcal{E}_t\big(\Pi_t^*, \mathcal{V}_t(\Pi_t^*, W_{m,t}^*)\big)$, the total distortion can be written as

$$\underbrace{\Big\{(W_{m,t}^* - W_{m,t})\hat{l}_{mt}(\Pi_t, W_{m,t}) + \big[(W_{m,t}^* - W_{m,t})\underline{\tau}_{mt}(\Pi_t) + (p_t^* - p_t)\underline{g}_t(\Pi_t) + (P_{c,t}^* - P_{c,t})\underline{Y}_{ct}(\Pi_t)\big]\hat{X}_t(\Pi_t, W_{m,t}) - (W_{m,t}^* - W_{m,t})\Big\}}_{\text{welfare change if given a lump-sum transfer}}$$
$$- \underbrace{\Big\{\mathcal{E}_t\big(\Pi_t^*, \mathcal{V}_t(\Pi_t, W_{m,t})\big) - \mathcal{E}_t\big(\Pi_t^*, \mathcal{V}_t(\Pi_t^*, W_{m,t}^*)\big)\Big\}}_{\text{actual welfare change}}.$$

The first bracketed term is the change in the budget resulting from the price difference between Π_t^* and Π_t, evaluated at the choices made under Π_t. This is the effective transfer received when the price change is induced by taxes or subsidies. If this was given as a lump-sum transfer, individuals would appreciate a welfare gain as if their income was increased by this amount.

The second bracketed term is the equivalent variation (EV), the difference between utilities $\mathcal{V}_t(\Pi_t^*, W_{m,t}^*)$ and $\mathcal{V}_t(\Pi_t, W_{m,t})$ in monetary terms using Π_t^* as the base price. The EV, a commonly used monetary measure of a welfare change, quantifies what income change (at the prices Π_t^*) would be equivalent to the price change in terms of its impact on utility.

Therefore, the total distortion is the difference between the hypothetical welfare change when the amount of transfer is distributed in a lump-sum manner (without affecting prices and individual choices) and the actual welfare change when the same amount is given through manipulated prices. Because the distortion is in monetary units, it is also the maximum amount of money individuals are willing to pay in order to eliminate the price distortion and instead receive a lump-sum transfer equivalent to the change in their budget.

D Additional Results

Table D-1: Child Investment Expenditure Shares by Parental Education for Subsample with Positive Child Care Expenditures (PSID, 2002)

		Mother's Education			
Expenditure Shares	All	HS dropout	HS graduate	Some College	College+
A. Single Mothers					
Mother's time	0.77	0.76	0.66	0.77	0.87
	(0.03)	(0.12)	(0.08)	(0.05)	(0.02)
HH goods	0.07	0.02	0.12	0.06	0.03
	(0.01)	(0.01)	(0.04)	(0.02)	(0.01)
Child care	0.16	0.23	0.22	0.16	0.10
	(0.02)	(0.11)	(0.04)	(0.04)	(0.02)
Sample size	57	2	15	24	16
B. Two-Parent Households					
Mother's time	0.49	0.29	0.45	0.52	0.49
	(0.02)	(0.05)	(0.05)	(0.04)	(0.03)
Father's time	0.37	0.31	0.42	0.35	0.38
	(0.02)	(0.07)	(0.06)	(0.04)	(0.04)
Total Parental time	0.86	0.60	0.88	0.86	0.87
	(0.01)	(0.01)	(0.02)	(0.03)	(0.02)
HH goods	0.04	0.17	0.03	0.05	0.04
	(0.01)	(0.10)	(0.01)	(0.02)	(0.00)
Child care	0.10	0.24	0.10	0.08	0.09
	(0.01)	(0.09)	(0.02)	(0.01)	(0.02)
Sample size	90	3	17	30	40

Notes: Samples restricted to children ages 0–12 from families with only 1 or 2 children ages 0–12, parents ages 18–65, mothers ages 16–45 when youngest child was born, and positive reported spending on child care. Table reports means (std. errors).

Table D-2: Weekly Hours of Child Investment Time by Mother's Education (2003–18 ATUS)

Time with Children (hours)	All	Mother's Education: HS dropout	HS graduate	Some College	College+
A. Single Mothers					
Mother's time	5.25	3.62	5.05	5.40	6.16
	(0.13)	(0.38)	(0.26)	(0.20)	(0.28)
	4,309	*321*	*1,197*	*1,655*	*1,136*
B. Two-Parent Households					
Mother's time	7.25	4.73	6.23	6.56	8.17
	(0.12)	(0.79)	(0.30)	(0.23)	(0.17)
	6,959	*217*	*1,018*	*1,836*	*3,888*
Father's time	6.06	3.28	4.99	5.60	6.86
	(0.13)	(1.56)	(0.31)	(0.25)	(0.18)
	6,026	*167*	*918*	*1,590*	*3,351*

Notes: Samples restricted to families with only 1 or 2 children ages 0–12, parents ages 18–65, mothers ages 16–45 when youngest child was born. Table reports means (std. errors) and *number of obs*.

Table D-3: Predicted probability of work (OLS)

	Single Mothers	Married Mothers	Married Fathers	Both Married Parents
Mother HS grad.	0.1860*	0.1976*	0.0454	0.1578*
	(0.0399)	(0.0412)	(0.0274)	(0.0445)
Mother some coll.	0.2176*	0.2047*	0.0410	0.1702*
	(0.0406)	(0.0426)	(0.0283)	(0.0458)
Mother coll+	0.3036*	0.2722*	0.0645*	0.2310*
	(0.0488)	(0.0445)	(0.0294)	(0.0478)
Mother's age	-0.0041	0.0001	0.0058*	0.0046
	(0.0023)	(0.0027)	(0.0018)	(0.0029)
Mother white	-0.0137	-0.0279	0.0825*	0.0186
	(0.0277)	(0.0202)	(0.0132)	(0.0215)
Num. children	-0.0161	-0.0200	-0.0096	-0.0286
age 0–5 in HH	(0.0449)	(0.0297)	(0.0195)	(0.0317)
Num. children	-0.0145	-0.0020	-0.0131	-0.0041
in HH	(0.0176)	(0.0137)	(0.0090)	(0.0147)
Age of youngest	0.0148	0.0122	-0.0016	0.0113
child in HH	(0.0088)	(0.0065)	(0.0042)	(0.0069)
Child 1 year old	0.1021	0.0272	-0.0299	0.0295
	(0.1169)	(0.1020)	(0.0665)	(0.1077)
Child 2 years old	0.0541	-0.0139	0.0295	0.0114
	(0.1121)	(0.1020)	(0.0666)	(0.1077)
Child 3 years old	0.0346	0.0688	-0.0281	0.0614
	(0.1168)	(0.1039)	(0.0678)	(0.1097)
Child 4 years old	0.2048	0.0294	-0.0381	0.0075
	(0.1157)	(0.1048)	(0.0684)	(0.1108)
Child 5 years old	0.2410*	0.0071	0.0075	-0.0079
	(0.1151)	(0.1052)	(0.0687)	(0.1112)
Child 6 years old	0.2315*	-0.0717	-0.0203	-0.0620
	(0.1126)	(0.1033)	(0.0675)	(0.1092)
Child 7 years old	0.2454*	0.0078	0.0021	-0.0002
	(0.1139)	(0.1048)	(0.0684)	(0.1107)
Child 8 years old	0.1842	0.0329	-0.0016	0.0357
	(0.1161)	(0.1057)	(0.0690)	(0.1117)
Child 9 years old	0.2161	-0.0099	-0.0092	-0.0059
	(0.1175)	(0.1064)	(0.0695)	(0.1125)
Child 10 years old	0.2439*	-0.0225	-0.0236	-0.0387
	(0.1206)	(0.1090)	(0.0710)	(0.1151)
Child 11 years old	0.2200	0.0001	-0.0183	-0.0131
	(0.1206)	(0.1104)	(0.0720)	(0.1167)
Child 12 years old	0.1647	0.0302	-0.0327	0.0101
	(0.1250)	(0.1126)	(0.0735)	(0.1190)
Year = 2002	0.0355	0.0711*	0.0763*	0.0992*
	(0.0292)	(0.0214)	(0.0140)	(0.0227)
Father HS grad.		0.1020*	0.0161	0.0921*
		(0.0344)	(0.0226)	(0.0368)
Father some coll.		0.0780*	0.0230	0.0863*
		(0.0377)	(0.0248)	(0.0403)
Father coll+		0.0105	0.0555*	0.0434
		(0.0384)	(0.0253)	(0.0411)
Father's age		-0.0020	-0.0045*	-0.0058*
		(0.0022)	(0.0014)	(0.0024)
Constant	0.4593*	0.4892*	0.7696*	0.4328*
	(0.1313)	(0.1204)	(0.0786)	(0.1274)
R-squared	0.101	0.056	0.066	0.052
N	1070	2251	2246	2220

Notes: Samples from 1997 and 2002 PSID CDS include parents of children ages 0–12 from families with no more than 2 children ages 0–12. Standard errors in parentheses. * statistically sig. at 0.05 level.

Table D-4: Log wage regressions for parents

	All Mothers	Single Mothers	Married Mothers	Married Fathers
Mother HS grad.	0.366			
	(0.312)			
Mother some coll.	0.561	0.133*	0.235*	
	(0.312)	(0.047)	(0.040)	
Mother coll+	0.833*	0.390*	0.510*	
	(0.313)	(0.058)	(0.039)	
Mother's age	0.053*	0.096*	0.035	
	(0.017)	(0.030)	(0.021)	
Mother's age-squared	-0.000	-0.001*	-0.000	
	(0.000)	(0.000)	(0.000)	
Mother white	0.008	0.104*	-0.039	0.169*
	(0.027)	(0.045)	(0.034)	(0.039)
Married	0.074*			
	(0.029)			
Father HS grad.				0.158*
				(0.059)
Father some coll.				0.364*
				(0.062)
Father coll+				0.621*
				(0.059)
Father's age				0.090*
				(0.015)
Father's age-squared				-0.001*
				(0.000)
Constant	0.478	0.144	1.227*	0.348
	(0.438)	(0.527)	(0.376)	(0.290)
R-squared	0.190	0.131	0.198	0.231
N	1814	606	1208	1589

Notes: Samples from 1997 and 2002 PSID CDS include parents of children ages 0–12 from families with no more than 2 children ages 0–12. Samples examining mothers (fathers) are limited to those with predicted probability of work at least 0.7 (0.85). Standard errors in parentheses. * statistically sig. at 0.05 level.

Table D-5: OLS & IV (instruments: state) estimates for mother time/goods relative demand with different sample restrictions on predicted probability of work

	OLS			Instrumental Variables		
	P(work)≥0.7	All Mothers	P(work)≥0.8	P(work)≥0.7	All Mothers	P(work)≥0.8
$\ln(\tilde{W}_{m,i})$	0.567*	0.596*	0.504*	0.778*	0.827*	0.725*
	(0.084)	(0.079)	(0.092)	(0.263)	(0.264)	(0.239)
Married	-0.173	-0.195*	-0.304*	-0.177	-0.196*	-0.307*
	(0.104)	(0.099)	(0.112)	(0.104)	(0.099)	(0.112)
Child's age	-0.096*	-0.107*	-0.089*	-0.095*	-0.108*	-0.088*
	(0.024)	(0.023)	(0.029)	(0.024)	(0.023)	(0.028)
Mother some	-0.101	-0.134	-0.228	-0.152	-0.201	-0.273*
college	(0.108)	(0.102)	(0.124)	(0.124)	(0.125)	(0.131)
Mother coll+	-0.185	-0.230*	-0.239	-0.281	-0.347*	-0.337*
	(0.119)	(0.113)	(0.133)	(0.164)	(0.171)	(0.164)
Mother's age	-0.010	-0.006	-0.009	-0.013	-0.009	-0.011
	(0.008)	(0.008)	(0.009)	(0.009)	(0.008)	(0.009)
Mother white	-0.201*	-0.143	-0.219*	-0.208*	-0.156	-0.223*
	(0.099)	(0.094)	(0.109)	(0.099)	(0.095)	(0.109)
Num. children	-0.037	0.018	-0.133	-0.042	0.005	-0.129
ages 0–5 in HH	(0.121)	(0.109)	(0.172)	(0.120)	(0.110)	(0.171)
Num. children	0.134	0.140*	0.187*	0.158*	0.165*	0.211*
in HH	(0.068)	(0.063)	(0.075)	(0.074)	(0.069)	(0.079)
Constant	2.633*	2.523*	2.754*	2.210*	2.076*	2.288*
	(0.378)	(0.356)	(0.420)	(0.627)	(0.604)	(0.624)
R-squared	0.109	0.119	0.115			
N	628	694	493	628	694	493

Notes: Samples from 2002 PSID CDS include parents of children ages 0–12 from families with no more than 2 children ages 0–12. Standard errors in parentheses. * statistically sig. at 0.05 level.

Table D-6: Estimates for parental time vs. goods relative demand (log wage fixed effects)

	OLS				Instrumental Variables			
	All Mothers	Single Mothers	Married Mothers	Married Fathers	All Mothers	Single Mothers	Married Mothers	Married Fathers
$\ln(\tilde{W}_{m,i})$	0.758*	0.745*	0.757*		0.752*	0.881*	0.673*	
	(0.098)	(0.213)	(0.109)		(0.258)	(0.343)	(0.289)	
Married	-0.176				-0.175			
	(0.103)				(0.102)			
Child's age	-0.115*	-0.112*	-0.116*	-0.072*	-0.115*	-0.115*	-0.114*	-0.072*
	(0.024)	(0.048)	(0.027)	(0.030)	(0.025)	(0.047)	(0.028)	(0.031)
Mother's log wage	-0.425*	-0.325	-0.457*		-0.422*	-0.408	-0.402	
fixed effect	(0.101)	(0.204)	(0.115)		(0.183)	(0.259)	(0.210)	
Mother white	-0.216*	-0.281	-0.207	-0.069	-0.216*	-0.286	-0.204	-0.068
	(0.097)	(0.193)	(0.113)	(0.137)	(0.097)	(0.190)	(0.112)	(0.138)
Num. children	0.061	-0.312	0.209	0.288*	0.060	-0.283	0.199	0.286*
ages 0–5 in HH	(0.116)	(0.243)	(0.130)	(0.139)	(0.120)	(0.245)	(0.133)	(0.141)
Num. children	0.120	0.098	0.143	0.110	0.119	0.103	0.136	0.110
in HH	(0.068)	(0.123)	(0.082)	(0.092)	(0.069)	(0.121)	(0.084)	(0.092)
$\ln(\tilde{W}_{f,i})$				0.679*				0.663
				(0.122)				(0.347)
Father's log wage				-0.165				-0.153
fixed effect				(0.134)				(0.270)
Constant	1.934*	2.056*	1.697*	1.124*	1.947*	1.740	1.892*	1.165
	(0.336)	(0.688)	(0.384)	(0.444)	(0.642)	(0.924)	(0.731)	(0.929)
Implied ρ	-3.132	-2.921	-3.117	-2.114	-3.036	-7.399	-2.061	-1.964
	(1.676)	(3.278)	(1.844)	(1.187)	(4.205)	(24.197)	(2.712)	(3.045)
R-squared	0.126	0.096	0.143	0.104				
N	618	193	425	470	618	193	425	470

Notes: Sample from 2002 PSID CDS includes children ages 0–12 from families with no more than 2 children ages 0–12. Samples examining mother (father) time are limited to those with predicted probability of work at least 0.7 (0.85). Standard errors in parentheses. * statistically sig. at 0.05 level.

Table D-7: Linear probability model estimates for positive child care expenditures

	All Households	Single Mothers	Two-Parent Households
$\ln(\tilde{P}_{c,i})$	0.032	0.042	0.037
	(0.023)	(0.036)	(0.030)
Child's age	-0.038*	-0.037*	-0.036*
	(0.004)	(0.007)	(0.005)
Mother HS grad.	-0.005	0.051	0.029
	(0.106)	(0.151)	(0.099)
Mother some coll.	0.076	0.150	0.097
	(0.106)	(0.152)	(0.099)
Mother coll+	0.077	0.180	0.100
	(0.106)	(0.154)	(0.100)
Mother's age	0.003*	0.004	0.009*
	(0.002)	(0.003)	(0.003)
Mother white	0.012	0.115*	-0.034
	(0.018)	(0.031)	(0.024)
Num. children	0.056**	0.042	0.065*
age 0–5 in HH	(0.020)	(0.033)	(0.025)
Num. children	-0.056*	-0.061*	-0.066*
in HH	(0.012)	(0.019)	(0.015)
Married	-0.003		
	(0.019)		
Year = 2002	-0.045*	0.015	-0.060*
	(0.018)	(0.030)	(0.022)
Father HS grad.			0.048
			(0.054)
Father some coll.			0.053
			(0.056)
Father coll+			0.060
			(0.057)
Father's age			-0.007*
			(0.003)
Constant	0.462*	0.306	0.480*
	(0.118)	(0.173)	(0.132)
R-squared	0.138	0.127	0.170
N	2,480	811	1671

Notes: Samples from 1997 and 2002 PSID CDS include children ages 0–12 from families with no more than 2 children ages 0–12. Samples for single mothers (two-parent households) are limited to those with predicted probability that the mother (both parents) work at least 0.7 (0.65). Standard errors in parentheses. * statistically sig. at 0.05 level.

Table D-8: GMM estimates for time/goods and child care/goods relative demand accounting for measurement error & unobserved heterogeneity (single mothers)

	No Instruments	Instruments: State
γ	-0.219	-0.223
	(0.267)	(0.828)
ρ	-1.072	-55.590
	(0.695)	(1119.952)
$(\phi_m - \phi_g)$:		
Constant	6.512*	138.485
	(1.857)	(2704.152)
Child's age	-0.181	-5.200
	(0.107)	(102.989)
Mother some coll.	0.261	-2.004
	(0.394)	(46.921)
Mother coll+	0.328	-6.029
	(0.449)	(129.985)
Mother's age	-0.047	-1.735
	(0.034)	(34.526)
Mother white	-0.650	-20.126
	(0.431)	(399.736)
Num. children	-1.026*	-29.578
ages 0–5 in HH	(0.517)	(585.233)
Num. children	0.212	6.877
in HH	(0.254)	(136.800)
ϕ_g:		
Constant	10.032	738.471
	(16.571)	(15358.668)
Child's age	0.457	21.490
	(0.522)	(440.428)
Mother some coll.	-1.730	-90.481
	(2.596)	(1859.913)
Mother coll+	-1.614	-90.277
	(2.661)	(1858.017)
Implied $\epsilon_{\tau,g}$	0.483	0.018
	(0.162)	(0.350)
Implied $\epsilon_{Y,g}$	0.821	0.818
	(0.180)	(0.553)
Objective Fun.	0.0001	0.0047
N	197	197

Notes: Sample from 2002 PSID CDS includes children ages 0–12 from families with no more than 2 children ages 0–12. Sample is limited to single mothers with predicted probability of work at least 0.7. Estimated coefficients related to measurement error in Equation (31) not shown. Standard errors in parentheses. * statistically sig. at 0.05 level.

Table D-9: GMM estimates for full child production function – $\tilde{\phi}_\theta$ and λ_{AP}

	No Borrowing/Saving	Unconstrained
λ_{AP}	1.22*	1.30*
	(0.05)	(0.05)
$\tilde{\phi}_\theta$:		
Const.	-1.14*	-1.39*
	(0.30)	(0.38)
Married	0.11*	0.05*
	(0.04)	(0.02)
Mother some coll.	0.94*	0.95*
	(0.02)	(0.02)
Mother coll+	-2.26*	-2.03*
	(0.87)	(1.06)
Father some coll.	-2.36*	-2.31*
	(0.67)	(0.78)
Father coll+	0.13	0.09
	(0.30)	(0.34)
Child's age	-2.05*	-2.13*
	(0.94)	(1.01)

Notes: Sample from PSID CDS includes children ages 0–12 from families with no more than 2 children ages 0–12. Moments using mother (father) time are limited to those with predicted probability of work at least 0.7 (0.85). Standard errors in parentheses. * statistically sig. at 0.05 level.

Table D-10: Elasticity of Total Investment Quantity with Respect to Input Prices

Price Change	Nested CES			Cobb-Douglas			% Difference between Cobb-Douglas and Nested CES		
	Wages	Goods	Child Care	Wages	Goods	Child Care	Wages	Goods	Child Care
A. Single Mothers									
10% Change	0.23	-0.06	-0.16	0.22	-0.06	-0.17	-1.57	2.37	3.84
50% Change	0.32	-0.06	-0.19	0.28	-0.08	-0.24	-13.29	18.97	23.00
B. Two-Parent Households									
10% Change	0.12	-0.03	-0.08	0.12	-0.03	-0.08	-4.73	3.07	-0.48
50% Change	0.18	-0.04	-0.10	0.15	-0.05	-0.11	-18.40	20.20	19.05