Leverage and the Foreclosure Crisis*

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Abstract

How much of the recent rise in foreclosures can be explained by the large number of high-leverage mortgage contracts originated during the housing boom? We present a model where heterogeneous households select from a set of mortgage contracts and choose whether to default on their payments given realizations of income and housing price shocks. The set of mortgage contracts consists of loans with high downpayments and loans with low downpayments. We run an experiment where the use of low-downpayment loans is initially limited by payment-to-income requirements but then becomes unrestricted for 8 years. The relaxation of approval standards causes homeownership rates, high-leverage originations and the frequency of high interest rate loans to rise much like they did in the US between 1998-2006. When home values fall by the magnitude observed in the US from 2007-08, default rates increase by over 180% as they do in the data. Two distinct counterfactual experiments where approval standards remain the same throughout suggest that the increased availability of high-leverage loans prior to the crisis can explain between 40% to 65% of the initial rise in foreclosure rates. Furthermore, we run policy experiments which suggest that recourse could have had significant dampening effects during the crisis.

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1 Introduction

The share of high-leverage loans in mortgage originations started rising sharply in the late 1990s.[1] Pinto (2010, see figure 1) calculates that among purchase loans insured by the Federal Housing Administration (FHA) or purchased by Government Sponsored Enterprises (GSEs) the fraction of originations with cumulative leverage in excess of 97% of the home value was under 5% in 1990 but rose to almost 40% in 2007. Gerardi et. al (2008) present similar evidence using a dataset of mortgages sold into mortgage-backed securities marketed as "sub-prime." Among these subprime loans, transactions with a cumulative loan-to-value (CLTV)[2] represented just 10 percent of all originations in 2000 but exceeded 50% of originations in 2006.[3]

The increased availability of loans with low downpayments made it possible for more households to obtain the financing necessary to purchase a house. At the same time however, because these contracts are characterized by little equity early in the life of the loan, they are prone to default when home prices fall. Not surprisingly then, (see, again, Gerardi et al., 2008 (Figure 4), or Mayer et. al., 2009, among many others) mortgages issued during the recent housing boom with high leverage have defaulted at much higher frequency than other loans since home prices began their collapse in late 2006.

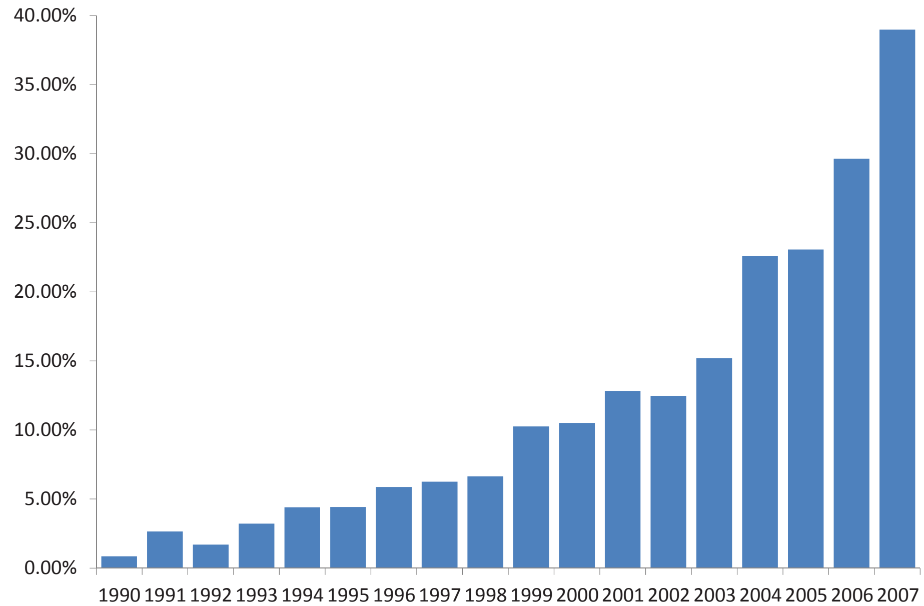
How much of the rise in foreclosures that started in 2007 can be attributed to the increased originations of high-leverage mortgages during the housing boom? To answer this question, we describe a housing model where the importance of high-leverage loans for default rates can be measured. Households move stochastically through three stages of life and make their housing and mortgage decisions in the middle stage. Two types of fixed-payment mortgages

[1] As Foote et al. (2012, section 2) among others point out, high-leverage loans are not new in the United States. Our paper is about the fact that their frequency increased in the years leading up to the foreclosure crisis.

[2] The CLTV at origination is the sum of the face value of all loans secured by the purchased property divided by the purchase price.

[3] Mayer et al. (2009) among others discuss similar evidence. These studies also point out that the use of secondary "piggy-back" loans increased markedly during that period. See also Duca et. al. (2011) and Bokhari et. al. (2013).

Figure 1: FHA and GSE purchase loans with a cumulative LTV above 97%

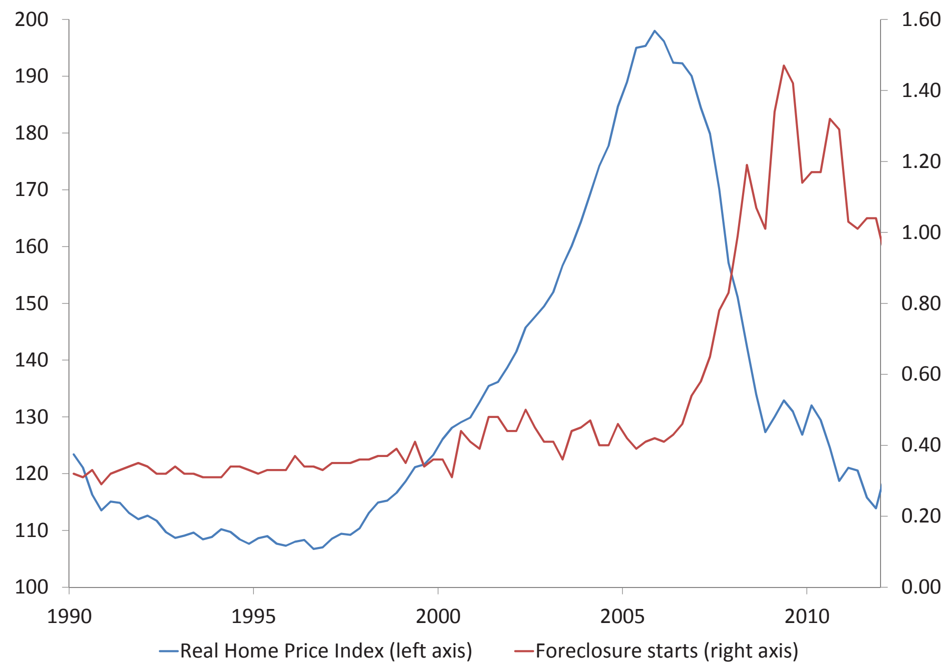


Source: Pinto (2010). Fannie Mae's purchase loans are the proxy for conventional loans. FHA loans are classified according to the first mortgage's size only.

are available to households: a contract with a 20% downpayment and a contract with no downpayment. Mortgage holders can terminate their contract before maturity. We consider a mortgage termination to be a foreclosure if it occurs in a state where the house value is below the mortgage's balance (that is, the agent's home equity is negative) as a result of aggregate and/or idiosyncratic home price shocks, or where the agent's income realization is such that they cannot make the mortgage payment they would owe for the period.

Foreclosures are costly for lenders because of the associated transaction costs and because they typically occur when home equity is negative. As a result, intermediaries demand higher yields from agents whose asset and income position make foreclosure more likely. In fact,

Figure 2: The housing boom and bust



Source: The real home price index is the US S&P/Case-Shiller Index. Foreclosure starts are from the Mortgage Bankers Association's National Delinquency Survey and are the reported number of mortgages for which foreclosure proceedings are started in a given quarter divided by the initial stock of mortgages.

intermediaries do not issue loans to some agents because expected default losses are too high. In particular, our model is consistent with the fact that agents at lower asset and income positions are less likely to become homeowners, face more expensive borrowing terms, and are more likely to default on their loan obligations.

We approximate the course of events depicted in figures 1 and 2 using a three-stage experiment. The first stage is a long period of moderate real house prices with "tight" approval standards that lasts until the late 1990s. Between 1998 and 2006, approval standards are relaxed and, at the same time, aggregate home prices rise. In 2007, aggregate home prices and approval standards return to their pre-boom level. We think of the beginning of this last stage as the crisis period.

We model changes in approval standards as exogenous changes in payment-to-income (PTI) requirements. A version of our model calibrated to capture key features of pre-boom US housing markets predicts that, following the relaxation of approval standards, the use of low-downpayment mortgages rises to a peak of 37% at the onset of the crisis, which is in line with the evidence displayed in figure 1. Likewise, home-ownership rates rise markedly as new households gain access to mortgage markets.

The aggregate home price collapse that takes place at the end of the boom stage in our model causes default rates to increase by 182% in the first two years of the crisis, accounting for over 98% of the rise in the data. In a counterfactual experiment where PTI requirements are left unchanged throughout the experiment, the use of high-downpayment loans changes little during the boom, and default rates only rise by 64%. In a second counterfactual where both approval standards and home values are left unchanged in the few years preceding the crisis, a price drop of the same relative magnitude as in the baseline experiment causes default rates to rise by 111%. In that sense, in our model, 40% to 65% of the initial spike in foreclosure rates can be attributed to the greater availability of high-leverage loans during the boom.

Importantly, while approval standards do change during the boom period we simulate, this does not imply that underwriting standards become poor during that period. At all times in this experiment, loans are priced correctly. Lenders fully understand the environment in which they are writing mortgages and, ex ante, all loans imply zero economic profits. Default rates spike when home values collapse because, for recently issued loans, an early home value correction is the worst possible realization of the underlying process.[4] We do not model the possibility that lenders had the wrong stochastic process in mind. Our results say that even with fully rational expectations, the large aggregate home value correction that took

[4] Since the worst-case scenario materializes during the crisis, the intermediary does experience ex-post losses on the loans it wrote shortly before the crisis. Had it known (or assigned a high probability to the possibility) that aggregate home prices were going to fall, the intermediary would have priced loans differently.

place in late 2007 was enough to generate a default spike of a magnitude quite similar to what transpired at least initially. An interesting question we leave for future work is whether incorrect expectations may have magnified the size of the crisis and the role of leverage further.

In the model, the increased use of high-downpayment loans magnifies the effect of the home price correction for two fundamental reasons. First and most obviously, more households find themselves in negative equity territory following the aggregate shock since average equity levels are lower before the shock when low downpayment mortgages are more frequent. But this equity effect is compounded by the selection effects associated with broadening access to mortgage markets. Relaxing approval standards allows agents with lower income and assets to enter mortgage markets. These new borrowers are inherently more prone to default. As discussed above, default typically involves a shock other than a pure home value shock. Selection effects compound the equity effect of high-leverage by populating mortgage markets with borrowers that are more likely to face payment difficulties.

We show in section 6.2 that new entrants into mortgage markets and households who opt for low-downpayment loans rather than high-downpayment loans when approval standards are relaxed account for much of the increase in default rates following the home-price collapse. Measuring the role of leverage in the foreclosure crisis requires predicting what housing choices and default decision these new entrants and loan-type switchers would have made during the boom period under stricter approval scenarios. We use our model makes predictions for these endogenous objects.

A fully articulated economic model also makes it possible to discuss the potential role of policy in the crisis. In addition to measuring the role leverage may have played in the crisis, we use our model to ask a policy question motivated by the observation that the extent of lender recourse varies significantly across economies. Feldstein (2008) and others have argued that the fact that recourse is highly limited in law or in practice in most US states greatly magnified the impact of the home value correction on default rates. In our model, broadening recourse to include non-housing assets turns out to have limited effects on default rates in the long-run. On the one hand, the risk of default falls due to harsher punishment for a given set of asset and income characteristics at origination and average recoveries rise, which lowers interest rates at origination. On the other hand, lower payments allow agents with lower income and assets to enter mortgage markets. This effect on the composition of the borrower pool turns out to mostly offset the direct, loan-level effect of recourse on default in the long-run, and long-run default rates only fall by 4% or so. This part of our paper is closely related to Hatchondo et. al. (2013). Like us they use a life-cycle model to simulate the effect of broader recourse on default rates, but they broaden recourse to include wage garnishment and, as a result, find a larger effect of recourse on default.[5]

The fact that the long-term effect of recourse on default is small in our model could suggest that broader recourse would have done little to mitigate the foreclosure crisis. That intuition turns out to be wrong, however. Repeating the same 3-stage experiment as above in an economy with recourse leads to a much smaller flare-up of default rates. By lowering interest rates and mortgage payments in the pre-boom period, broader recourse means that relaxed approval standards have much less impact on households' ability to participate in mortgage markets. Furthermore, given the increased cost of default for borrowers, the use of low-downpayment loans becomes riskier. Leverage is thus less prevalent at the onset of the crisis, and, as a result, the impact of the home value correction on default rates falls.

As figure 2 shows, default rates briefly retreated in late 2007 before experiencing a second spike, even though home prices appear to have stabilized by that point. A possible explanation for this second spike is the fact that, as Saez (2013) among many others document, the housing crisis was followed by decline in average household income of over 15% between 2007 and 2009. We show that an aggregate income shock of that size can cause a second spike in default rates of the right magnitude. Furthermore, the same counterfactuals as above suggest that the same aggregate income shock would have caused a much smaller increase in defaults had access to high-leverage loans been more restricted during the boom. In other words, the frequency of high-leverage loans during the boom increased the sensitivity of default rates to both aggregate home price and aggregate income shocks.

[5] They also simulate the impact of down-payment regulations on default rates.

Our paper is related to several other structural models. The idea that mortgage innovation may have implications for foreclosures is taken up in Garriga and Schlagenhauf (2009). They quantify the impact of an unanticipated aggregate house price decline on default rates where there is cross-subsidization of mortgages **within** but not **across** mortgage types. A key difference between our paper and theirs is that we consider a menu of different terms on contracts **both** within **and** across mortgage types.[6] Our paper is more closely related to Guler (2008) where intermediaries offer a menu of FRMs at endogenously chosen downpayment rates or Chatterjee and Eyigungor (2009) where intermediaries offer a menu of infinite maturity interest-only mortgage contracts in which borrowers accumulate no equity over time. Guler studies the impact of an innovation to the screening technology on default rates and Chatterjee and Eyigungor study the effect of an endogenous price drop arising from an overbuilding shock. Mitman (2012) considers the interaction of recourse and bankruptcy on the decision to default in an environment with one period mortgages and costless refinance. Campbell and Cocco (2012) study the effect of differences in loan-to-value and loan-to-income on the default decision in an environment with a rich structure of aggregate shocks but where households are identical at the time of contract selection.

Section 2 lays out the economic environment. Section 3 describes optimal behavior on the part of all agents and defines an equilibrium. Section 4 discusses our parameterization procedure. Section 5 characterizes equilibrium behavior across different long run equilibria to understand contract selection, default, mortgage pricing, and the role of recourse policy. Section 6 presents our main transition experiment. Section 7 concludes.

[6] Here we extend the one-period pricing framework of Chatterjee, et al. (2007) to long term but finite contracts.

2 Environment

2.1 Demographics, Tastes, and Technologies

Time is discrete and infinite. The economy is populated by a continuum of households and by a financial intermediary. Each period a constant mass of households are born. We normalize this constant mass so that the unique invariant size of the population is one. Households move stochastically through four stages: youth ($\mathcal{Y}$), mid-age ($\mathcal{M}$), old-age ($\mathcal{O}$), and death ($\mathcal{D}$). At the beginning of each period, young households become mid-aged with probability ρ^M, mid-age households become old with probability ρ^O, and old households die with probability ρ^D and are replaced by young households.

Each period when young or mid-aged, households receive an idiosyncratic shock to their earnings y_t denominated in terms of the unique consumption good. For $\eta \in \{\mathcal{Y}, \mathcal{M}\}$, these income shocks follow a Markov process with finite support $Y^\eta \subset I\!R_+$ and transition matrices P^η. Earnings shocks obey a law of large numbers. Agents begin life at an income level drawn from the unique invariant distribution associated with the young agent's income process. When old, agents earn a fixed, certain amount of income $y^{\mathcal{O}} > 0$. Where convenient, we will write $Y \equiv Y^{\mathcal{Y}} \cup Y^{\mathcal{M}} \cup \{y^O\}$ for the set of all possible income values.

Households can save by depositing $a_t \geq 0$ with the intermediary and earn the risk-free storage return r with certainty on these savings. For old agents, returns are annuitized so that surviving households earn return $\frac{1+r}{1-\rho^D} - 1$ on their deposits while households who die do so with no wealth.

Households value consumption and housing services. They can obtain housing services from the rental market or from the owner-occupied market. On the first market, they can rent quantity $h^1 > 0$. When they become mid-aged, agents can choose to purchase quantity $h_t \in \{h^2, h^3\} \in I\!R^2_+$ of housing capital. We refer to this asset as a house. While mid-aged, a household which is currently renting has an exogenous opportunity to purchase a house with probability γ.

Our economy is subject to aggregate uncertainty at date t denoted $s_t \in S \equiv \{L, N, H\}$. We take the unit price q_{s_t} of homes as the exogenous realization of a Markov process defined on:

$$\mathcal{Q} \equiv \{q_L, q_N, q_H\} \in \mathbb{R}^3_+$$

where $q_L < q_N < q_H$ with transition matrix P^q.[7] Rental rates respond to the same aggregate uncertainty so we assume three distinct values $\{R_L, R_N, R_H\}$ which we calibrate to match the pertinent evidence on price-to-rent ratios.

Once agents own a house of size $h_t \in \{h^2, h^3\}$, the market value of the housing capital they own in any given aggregate state s is $q_{s_t}\epsilon_t h_t$ where ϵ_t is an idiosyncratic shock drawn from

$$\mathcal{E} \equiv \{\epsilon_b, 1, \epsilon_g\}$$

which follows a Markov process with transition matrix P^ϵ. The idiosyncratic shock process is independent of aggregate shocks and obeys a law of large numbers. One possible interpretation of these shocks is "neighborhood effects[8]" which change the market value of the house to a potential buyer independent of aggregate housing price changes. We introduce these idiosyncratic shocks so that even when aggregate home prices are stable, some homeowners experience negative equity after house purchases while other homeowners experience positive capital gains on their houses. We will specify the ϵ process to match the relevant evidence on the dispersion of housing capital gains in the United States.

Households thus face aggregate uncertainty about home prices and three sources of idiosyncratic uncertainty – aging shocks, income shocks, and house-specific price shocks. For

[7] In a previous version of this paper, we assumed a linear technology for transforming consumption goods into housing capital in which case under perfect competition aggregate prices were simply given by the inverse of aggregate housing total factor productivity shocks.

[8] While we assume that idiosyncratic shocks obey a law of large numbers, we do not assume that these shocks are independent across households so that clusters of agents one could think of as geographical locations may have ex-ante correlated house values.

every household of age $t \in \{0, +\infty\}$, histories are thus elements of

$$\left[S \times \{\mathcal{Y}, \mathcal{M}, \mathcal{O}, \mathcal{D}\} \times Y \times \mathcal{E} \right]^t.$$

Households order history-contingent processes $\{c_t, h_t\}_{t=0}^{+\infty}$ according to the following utility function:

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, h_t)$$

where for all $t \geq 0$, $c_t \geq 0$, $h_t \in \{h^1, h^2, h^3\}$, and

$$u(c_t, h_t) \equiv \log c_t + \log[h_t \times \theta(h_t)]$$

with

$$\theta(h^3) = \theta(h^2) > 1 = \theta(h^1)$$

so that homeowners enjoy a proportional utility premium over renters. We think of θ as capturing any enjoyment agents derive from owning rather than renting their home, but it also serves as a proxy for any pecuniary benefit associated with owning which we do not explicitly model.

For all date t, owners of a house of size $h_t \in \{h^2, h^3\}$ bear maintenance costs $\delta q_{s_t} h_t$ where $\delta > 0$.[9] Owners who turn old must sell their house. Since this is the only source of exogenous sales in our model one could think of this possibility as capturing events such health shocks or divorce that constrain agents to sell their home and experience a permanent change in their income prospects.

The financial intermediary is an infinitely-lived risk-neutral agent that holds household savings and can store these savings at net return $r \geq 0$ at all dates. At date t, it can also buy existing homes at unit price $q_t \epsilon_t$, transform the resulting housing capital into the consumption

[9] We assume that maintenance costs depend only on the aggregate state of the economy (i.e. q_s) and do not include idiosyncratic shocks (ϵ). Assuming that idiosyncratic shocks also affect maintenance costs does not have a significant impact on our results.

good at rate $\frac{1}{q_t \epsilon_t}$, or rebundle it to rent or sell to new homebuyers.[10] When it buys an existing home of size h_t at market value $q_{s_t}\epsilon_t h_t$, rebundling either requires an expenditure of $q_{s_t}(1-\epsilon_t)h_t$ when $\epsilon_t < 1$ to return the home to marketable value or entails a windfall $q_{s_t}(\epsilon_t - 1)h_t$ when $\epsilon_t > 1$.

2.2 Mortgages

Households that purchase a house of size $h_t \in \{h^2, h^3\}$ at time t must finance this purchase with a fixed rate mortgage contract of maturity T with downpayment fraction $\nu_t \in \{LD, HD\}$. Specifically, the mortgage requires a downpayment of size $\nu_t q_{s_t} h_t$ and stipulates an interest rate $r_t^\nu(a_t, y_t, h_t; s_t)$ that depends on the household's wealth and income characteristics at time of origination, the size of the loan (which obviously depends on house prices q_{s_t} and the size of the house h_t), and state dependent mortgage approval standards parameterized by α_{s_t}. Given this interest rate, constant payments $m_t^\nu(a_t, y_t, h_t; s_t)$ and a principal balance schedule $\{b_{t,n}^\nu(a_t, y_t, h_t; s_t)\}_{n=0}^{T-1}$ can be computed using standard fixed annuity calculations, where $n = 0, 1, ..., T-1$ denotes the period following origination.[11]

A simple way to specify approval standards on mortgages originated at date t is to assume that a household applying for a mortgage must meet a payment-to-income (PTI) requirement. Specifically, in order to qualify for a mortgage with downpayment ν_t at time t, a mid-aged household of type (a_t, y_t) who wants a loan of size $(1-\nu_t)q_{s_t}h_t$ must satisfy

$$\frac{m_t^\nu}{y_t} \le \alpha_{s_t} \tag{2.1}$$

[10] Rebundling keeps the dimension of the state space manageable. Note that the fact that each agent's housing choice set is discrete does not impose an integer constraint on the intermediary since it deals with a continuum of households.

[11] Suppressing initial characteristics for notational simplicity, then

$$m_t^\nu = \frac{r_t^\nu}{1-(1+r_t^\nu)^{-T}}(1-\nu_t)q_{s_t}h_t$$

and

$$b_{t,n+1}^\nu = b_{t,n}^\nu(1+r_t^\nu) - m_t^\nu,$$

where $b_{t,0}^\nu = (1-\nu_t)q_{s_t}h_t$.

where $\alpha_{s_t} > 0$ for all aggregate states. Despite assuming that the PTI requirement is the same across downpayment sizes[12], a given household is less likely to qualify for a high-leverage loan than a low-leverage loan since they carry a higher interest rate in equilibrium and start with a higher balance.

Varying α will enable us to generate fractions of high leverage loans that mimic Figure 1 and trace the consequences of this change. While we do not view this specific aspect of our model as a deep theory for why the frequency of high leverage loans started increasing in the late 1990s, all evidence is that PTIs did rise markedly during the housing boom. For instance, Bokhari et. al. (2013) calculate that among single-family home loans purchased by Fannie Mae, the share of loans with PTIs above 42% rises from around 5% in 1990 to over 40% in 2007. Similarly, according to data released by the FHA in 2011[13], the fraction of first-lien, single family mortgages acquired by government sponsored enterprises with a PTI above 28% or a total monthly debt to income (or "back-end DTI") above 36% doubled from 38% in 1998 to 77% in 2007. Finally and as we discuss in further detail in our calibration section, Survey of Consumer Finance data suggest that loan-to-income ratios rose noticeably on purchase loans during the housing boom. While our model generates such an increase for several reasons – including the fact that the relaxation of approval standards allows lower income households to obtain a mortgage – such a significant increase in loan-to-income ratios is consistent as we will argue with a change in PTI requirements.

The set of mortgage terms from which a given household can choose is endogenous in our model and must be consistent in equilibrium with certain conditions. Specifically, let $K_t(a_t, y_t, h_t; s_t) \subset \{LD, HD\}$ be the set of feasible downpayment options on a mortgage offered to a household with characteristics (a_t, y_t) which wants to purchase a house of size h_t at price q_{s_t} under approval standards α_{s_t}. The set K_t must satisfy the following conditions in equilibrium: (i) the downpayment must budget feasible given household wealth; (ii) the

[12] FHA loans, which account for most high leverage loans in figure 1 prior to 1998, had formal PTI limits in the 1990s that were only slightly lower than those typical of conventional loans (see Bunce et. al., 1995, for a discussion).

[13] See Mortage Market Note 11-02, available at http://www.fhfa.gov/webfiles/20686/QRM_FINAL_ALL.pdf.

payment-to-income requirement is satisfied; and (iii) the lender must expect to make zero profits on such mortgages (these conditions will be made rigorous below). Of course, in equilibrium, the set K_t may be empty.

A mortgage-holder can terminate the contract at the beginning of any period, in which case the house is sold. We will consider a termination to be a foreclosure when the outstanding principal exceeds the house value or when the agent's state is such that it cannot meet its mortgage payment in the current period. In the event of foreclosure, fraction $\chi > 0$ of the sale value is lost in transaction costs (e.g. legal costs, costs of restoring the property to saleable conditions, etc.).[14] If the mortgage's outstanding balance at the time of default $t+n$ is $b^{\nu}_{t,n}$, the intermediary collects $\min\{(1-\chi)q_{s_{t+n}}\epsilon_{t+n}h_t, b^{\nu}_{t,n}\}$ where $q_{s_{t+n}}\epsilon_{t+n}h_t$ is the house value at date $t+n$, while the household receives $\max\{(1-\chi)q_{s_{t+n}}\epsilon_{t+n}h_t - b^{\nu}_{t,n}, 0\}$.

More formally, default arises in two cases at a given date $t+n$. First, if it is not budget feasible for the household to meet its mortgage payment:

$$y_{t+n} + a_{t+n}(1+r) - m^{\nu}_t - \delta q_{s_{t+n}} h_t < 0, \tag{2.2}$$

the household is constrained to terminate its mortgage. A second form of default occurs when the household can meet their mortgage payment (i.e. (2.2) does not hold) but the household chooses to sell with negative home equity:

$$q_{s_{t+n}}\epsilon_{t+n}h_t - b^{\nu}_{t,n} < 0, \tag{2.3}$$

In either default case, we will write $D_{t+n} = 1$ while $D_{t+n} = 0$ otherwise. Defaulting agents include households who become old at the start of the period and must sell a home with negative equity. Naturally, mortgage holders may also choose to sell their house even when they can meet the payment and have positive equity, for instance because they are borrowing constrained in the current period. We will think of such a transaction as a regular sale.

[14] For more discussion of these costs, see http://www.nga.org/Files/pdf/0805FORECLOSUREMORTGAGE.PDF.

2.3 Timing

The timing in each period is as follows. At the beginning of the period, agents discover whether or not they have aged and receive a perfectly informative signal about their income draw. Aggregate and idiosyncratic house price shocks are also realized at the beginning of the period. Owners then decide whether to remain owners or to become renters either by selling their house or through foreclosure. Renters discover whether or not home-buying is an option at the beginning of the period. Agents who just turned mid-aged get this option with probability one. Agents who get the home-buying option make their housing and mortgage choice decisions at the beginning of the period, after all uncertainty for the period is resolved. Downpayments are thus made at the beginning of the period. At the end of the period, agents receive their income, mortgage payments are made, and consumption takes place.

3 Equilibrium

This section describes a recursive competitive equilibrium for our economy. To ease notation, we drop all time markers using the convention that, for a given variable x, $x_t \equiv x$ and $x_{t+1} \equiv x'$.

3.1 Household Problem

3.1.1 Old agents

In aggregate state s, the individual state of old households is fully described by their asset position $a \geq 0$. The value function for an old agent with assets $a \in \mathbb{R}_+$ solves

$$V_O(a; s) = \max_{a' \geq 0} \left\{ u\left(c, h^1\right) + \beta(1 - \rho^D) E_{s'|s} V_O(a'; s') \right\}$$

s.t.

$$c = a\frac{(1+r)}{1-\rho^D} + y^O - h^1 R_s - a' \geq 0.$$

Note that even though old agents do not own homes, the aggregate value of the housing good affects their welfare because it moves the rental rate.

3.1.2 Mid-aged Agents

For mid-aged agents we need to consider three distinct cases depending on housing status.

Case 1: Renter

If the mid-aged household enters the period as a renter (R), the value function is:

$$\begin{aligned} V_M^R(a, y; s) \quad = \quad & \max_{c' \geq 0, a' \geq 0} u(c, h^1) + \beta\rho^O E_{s'|s}\left[V_O(a'; s')\right] \\ & + \beta(1-\rho^O) E_{y', s'|y, s} \left[\begin{array}{c} (1-\gamma)V_M^R(a', y'; s') \\ +\gamma V_M(a', y', n=0; s') \end{array} \right] \end{aligned}$$

s.t.

$$c + a' = y + a(1+r) - R_s h^1$$

where $V_M(a', y', n=0; s')$, defined below, is the value function for mid-aged agents who have the option to buy a home given their assets and income and given the aggregate state.

Case 2: Existing Homeowners

Households who already own a home have to decide whether to remain homeowners or to become renters. As in the case of renters, their value function depends on their asset, their income and aggregate conditions, but it also depends on the current market value of their home (hence on ϵ) and on the choices they made when their mortgage was originated. Let (ν, κ) be the tuple of mortgage characteristics at origination where $\kappa = (\widehat{a}, \widehat{y}, \widehat{h}; \widehat{s})$ denotes the origination state. This original information pins down mortgage payments $m^\nu(\kappa)$ and the remaining balance $b_n^\nu(\kappa)$ under the existing contract. Equipped with this notation, we can now define the value function of a homeowner ($n \geq 1$):[15]

$$V_M^{(\nu,\kappa)}(a, y, \epsilon, n; s) = \max_{c\geq 0, a'\geq 0, h\in\{h^1, \widehat{h}\}} \quad u(c,h) + \beta\rho^O E_{\epsilon', s'|\epsilon, s}\left[V_O(a' + 1_{\{h=\widehat{h}\}} S_{n+1}^{(\nu,\kappa)}(\epsilon', s'); s')\right] + \beta(1-\rho^O) E_{y',\epsilon',s'|y,\epsilon,s}\left[\begin{array}{c} 1_{\{h=h^1\}} V_M^R(a', y'; s') \\ +1_{\{h=\widehat{h}\}} V_M^{(\nu,\kappa)}(a', y', \epsilon', n+1; s') \end{array}\right]$$

where if $h = \widehat{h}$, then

$$s.t.\ c + a' = y + a(1+r) - m^\nu(\kappa)1_{\{n<T\}} - \delta q_s h$$

and if $h = h^1$, then

$$\begin{aligned} c + a' &= y + (1+r)\left[a + S_n^{(\nu,\kappa)}(\epsilon, s)\right] - R_s h^1 \\ S_n^{(\nu,\kappa)}(\epsilon, s) &= \max\left\{(1 - D^{(\nu,\kappa)}(a, y, \epsilon, n; s)\chi) q_s \epsilon \widehat{h} - b_n^\nu(\kappa), 0\right\} \\ D^{(\nu,\kappa)}(a, y, \epsilon, n; s) &= 1 \text{ if } y + a(1+r) - m^\nu(\kappa)1_{\{n<T\}} - \delta q_s h < 0 \text{ or } q_s\epsilon\widehat{h} - b_n^\nu(\kappa) < 0. \end{aligned}$$

The budget constraint depends on whether or not the household keeps its house. When households become renters, their asset position is increased by the homeowner's share of the salvage value of the house, denoted $S_n^{(\nu,\kappa)}(\epsilon, s)$, net of their outstanding principal and, in the event of default, net of transaction costs. When households become renters, their asset position is increased by the proceeds from selling the house net of the outstanding principal and, in the event of default, net of transaction costs. $S_n^{(\nu,\kappa)}(\epsilon, s)$ denotes these net proceeds. Their housing expenses are the sum of mortgage and maintenance payments if they keep the house or the cost of rental otherwise. The final constraint states that selling the house without incurring default costs is only possible if the household is able to meet its mortgage obligations and has positive equity.

[15] 1_x denotes the indicator function which takes the value 1 if x is true.

Case 3: The Option to Buy a House

A renter who receives the option to buy a home at the start of a given period must decide whether to exercise that option, and if they become homeowners, what mortgage to use to finance their house purchase. Let $K(\kappa) \subset \{LD, HD\}$ be the set of feasible downpayment options on a mortgage offered to a household given contract-relevant characteristics $\kappa = (a, y, h; s)$ at origination.

The household's value function solves:

$$\begin{aligned} V_M(a, y, n = 0; s) \quad = \quad & \max_{c \geq 0, a' \geq 0, h \in \{h^1, h^2, h^3\}, \nu \in K} u(c, h) + \beta \rho^O E_{\epsilon', s' | 1, s} \left[V_O(a' + 1_{\{h \in \{h^2, h^3\}\}} S_{n=1}^{(\nu,\kappa)}(\epsilon', s'); s') \right] \\ & + \beta(1 - \rho^O) E_{y', \epsilon', s' | y, 1, s} \left[\begin{array}{c} 1_{\{h = h^1\}} \left(\begin{array}{c} (1 - \gamma) V_M^R(a', y'; s') \\ + \gamma V_M(a', y', n = 0; s') \end{array} \right) \\ + 1_{\{h \in \{h^2, h^3\}\}} \left(V_M^{(\nu,\kappa)}(a', y', \epsilon', n = 1; s') \right) \end{array} \right] \end{aligned}$$

where if $h = h^1$, then

$$s.t. \ c + a' = y + a(1 + r) - R_s h^1$$

and if $h \in \{h^2, h^3\}$, then the following conditions must hold

$$\begin{aligned} c + a' &= y + (1 + r)\left[a - \nu q_s h\right] - m^\nu(\kappa) - \delta q_s h \\ a &\geq \nu q_s h \qquad (3.1) \\ \frac{m^\nu(\kappa)}{y} &\leq \alpha_s. \qquad (3.2) \end{aligned}$$

3.1.3 Young Agents

The value function of a young household depends only on their assets and income and on aggregate conditions. It solves:

$$V_Y(a, y; s) \quad = \quad \max_{c \geq 0, a' \geq 0} u(c, h^1) + \beta E_{y', s' | y, s} \left[(1 - \rho^M) V_Y(a', y'; s') + \rho^M V_M(a', y', n = 0; s') \right]$$

s.t.

$$c + a' = y + a(1+r) - R_s h^1.$$

3.2 Intermediary's Problem

All possible uses of loanable funds must earn the same return for the intermediary.[16] This implies that the expected return on originated mortgages net of expected foreclosure costs must cover the opportunity cost of funds. The intermediary incurs mortgage service costs which we model as a premium $\phi > 0$ on the opportunity cost of funds loaned to the agent for housing purposes.

To make the resulting condition precise, given discount rate $r + \phi$, denote the expected present value to the intermediary of an existing mortgage contract with origination characteristics (ν, κ) held by a mid-aged home-owner with current characteristics (a, y, ϵ, n) and given the aggregate state s by $W_n^{(\nu,\kappa)}(a, y, \epsilon; s)$. If the mortgage is paid off ($n \geq T$) then $W_n^{(\nu,\kappa)} = 0$. Otherwise,

$$\begin{aligned} W_n^{(\nu,\kappa)}(a, y, \epsilon; s) \quad = \quad & 1_{\{h^{(\nu,\kappa)}(a,y,\epsilon,n;s)=h^1\}} \min\{(1 - D^{(\nu,\kappa)}(a, y, \epsilon, n; s)\chi)q_s \epsilon \widehat{h}, b_n^{\nu}(\kappa)\} \\ & +1_{\{h^{(\nu,\kappa)}(a,y,\epsilon,n;s)=\widehat{h}\}} \left(\frac{m^{\nu}(\kappa)}{1 + r + \phi} + E_{y',\epsilon',s'|y,\epsilon,s} \left[\frac{W_{n+1}^{(\nu,\kappa)}(a', y', \epsilon'; s')}{1 + r + \phi} \right] \right). \end{aligned}$$

Indeed, in the event of a sale (i.e. $h^{(\nu,\kappa)}(a, y, \epsilon, n; s) = h^1$), the bank either recovers the loan's balance or, if lower and in the event of default, foreclosure proceeds. If the homeowner stays

[16] While we take home and rental prices as driven by exogenous processes, an arbitrage condition must implicitly hold between these two activities. Since homes are subject to valuation shocks, it is natural to assume that rental units are subject to valuation shocks as well. Any exogenous specification of rental rates and home prices pins down what the expected value of these shocks must be in equilibrium because the intermediary is free to direct the capital it holds to either the rental or the owner-occupied market. To see this, let Δ be the gross rate of valuation growth of rental units, a random variable which can depend on the aggregate state of home prices. Arbitrage requires that at all dates and for all current states q of aggregate prices,

$$q = \frac{R(q) + E_q[\Delta q]}{1 + r}.$$

Any specification of the process for q and R implies a specification for $E_q[\Delta q]$.

in her home (i.e. $h^{(\nu,\kappa)}(a,y,\epsilon,n;s)=\widehat{h}$), the bank receives the mortgage payment and the mortgage ages by one period.

If the household was a renter and receives an exogenous opportunity to purchase a house in state $(\widehat{a},\widehat{y};\widehat{s})$, the household qualifies for a mortgage with downpayment ν on a house of size $\widehat{h}\in\{h^2,h^3\}$ only provided it can make the associated downpayment (i.e. constraint (3.1) is satisfied) and it meets the PTI requirement (i.e. constraint (3.2)) If either (3.1) or (3.2) is violated at origination, we normalize the present value $W_0^{(\nu,\kappa)}=0$. If the household does meet both qualification constraints, then:

$$W_0^{(\nu,\kappa)}(\widehat{a},\widehat{y},\epsilon=1;\widehat{s})=\frac{m^\nu(\kappa)}{1+r+\phi}+E_{y',\epsilon',s'|y,1,s}\left[\frac{W_1^{(\nu,\kappa)}(a',y',\epsilon';s')}{1+r+\phi}\right].$$

Given the interest rate schedule $r^\nu(\kappa)$ which implies $m^\nu(\kappa)$, the intermediary expects to earn zero profit on a loan contract with characteristics κ if

$$W_0^{(\nu,\kappa)}(\widehat{a},\widehat{y},1;\widehat{s})-(1-\nu)\widehat{q}_s\widehat{h}=0. \tag{3.3}$$

Assuming free-entry into intermediation activities, it must be in equilibrium that the set $K(\kappa)$ of mortgage contracts and interest rate schedules $r^\nu(\kappa)$ available for the purchase of a home of size $\widehat{h}\in\{h^2,h^3\}$ for a household with characteristics $(\widehat{a},\widehat{y})$ in aggregate state s satisfy condition (3.3) along with (3.1)-(3.2).[17]

[17] As discussed at length by Quintin (2012), there may be several interest rate offerings that produce zero expected profits, even at equal downpayment, since the endogeneity of default generically makes W_0 discontinuous and non-monotonic. Computationally, we need to make sure that among rates that satisfy the zero-profit constraint for a given set of origination characteristics, the most favorable to the household prevails, which prevents us from using geometrically convergent search methods such as bisection. Instead, we start the search for the best possible rate from $r+\phi$ and crawl forward until condition (3.3) is met. This is the most time-consuming part of the algorithm we describe in an online appendix.

3.3 Cross-Sectional Distribution

From any given set of initial conditions and given any given realization of home prices and rental rates, our model implies a sequence of distributions of household states. This section makes the mapping from aggregate price shocks to distributions precise. Much of our upcoming calibration entails matching moments of these cross-sectional distributions with the relevant data.

The set of possible histories of aggregate shocks up to date t is S^t. An element $s^t \in S^t$ implies a path for home prices, approval standards and rental rates. Now recall that old households who die are immediately replaced by young households. Therefore, the transition matrix across ages is effectively given by:

$$\begin{bmatrix} (1-\rho^M) & \rho^M & 0 \\ 0 & (1-\rho^O) & \rho^O \\ \rho^D & 0 & 1-\rho^D \end{bmatrix}.$$

Let $(\psi^{\mathcal{Y}}, \psi^{\mathcal{M}}, \psi^{\mathcal{O}})$ be the corresponding invariant distribution of ages. Making the mass of agents born each period $\mu^0 \equiv \psi^{\mathcal{O}}\rho^D$ normalizes the total population size to one. Recall in addition that newborns start their life with no assets and that their income is drawn from the unique invariant income distribution p^0 associated with $P^{\mathcal{Y}}$.

There are five fundamental types of agents in our environment. Old agents are distributed over the set of possible assets

$$\Omega^{\mathcal{O}} = I\!R_+.$$

Denote the distribution of individual states for old households at the start of date t given a history s^{t-1} of aggregate shocks up to the preceding period by

$$\mu^t_{\mathcal{O}}(\cdot|s^{t-1}) : B\left(\Omega^{\mathcal{O}}\right) \mapsto [0,1]$$

where $B\left(\Omega^{\mathcal{O}}\right)$ is the set of Borel measurable subsets of $\Omega^{\mathcal{O}}$. By convention, we will define this and all state distributions at a given date after all shocks are realized but before housing choices are made.

Young agents are distributed on the following set of possible states:

$$\Omega^{\mathcal{Y}} = \left\{(a, y) \in I\!R_+ \times Y^{\mathcal{Y}}\right\}.$$

Denote by $\mu^t_{\mathcal{Y}}(\cdot|s^{t-1})$ the corresponding cross-sectional distribution of individual states for the young given a history s^{t-1} of past shocks.

Mid-aged renters and mid-aged households with the option to buy ($n = 0$) are distributed over the same asset-income space

$$\Omega^{\mathcal{M},R} = \Omega^{\mathcal{M},n=0} = \left\{(a, y) \in I\!R_+ \times Y^{\mathcal{M}}\right\}$$

and we denote their respective history-conditional distributions at date t by $\mu^t_{\mathcal{M},R}(\cdot|s^{t-1})$, and $\mu^t_{\mathcal{M},n=0}(\cdot|s^{t-1})$.

The fifth and final type of agents are mid-aged homeowners. Their state includes not only their asset and income position, but also their idiosyncratic house price shock (ϵ), their mortgage type (κ, ν) and the age (n) of their contract. The corresponding space is:

$$\Omega^{\mathcal{M},n\geq 1} = \left\{(a, y, \epsilon, n, \nu, \kappa) \in I\!R_+ \times Y^{\mathcal{M}} \times \mathcal{E} \times I\!N_{++} \times \{LD, HD\} \times \mathcal{K}\right\}$$

where $\mathcal{K} = \left\{I\!R_+ \times Y^{\mathcal{M}} \times \{h^2, h^3\} \times S\right\}$ is the set of possible mortgage characteristics at origination. Let $\mu^t_{\mathcal{M},n\geq 1}(\cdot|s^{t-1})$ be the associated distribution.

With this notation in hand, we can define transition functions for distributions of individual states. Consider first the young. Let A' be any Borel subset of $I\!R_+$ and pick any $y' \in Y^{\mathcal{Y}}$. An agent is young at the start of a period if: (i) they were just born; or (ii) they were young in the previous period and did not age. It follows that for any t,

$$\mu_{\mathcal{Y}}^{t+1}(A', y'|s^t) = \mu^0 p^0(y') 1_{\{0 \in A')\}} + (1-\rho^M) \int_{\omega \in \Omega^{\mathcal{Y}}} 1_{\{a'_{\mathcal{Y}}(\omega; s^t) \in A')\}} P^{\mathcal{Y}}(y'|y) d\mu_{\mathcal{Y}}^t(\omega|s^{t-1}).$$

Here, $a'_{\mathcal{Y}}(\omega; s^t)$ is the agent's savings choice given his individual state $\omega = (a, y) \in \Omega^{\mathcal{Y}}$ and aggregate history $s^t \in S^t$. Transitions are similarly defined for the old and we omit them for conciseness.

Agents are mid-aged renters at the start of a given period if: (i) they were mid-aged renters, did not get the option to buy, and did not age; or (ii) if they had the option to become homeowners in the previous period, chose to forego that option, and did not age. For any measurable subset (A', y') of $\Omega^{\mathcal{M},R}$ and history, the transition is given by

$$\begin{aligned} \mu_{\mathcal{M},R}^{t+1}(A', y'|s^t) &= (1-\rho^O)(1-\gamma) \int_{\omega \in \Omega^{\mathcal{M},R}} 1_{\{a'_{\mathcal{M},R}(\omega; s^t) \in A')\}} P^{\mathcal{M}}(y'|y) d\mu_{\mathcal{M},R}^t(\omega|s^{t-1}) \\ &+ (1-\rho^O) \int_{\{\omega \in \Omega^{\mathcal{M},n=0} : h_{\mathcal{M},n=0}(\omega; s^t) = h^1\}} 1_{\{a'_{\mathcal{M},n=0}(\omega; s^t) \in A')\}} P^{\mathcal{M}}(y'|y) d\mu_{\mathcal{M},n=0}^t(\omega|s^{t-1}). \end{aligned}$$

Here, $h_{\mathcal{M},n=0}(\omega; s^t)$ is the household's housing choice given its individual state $\omega = (a, y) \in \Omega^{\mathcal{M},n=0}$ and aggregate history $s^t \in S^t$.

Households start a period with the option to buy if: (i) they were mid-aged renters in the previous period and received the option to buy; or (ii) if they just became mid-aged. This gives, for any measurable subset (A', y') of $\Omega_{\mathcal{M},n=0}$ and history,

$$\begin{aligned} \mu_{\mathcal{M},n=0}^{t+1}(A', y'|s^t) &= (1-\rho^O)\gamma \int_{\omega \in \Omega^{\mathcal{M},R}} 1_{\{a'_{\mathcal{M},R}(\omega; s^t) \in A')\}} P^{\mathcal{M}}(y'|y) d\mu_{\mathcal{M},R}^t(\omega|s^{t-1}) \\ &+ \rho^M \int_{\omega \in \Omega^{\mathcal{Y}}} 1_{\{a'_{Y}(\omega; s^t) \in A')\}} P^{\mathcal{Y}}(y'|y) d\mu_{\mathcal{Y}}^t(\omega|s^{t-1}) \end{aligned}$$

Finally, the cross-sectional distribution of homeowners evolves according to whether they were: (i) homeowners in the previous period and did not age or choose to sell or default; or (ii) were given the option to buy, took it, and did not change age state. Consider any Borel subset A' of $I\!R_+$, $y' \in Y^{\mathcal{M}}$, $\varepsilon' \in \mathcal{E}$, $n' \in I\!N_{++}$, $\widehat{\nu} \in \{LD, HD\}$ and $\widehat{\kappa} = (\widehat{a}, \widehat{y}, \widehat{h}; \widehat{s}) \in \mathcal{K}$ at any date t and in any history s^t. Denote by $\Omega_n^{(\kappa,\nu)}$ the subset of $\Omega^{\mathcal{M},n\geq 1}$ of homeowners with mortgage characteristics (κ, ν) and mortgage age $n \in I\!N$. Then,

$$\mu^{t+1}_{\mathcal{M},n\geq 1}(A', y', \epsilon', n', \widehat{\nu}, \widehat{\kappa}|s^t) =$$

$$(1-\rho^O) \times \Bigg\{ \int_{\omega \in \Omega^{(\kappa,\nu)}_{n'-1}} 1_{\{a'_{\mathcal{M},n\geq 1}(\omega;s^t)\in A', h_{\mathcal{M},n\geq 1}(\omega;s^t)=\widehat{h}\}} P^{\mathcal{M}}(y'|y) P^{\epsilon}(\epsilon'|\epsilon) d\mu^t_{\mathcal{M},n\geq 1}(\omega|s^{t-1})$$
$$+ \int_{\omega\in\Omega^{\mathcal{M},n=0}} 1_{\{a'_{\mathcal{M},n=0}(\omega;s^t)\in A', h_{\mathcal{M},n=0}(\omega;s^t)=\widehat{h}, \nu_{\mathcal{M},n=0}(\omega;s^t)=\widehat{\nu}, s^t_t=\widehat{s}\}} P^{\mathcal{M}}(y'|y) P^{\epsilon}(\epsilon'|1) d\mu^t_{\mathcal{M},n=0}(\omega|s^{t-1}) \Bigg\}$$

where s^t_t is the date t realization of aggregate shock history s^t.

3.4 Definition of Equilibrium

Given an an initial distribution of household states $\{\mu^0_{\mathcal{Y}}, \mu^0_{\mathcal{M},n=0}, \mu^0_{\mathcal{M},n\geq 1}, \mu^0_{\mathcal{M},R}, \mu^0_{\mathcal{O}}\}$, an equilibrium is a set of recursive household policy functions, a menu K of mortgages, and a sequence $\{\mu^t_{\mathcal{Y}}, \mu^t_{\mathcal{M},n=0}, \mu^t_{\mathcal{M},n\geq 1}, \mu^t_{\mathcal{M},R}, \mu^t_{\mathcal{O}}\}_{t=1}^{+\infty}$ of cross-sectional distributions for all possible histories of aggregate shocks such that:

1. Policies solve the household problems in subsection 3.1 given the mortgage menu K;

2. Given household policies and for all possible $\omega \in \Omega_{\mathcal{M},n=0}$ and aggregate histories s^t, mortgage options in $K(\omega, s^t)$ satisfy the intermediary's zero-profit condition (3.3) in subsection 3.2;

3. The contingent history of cross-sectional distributions is the one implied by optimal household policies for all possible histories of aggregate shocks, as detailed in subsection 3.3.

In all the equilibria we discuss below, aggregate household assets vastly exceed the balance on outstanding mortgage hence storage investments are strictly positive in all periods. Since all mortgage loans are priced in such a way that the intermediary is indifferent between storage and funding mortgages, loan markets trivially clear and our economy is effectively closed.

4 Parameter Selection

Our main quantitative goal is to simulate a course of aggregate home price shocks that is consistent with the pattern displayed in figure 3 under various scenarios for approval standards. To that end, we first need to parameterize the model. We take a model period to be 2 years long so that we only need to keep track of 15 model periods when considering 30-year mortgages.

4.1 Parameters Selected Independently

As evident in figure 3, real home values were relatively stable between 1890 and 2013 with two key exceptions: a roughly 25-year span of low relative home values that begins around 1918, and the recent boom period between 1999 and 2006. In order to approximate these data with our three-point process, we will treat the 1890-1917 and 1944-1998 time span as periods where real home values are at their intermediate, "normal" level q_N, while home values are at their low level q_L between 1918 and 1943 and at their high level q_H between 1999 and 2006.

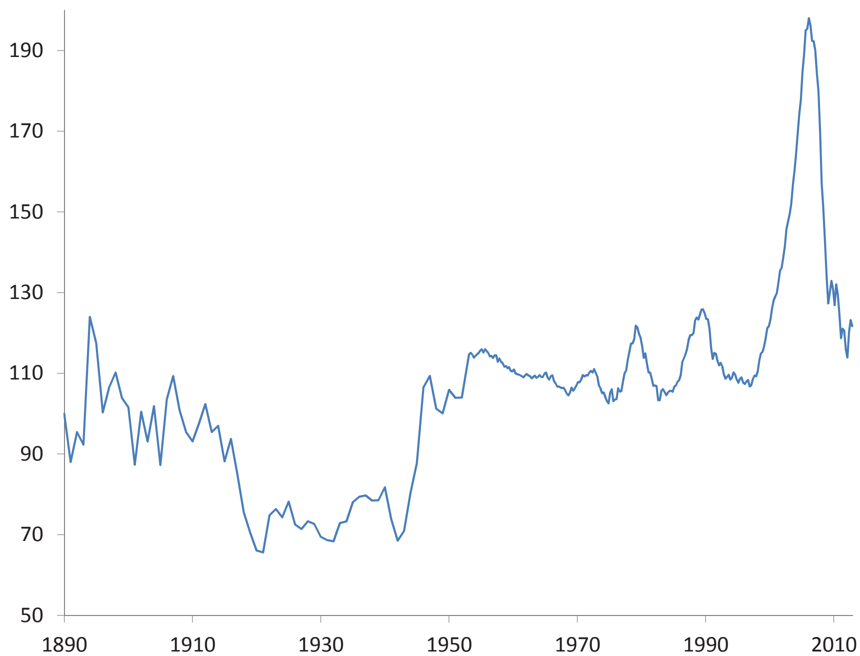
The normal level q_N of home values will be selected below when we target pre-housing boom moments. To match the magnitude of historical deviations from this typical level, we specify

$$\mathcal{Q} = q_N \times (0.7, 1, 1.45)$$

since home values are roughly 30% below their normal time average value between 1918 and 1943 and are, on average, 45% above their normal level between 1999 and 2006.[18]

[18]Real home values peak at near 85% above their previous trough in 2006 but since we are approximating the entire 1999-2006 period with one q level, we are effectively calibrating q_H to its mid-point value during the boom. Another virtue of this calibration is that it implies a 30% decline in values in the first two years of the crisis which roughly matches the decline in the real US Case-Shiller index between the last quarter of 2006 and the last quarter of 2008.

Figure 3: Real US home price index since 1890



Source: Shiller (2000), updated data available at http://www.econ.yale.edu/ shiller/data.htm.

We then calibrate the transition matrix P^q so that: 1) two deviations from normal value levels are expected in any given century; 2) deviations to q_L are expected to last 25 years; and 3) deviations to q_H are expected to last 8 years.[19] Since we think of a model period as

[19]Obviously, calibrating the expected length of each deviation to match the exact duration of their unique respective counterpart is but one of many ways to pin down expectations but it seems to be the natural starting point. Furthermore, our results are not sensitive to that assumption: calibrating P^q so that the boom is expected to last 20 years rather than 8 years barely changes our main quantitative findings.

lasting two years, the transition matrix for the aggregate shock for all t is:

$$P^q = \begin{bmatrix} 0.90 & 0.10 & 0.00 \\ 0.02 & 0.96 & 0.02 \\ 0.00 & 0.25 & 0.75 \end{bmatrix}.$$

As for rental rates, Davis et. al (2008) calculate that the ratio of yearly gross rents to house prices is around 5% for much of the 1960-2008 time period with the exception of the boom period when the ratio falls to about 3.5%. Correspondingly, and given our two-year convention, we set $R_N = .10 \times q_N$ and $R_H = .07 \times q_H$. Since rent-to-price data do not exist to our knowledge for earlier periods, we simply assume that the ratio is also around its typical 10% during period of low prices hence set $R_L = .1 \times q_L$.

Next we set demographic parameters to $(\rho^M, \rho^O, \rho^D) = (\frac{1}{7}, \frac{1}{15}, \frac{1}{10})$ so that, on average, agents are young for 14 years starting at 20, mid-aged for 30 years, and old aged (retired) for 20 years. Recall that becoming old in our environment has two key consequences: agents are forced to sell and their income expectation are permanently altered. In particular, a recent homeowner who experiences that shock early in the life of their mortgage loan can be constrained to default as a result of this exogenous shock. Roughly speaking, setting $\rho^O = \frac{1}{15}$ means that households expect to be constrained to sell for exogenous (non-income) reasons once every 30 years.

The income process is calibrated using the Panel Study of Income Dynamics (PSID) survey. We consider households in each PSID sample whose head is between 20 and 34 years of age to be young while households whose head is between 35 and 64 years are considered to be mid-aged. Each demographic group in the 1997 and 1999 PSID surveys is then split into income quartiles. The support for the income distribution is the average income in each quartile in the two surveys, normalized by the median income value for the mid-aged group.

This yields a support for the income distribution of young agents of:

$$Y^{\mathcal{Y}} = \{0.1452, 0.5725, 0.9216, 1.8533\},$$

while the support for mid-aged agents is

$$Y^{\mathcal{M}} = \{0.1543, 0.7199, 1.3320, 2.8555\}.$$

We then equate the income transition matrix for each age group to the frequency distribution of transitions across quartiles for households which appear in both the 1997 and 1999 survey. The resulting transition matrix for young agents is:

$$P^{\mathcal{Y}} = \begin{bmatrix} 0.5920 & 0.2759 & 0.1034 & 0.0287 \\ 0.1292 & 0.5015 & 0.2769 & 0.0923 \\ 0.0512 & 0.1898 & 0.491 & 0.2681 \\ 0.0317 & 0.0762 & 0.1238 & 0.7683 \end{bmatrix},$$

while, for mid-aged agents, it is:

$$P^{\mathcal{M}} = \begin{bmatrix} 0.7490 & 0.1926 & 0.0393 & 0.0190 \\ 0.1787 & 0.6388 & 0.1559 & 0.0266 \\ 0.0546 & 0.1615 & 0.6394 & 0.1445 \\ 0.0202 & 0.0303 & 0.1573 & 0.7921 \end{bmatrix}.$$

Income in old age is $y^O = 0.40$. This makes retirement income 40% of median income among the mid-aged, which is consistent with standard estimates of replacement ratios.

We next let the (two-year) risk-free rate be $r = 0.08$ and choose the maintenance cost (δ) to be 5% in order to match the yearly gross rate of depreciation of housing capital, which is 2.5% annually according to Harding et al. (2007).

The down-payment ratio ν^{HD} is 20% while the maturity T is 15 periods (=30 years).

Table 1: **Benchmark parameters**

Parameters determined independently			*Parameters determined jointly*		
ρ^M	Fraction of young agents who become mid-aged	1/7	θ	Owner-occupied premium	1.767
			β	Discount rate	0.849
ρ^O	Fraction of mid-aged agents who become old	1/15	$\alpha_N (= \alpha_L)$	PTI level in normal (low) state	0.200
			ϕ	Mortgage service cost	0.058
ρ^D	Fraction of old agents who die	1/10	χ	Foreclosing costs	0.499
			h^1	Size of rental unit	1.000
r	Storage return	8%	h^2	Size of small house	1.225
δ	Maintenance rate	5%	h^3	Size of large house	1.879
ν^{HD}	High downpayment	20%	λ	Home-value shock probability	0.217
T	Mortgage maturity	15	$\tilde{\epsilon}$	Size of home value shock	0.351
$\frac{q_L}{q_N}$	Low home value level	0.7	q_N	Normal home value level	0.864
$\frac{q_H}{q_N}$	High home value level	1.45			

Low-downpayment contracts have the same 30-year term but require no downpayment (i.e. $\nu^{LD} = 0$). The PTI requirement is assumed to be the same for both mortgages and the same whether $q = q_L$ or $q = q_N$. During the boom (when $q = q_H$), PTI constraints are fully relaxed. We will think of the second stage of our transition experiment as a period of high prices and relaxed approval standards, and compare the model's prediction for that stage to the relevant data from the 1999-2006 period. The PTI level when $q \in \{q_L, q_N\}$ will be selected in the joint part of the calibration, to which we now turn.

4.2 Parameters Selected Jointly

Our strategy to jointly select remaining parameters is to think of the few years that preceded the housing boom as following a long period of normal aggregate home values. We make the assumption that the distribution of household states at that point is near the long-run distribution that would obtain following a infinitely long draw $\{q_t = q_N\}_{t=0}^{+\infty}$ of normal aggregate home values.[20]

We make the strong assumption that buying a home is a one-time-only option for computational tractability (i.e. $\gamma = 0$). Forcing agents who have sold their home or defaulted to become renters for the rest of their life enables us to price mortgage contracts for each possible asset-income-house size position at origination independently from rates offered to borrowers with different characteristics. If agents had the option to take another mortgage after they terminate their first contracts, their decisions to default – hence the intermediary's expected profits – would depend on future contracts, which would mean we need to jointly solve a high-dimensional set of fixed points. We emphasize, however, that this does not imply that all home-buyers are identical. Since agents become mid-aged stochastically, the model generates an endogenous distribution of asset-holdings among potential home-buyers. As we will argue in the next section, this heterogeneity in the pool of borrowers matters critically for contract selection.

Remaining parameters include the owner-occupied premium (θ), the household discount rate (β), the normal level (q_N) of home prices, the mortgage service premium (ϕ), the PTI level $\alpha_N = \alpha_L$, the foreclosure transaction cost (χ), and the housing commodity space (h^1, h^2, h^3). We normalize the location of the housing space by making $h^1 = 1$ since a parallel shift in (h^1, h^2, h^3) together with an offsetting shift in q_N leaves the equilibrium allocation unchanged. As for the idiosyncratic home price shock, we specify

$$\mathcal{E} = \{1 - \tilde{\epsilon}, 1, 1 + \tilde{\epsilon}\}$$

[20] Alternatively and given the data shown in figure 3, one could compute model counterparts for pre-1998 moments by starting at the US economy at this long run distribution in 1918, assuming that 12 or 13 model periods (24 or 26 years) of low prices followed, and that 27 periods (44 years) of normal home price levels followed that low price phase. Predicted model moments are virtually unchanged under that alternative strategy.

and

$$P^{\epsilon} = \begin{bmatrix} \lambda & 1-\lambda & 0 \\ \lambda & 1-2\lambda & \lambda \\ 0 & 1-\lambda & \lambda \end{bmatrix},$$

which adds two parameters to be calibrated: $(\tilde{\epsilon}, \lambda) \in [0, 1] \times [0, \frac{1}{2}]$. Note that this symmetric specification of the idiosyncratic process implies that households expect zero capital gains absent aggregate shocks. It also implies that the standard deviation of idiosyncratic gains over the first two years of home ownership is $\sqrt{2\lambda}\tilde{\epsilon}$. We will use a data counterpart for that moment to discipline the parameterization of P^{ϵ}.

We select the ten remaining parameters via a simulated method of moments so that, at the long-run distribution associated with aggregate state N, our model best approximates pre-1998 US data counterparts for eleven targets. Our first target is the ownership rate among households whose head is between 30 and 55 years old, and use as model counterpart for this statistic the rate of ownership among agents who have been mid-aged for thirteen periods or fewer. According to Census data, the home-ownership rate is roughly 66% for that age range on average between 1990 and 1998.

Our second target is the average ratio of non-housing assets to income among homeowners whose head age is between 35 and 64 in the 1998 Survey of Consumer Finance (SCF) survey. The yearly value of this ratio is 2.84, which corresponds to a ratio of assets to two-years worth of income of 1.42.[21]

We then set three housing spending share targets meant to identify the parameterization of (h^2, h^3) and q_N. First, according to the evidence available from the Bureau of Economic Analysis' Personal Consumption Expenditure data, the ratio of rents (in imputed terms for owners) to overall expenditures is near 15%. Second, Green and Malpezzi (1993, p11) calculate that renters in the bottom income deciles in the US spend between 40% and 50% of their

[21] Because agents only have one asset in our model besides a house, we interpret a as net assets. Our measure of net assets does not include housing-related assets or debts, such as home equity or mortgages. Since agents are not allowed to have negative assets in our model, households who have negative non-housing assets are assumed to have zero assets in the calculation.

income on rent. Correspondingly, we target a ratio of rents to income for renters at income level y_1 of 45%. Third, according to the 1998 Consumer Expenditure Survey, expenditures on shelter account for 17.3% of the expenditures of home-owners.

Next, we target an average yield of 14.5% for the high-down-payment mortgage over a two-year period, which corresponds to 7.25% a year. This target implies a 300 basis points spread in bond-equivalent terms between 30-year conventional fixed-rate mortgage rate and 1-year treasury rates which is consistent with data available from the Federal Housing Finance Board for the 1990-98 time period.

Pinto (2010) estimates that among conventional and FHA mortgages, loans with a CLTV in excess of 97% accounted for roughly 4% of home purchase origination between 1990 and 1998. Glaeser et. al. (2012) estimate on the other hand that mortgages with zero equity at origination accounted for "at least 10%" of purchase originations by 1998. We choose to target a pre-boom fraction of LD originations at the midpoint of those two estimates, namely 7%.

Since mortgage pricing obviously depends on expected losses in the event of default, we include in our set of targets a loss severity rate defined as the present value of all losses on a given foreclosed loan as a fraction of the default date balance. As Hayre and Saraf (2008) explain, these losses are caused both by transaction and time costs associated with the foreclosure process, and by the fact that foreclosed properties tend to sell at a discount relative to other, similar properties. Using a dataset of 90,000 first-lien liquidated loans, they estimate that loss severity rates range from around 35% among recent mortgages to as much as 60% among older loans. Based on these numbers we choose parameters so that in the event of default and on average,

$$\frac{\min\{(1-\chi)qh, b)}{b} = 0.5$$

where b is the outstanding principal at the time of default and qh is the house value. In other words, on average, the intermediary recovers 50% of the outstanding principal it is owed on defaulted loans.

While estimates vary across studies (see Pennington-Cross, 2006, for a review), a typical finding is that foreclosed properties sell for a price that is around a quarter lower than that of observably similar properties. We therefore target a market discount on foreclosed properties of 75%. We define this discount to be the average price of foreclosed properties divided by the average price of regular home sales, after conditioning on size at origination.

The average foreclosure discount and the average loss severity rates are related since part of the loss incurred by intermediaries in the event of default stems from the fact that foreclosed properties tend to be devalued properties. However, a loss in market value of 25% alone could not account for an average loss severity rate of 50%. In the data, this discrepancy reflects the transaction costs associated with foreclosure. Our transaction cost parameter χ proxies for these costs and we use this parameter in our calibration to bridge the gap between the foreclosure discount and the total loss associated with foreclosure.

Our final two targets are intended to inform the parameterization of the idiosyncratic process. First, we use data from the 1998 SCF to estimate a standard deviation of roughly 22% of reported capital gains on homes purchased in 1996 or 1997 by households whose head is between 35 and 64 years old.[22] Second, we target the rate of mortgage terminations caused by default in 1998. To measure that number, we begin with data from the Mortgage Bankers Association (MBA)'s National Delinquency Survey on the fraction of mortgages that enter the foreclosure process in a given quarter. As Jeske et. al. (2011) or Herkenhof and Ohanian (2012) among others have pointed out, many foreclosures started do not end up leading to termination and eviction as they do in our model. Jeske et. al. (2011) point to a 2006 survey that suggest that around a quarter of foreclosure started end up in liquidation. To get more direct evidence on the eventual outcome of foreclosure starts we obtained a dataset[23] that

[22]In all our SCF computations, we use the exact calculation methods used by the Federal Reserve Board to produce its biennial *Federal Reserve Bulletin* SCF summary. To minimize the effect of outliers, we remove observations with reported 2-year gains in excess of 200% or 2-year losses in excess of 75%. These represent 3% of house capital gains reports in SCF-weighted terms in the 1998 survey.

[23]We purchased the data from Illinois' Record Information Services (RIS). RIS collects and compile these data from case files of county circuit court. See http://www.public-record.com/ for details. The dataset covers the following counties: DeKalb, DuPage, Kane, Kendall, Lake, McHenry and Will, and includes over 180,000 foreclosure start records between 1998 and 2011. The number of yearly starts hovers between 4,000

Table 2: Long-run moments

	Pre-98 data	Pre-98 benchmark	Long boom
Home-ownership rate	0.66	0.65	0.72
Ex-housing asset to income ratio	1.42	1.53	1.46
Housing expenditure share	0.15	0.15	0.15
Rent to income ratio	0.45	0.56	0.57
Homeowner housing share	0.173	0.183	0.277
Interest rate on HD loans	0.145	0.148	0.161
Foreclosure rate	1.45	1.41	2.52
Foreclosure discount	0.75	0.70	0.72
Recovery rate	0.50	0.50	0.45
Fraction of LD loans	0.07	0.07	0.33
Standard error of 2-year capital gains	0.22	0.23	0.23

tracks all foreclosures started in seven Chicago-area counties between 1998 and 2011 from the time the procedure is legally initiated to its end either by auction or non-auction resolutions. In these data, 52.7% of foreclosures started in 1998 end up leading to an auction. Over the entire 1998-2011 period the ratio of auctions to start is 49.3%. In light of these statistics, we chose to target half the rate of foreclosure starts reported by the MBA in 1998 which gives a two-year target default rate of 1.45 (or roughly 0.18 on the standard quarterly basis.)

4.3 Model Fit

Tables 1 and 2 show the outcome of minimizing the distance between our model's predictions and these targets. The estimate of normal and low state PTI is $\alpha = 20\%$ which, at first glance, seems low given that the traditional ratio typically used in mortgage underwriting in the 1990s was closer to 30% (29% for low-downpayment FHA loans.)[24] However, in practice, that ratio includes property taxes, insurance and other owning costs such as home-owner association fees which we do not model. Furthermore, in our environment, the PTI must

and 10,000 until 2006 but then increases to reach a peak of nearly 33,000 in 2010.

[24]See Bunce et. al., 1995.

proxy for all underwriting criteria that governed approval decisions before the boom. Given these considerations, our model PTI does not seem unreasonable.

Overall, the method of moments we employ reaches a set of ten parameters that produces model moments that are very close to the eleven targets we described in the previous section. In our transition experiment, we will show that the model also makes reasonable predictions for key aspects of the post-98 boom period which we did not explicitly target, including the behavior of home-ownership rates and the rise of high-priced loans during the transition. This section considers another set of predictions we didn't explicitly target: how the use of leverage co-varies with borrower characteristics in the cross-section.

Table 3 compares the cross-sectional predictions of our model for the use of leverage as proxied by loan-to-income (LTY) ratios at origination and the fraction of loans with cumulative loan-to-value ratios above 95% to the corresponding evidence from the 1998, 2007, and 2010 Survey of Consumer Finance. The model counterparts are the pre-98 benchmark, the last period of the boom in the transition experiment we will describe in detail in section 6, and the first period of the crisis in that same experiment. All data moments are computed for homeowners who bought a home in the two years preceding the survey.

Predicted loan-to-value ratios are below their SCF counterparts in 1998 but are at the right level in 2007, at the end of the boom period. The same pattern holds for the frequency of high-CLTV loans which is almost as high in the SCF in 1998 as it is in 2007.[25] Given our calibration strategy (we target a 7% frequency of high-LTV loans in 1998), the model under-predicts the use of high-leverage loans in 1998 but over-predicts it in 2007. As we will discuss in the transition section however, our model's predictions for the overall fraction of high leverage loans at the onset of the crisis accords well with other sources of data.

[25] The fraction of loans with CLTV higher than 95% rises from 13.5% to 15.5%. At the same time, these same data contain a high number of unreasonably high self-reported CLTVs on recent loans which suggests that the SCF is not a reliable source for measuring average CLTVs at origination. Here we use these data to evaluate where whether our model makes reasonable predictions for how leverage should covary with observable characteristics.

Table 3: Cross-sectional predictions

	1998 survey				2007 survey				2010 survey			
	LTY		High-LTV		LTY		High-LTV		LTY		High-LTV	
	Data	Model	Data	Model	Data	Model	Data	Model	Data	Model	Data	Model
Income												
Below median	1.27	0.87	0.16	0.14	1.92	1.91	0.21	0.55	1.57	0.85	0.22	0.25
	(0.04)		*(0.02)*		*(0.06)*		*(0.01)*		*(0.06)*		*(0.01)*	
Above median	0.71	0.46	0.10	0.00	1.07	0.76	0.10	0.12	1.01	0.46	0.21	0.00
	(0.01)		*(0.01)*		*(0.02)*		*(0.01)*		*(0.02)*		*(0.01)*	
Asset-to-income												
Below median	0.95	0.68	0.19	0.14	1.49	1.34	0.22	0.58	1.16	0.65	0.28	0.25
	(0.02)		*(0.02)*		*(0.04)*		*(0.02)*		*(0.03)*		*(0.02)*	
Above median	0.97	0.64	0.08	0.00	1.47	1.33	0.09	0.08	1.38	0.65	0.14	0.00
	(0.02)		*(0.01)*		*(0.04)*		*(0.01)*		*(0.05)*		*(0.01)*	
Age												
Below 35	0.99	0.66	0.17	0.10	1.60	1.33	0.24	0.43	1.24	0.65	0.25	0.19
	(0.03)		*(0.02)*		*(0.04)*		*(0.02)*		*(0.04)*		*(0.02)*	
Above 35	0.95	0.68	0.11	0.04	1.37	1.36	0.10	0.20	1.30	0.66	0.18	0.04
	(0.02)		*(0.01)*		*(0.04)*		*(0.01)*		*(0.04)*		*(0.01)*	
Loan size												
Below median	0.79	0.54	0.06	0.11	1.08	1.33	0.16	0.29	0.89	0.54	0.16	0.18
	(0.03)		*(0.02)*		*(0.03)*		*(0.01)*		*(0.02)*		*(0.01)*	
Above median	1.10	0.74	0.21	0.04	1.79	1.31	0.15	0.39	1.57	0.72	0.26	0.07
	(0.02)		*(0.01)*		*(0.04)*		*(0.01)*		*(0.05)*		*(0.01)*	

Notes: All numbers are for purchase loans originated in the two years preceding each survey. LTY entries are the average loan-to-income ratio in each categories and include all reported liens. High-LTV entries are the fraction of contracts with cumulative leverage in excess of 95% of reported home value. SCF sample weights are used throughout, standard errors are in parentheses. In the model, "Below 35 years" refers to homeowners who become middaged after 12 periods of youth or fewer.

Loan-to-income ratios are lower in the 2010 survey than in the 2007 survey in all subgroups, most notably among poor and young home-owners. Our model delivers a fall in LTYs as well since approval standards become tighter when the boom end but the model's predicted correction is more abrupt than in the data.

The table shows that our model correctly predicts that loan-to-income ratios and the use of high-CLTV loans should fall with income. There seems to be little relationship between asset holding or age and LTYs in either the model or the data. The model correctly predicts that low-downpayment loans should be highly concentrated at the bottom of the asset distribution and among young borrowers.[26] Quantitatively, our model exaggerates the concentration of high-leverage loans among asset-poor agents, which may owe to the fact that we only give agents one ex-housing investment alternative. Introducing investment options with different risk-return characteristics could lead more asset-rich agents to take advantage of high-leverage mortgage options.

The third section of the table breaks down home-owners according to whether or not the head is 35 years of age or younger. In the model, we classify a home buyer to be under 35 years of age if they become mid-aged in 12 periods or fewer. There is no clear relationship between age and LTYs but younger households select high-LTV loans more frequently both in the model and in the data.

The use of leverage is higher for loans of above-median size pre-98, both in the data and our model. In the 2007 SCF, LTYs fall with loan size. The model fails to fully capture this apparent reversal but it does go from predicting a positive relationship between loans size and LTYs to predicting no relationship.

In summary then, the model misses on some of the quantitative features of cross-sectional patterns in the SCF which is not surprising given that it only contains so many sources of heterogeneity among households. At the same time, the model correctly picks up the parts

[26] Since $\gamma = 0$, our simulations do not make predictions for repeat buyers but, to the extent that these would be tend to older buyers in the model, these repeat buyers would presumably be less likely to select high-leverage loans.

of the income, asset and age distributions where leverage should be the most prevalent.

5 Long-run Consequences of Rising Leverage

This section compares properties of the long run equilibrium associated with normal times (our benchmark) to a long run equilibrium associated with a boom in house prices and relaxed approval standards. The last column of table 2 displays equilibrium moments at the long-run distribution that follows a "long boom" period $\{q_t = q_H, \alpha_t = \infty\}_{t=0}^{+\infty}$. Comparing these two long-run equilibria highlights the impact of leverage on housing and default decisions and paves the way for interpreting the transition experiments we run in Section 6.

5.1 Contract Selection and Home-Ownership

The last column of table 2 shows that the change in approval standards causes home-ownership rates to rise from 65% to 72% and that the use of LD loans rises dramatically (from 7% of originations to 33%). Put another way, the greater availability of LD loans in the long-boom equilibrium more than offsets the fact that homes become more expensive.

Table 4 displays contract selection and housing decisions in the benchmark and in the boom state conditional on a household's asset and income position at origination. In the benchmark, none of the households at the very bottom of the income distribution are able to become home-owners. During the boom, some of the lowest income households whose assets are sufficiently high for the intermediary to expect to break even are able to enter mortgage markets. But all of them make a 20% down-payment because the interest rate the intermediary would need to impose on those households would be prohibitively high if they started with no equity. Poor homeowners are mostly found in the second income quartile. For those households, the asset threshold above which they become home-owners falls from 1.77 to 0.63 as we go from the benchmark to the boom and low-asset agents use the LD mortgage to become home-owners. Households in the third income quartile who have low

Table 4: Housing and mortgage decisions by asset and income

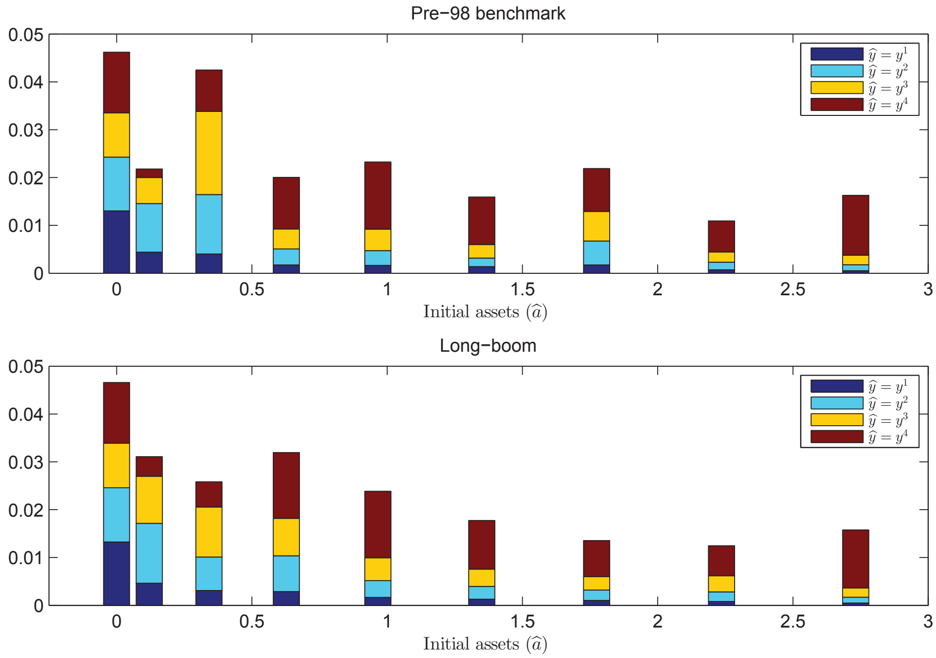
		Rent	LD loan		HD loan	
		h^1	h^2	h^3	h^2	h^3
	y^1	all a_0	–	–	–	–
Normal state	y^2	$a_0 < 1.77$	–	–	$1.77 \leq a_0$	–
($q = q_N$, $PTI = 20\%$)	y^3	–	$a_0 < 0.34$	–	–	$0.34 \leq a_0$
	y^4	–	–	$a_0 < 0.34$	–	$0.34 \leq a_0$
	y^1	$a_0 < 1.35$	–	–	$1.35 \leq a_0 < 3.26$	$3.26 \leq a_0$
Boom state	y^2	–	$a_0 < 0.63$	–	$0.63 \leq a_0 < 1.35$	$1.35 \leq a_0$
($q = q_H$, no PTI)	y^3	–	–	$a_0 < 0.63$	–	$0.63 \leq a_0$
	y^4	–	–	$a_0 < 0.63$	–	$0.63 \leq a_0$

assets ($a_0 < 0.34$) now opt for larger homes using an LD-loan while those in the intermediate range ($a_0 \in (0.34, 0.63)$) continue to buy large homes but use LD loans in the boom since the price rise makes the downpayment expensive. At the top of the income distribution, all agents buy large homes but those with low assets now opt more frequently for LD loans.

The frequency of different housing contracts depends both on the decision rules displayed in Table 4 and on the cross-sectional income and asset distribution. Figure 4 shows the endogenous asset distribution among households who just became mid-aged in the pre-98 benchmark and in the long boom (i.e. $\mu_{\mathcal{M},n=0}(A, y|s)$). It is clear from the figure that low income but high asset agents are a rare type of homeowner during boom times. They turn out to represent less than 5% of home-owners. All but 15% of households who have the lowest income choose to remain renters. One key reason why low-income but high-asset households are rare in this economy is that the income process is highly persistent so that assets and income are highly correlated. Low income households are heavily concentrated at the bottom of the asset distribution in both equilibria.

In summary then, LD loans tend to allow agents with lower assets to start participating in mortgage markets, to take on bigger loans, or to put less money down even when they choose to remain at the same loan size. Because the income process is highly persistent, assets and income tend to be correlated hence the greater frequency of LD loans implies a

Figure 4: Distribution of assets upon entering mid-age



greater frequency of low income borrowers. Combined with the direct effect of the relaxation of PTI constraints, the composition of the pool of borrowers shifts towards the low income and low asset part of the state space.

5.2 Default

The last column of table 2 shows a large long-run increase in overall default rates (by over 75%) when approval standards are relaxed. Default rates increase during booms as a result of two complementary factors. First, the greater availability of LD mortgages enable agents at the bottom of the asset and income distribution to select into homeownership. These are high-default risk agents because they are more likely to find themselves unable to meet their mortgage payments at some point over the life of the contract. Second, even at equal asset and income conditions at origination, LD loans are associated with higher default rates because agents are slower to build up home equity.

To illustrate the second "equity" channel, figure 5 displays the evolution of home equity as a function of maturity for both types of contracts of size $q_N h^3$ and given a contract rate of 14.5%. LD loans feature less equity at all maturities than HD loans for obvious reasons. The key consequence of this fact is that a devaluation shock is more likely to make net equity negative for the first type of loans, as the bottom panel of the figure illustrates. The dotted lines show home equity following a negative idiosyncratic shock. The shock causes equity to become negative on high-downpayment loans in the first eight periods while for high-leverage loans equity become negative following the same shock for the first eleven periods.

Figure 5: Mortgage debt and home equity by contract type

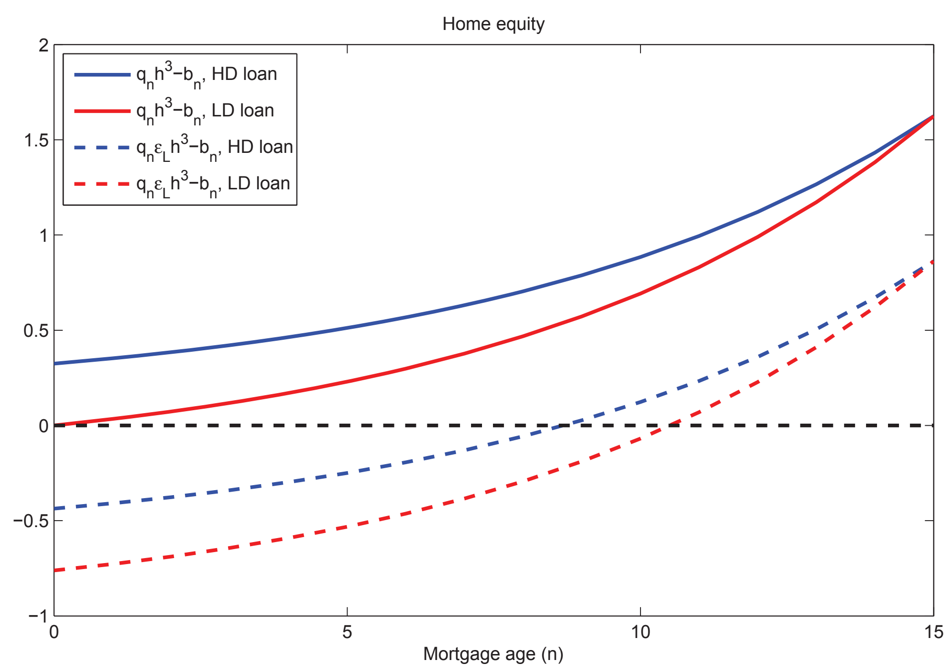


Figure 6 illustrates the link between loan types and default rates by plotting average hazard rates (the fraction of remaining loans that default at all possible loan ages) in the benchmark equilibrium and in the long-boom equilibrium based on random samples of 50,000 loans in each case. LD loans have a higher propensity to default at all ages in both equilibria. Nevertheless, the gap is very small in the benchmark until at least half-way to maturity. This is because, as we documented above, pre-boom approval standards are such that low income agents do not qualify for high-leverage loans. As a result, as a group, LD borrowers are not much riskier than their HD counterparts. After 8 periods however, the two plots diverge noticeably. This is because by that time high-leverage borrowers remain in positive equity territory even if their house devalues while a bad idiosyncratic shocks continues to send LD borrowers into negative equity territory. In summary then, high-leverage mortgages are not much riskier than HD loans in the pre-boom economy because selection effects are muted by tight approval standards.

In sharp contrast, hazard rates are completely different for the two types of loans in the long boom economy. Agents at the bottom of the asset and income distribution take advantage of weakened approval standards to become home-owners. LD borrowers are at a much higher risk of being unable to meet mortgage payments hence default at a much higher frequency, even early in the loan's life. Since low downpayments become much more frequent during the boom and average default rates on those loans rise noticeably, average defaults rates are bound to rise, which is what table 2 shows.

To document more systematically the effects of selection and equity effects on default, we generated a random sample of 50,000 loans originated in the long-boom equilibrium and estimated a competing risk model with the following covariates for borrower $i \in \{1, \ldots, 50000\}$:

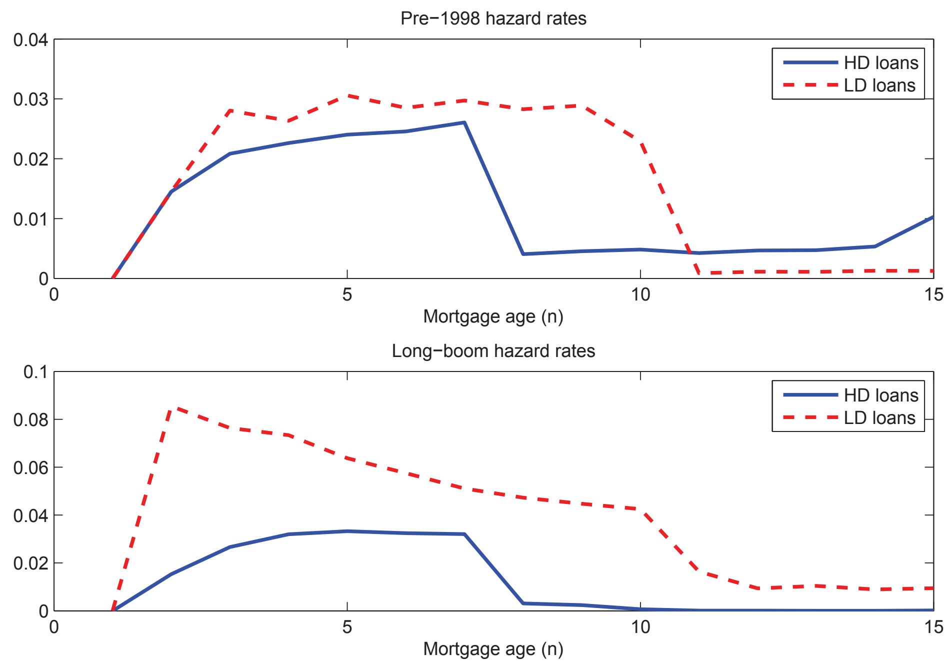
1. the mortgage type ($1_{LD} = 1$ if the borrower selects a zero-downpayment loan, 0 otherwise);

2. the loan-to-income ratio at origination (LTY_i);

3. the asset-to-loan ratio at origination (ATL_i);

4. a measure $HE_{i,n} \equiv h\epsilon_{i,n}q_H - b_n$ of home-equity on each loan at each possible age $n = 0, \ldots 14$ where ϵ_n is the value of the idiosyncratic value shock on loan i in period n

Figure 6: Average default hazard rates



while b_n is the loan balance;

5. the income $INC_{i,n}$ of borrower i in period n.

Note that the first three covariates are fixed at origination while the last two covariates vary with loan age.

Let $\xi^e_{n,i}$ be the hazard rate at loan age n for homeowner i due to event $e \in \{D, S\}$, where D stands for default while S stands for sale. We adopt a standard Cox proportional hazard specification for hazard rates, namely:

$$\xi^e_{n,i} = H\left(\omega^e_n \times \exp\left\{\beta^e_{LD} 1_{LD,i} + \beta^e_{LTY} LTY_i + \beta^e_{ATL} ATL_i + \beta^e_{HE} HE_{n,i} + \beta_{INC} INC_{n,i}\right\}\right),$$

where $H(x) = \exp(-\exp x)$ for all $x > 0$, and ξ^e_n is the baseline hazard rate at loan age n.[27]

[27] Most of the empirical literature on default rates adopts a version of this proportional hazard specification.

Table 5 shows the result of the estimation. As expected, estimated hazard rates into default rise with loan-to-income at origination and fall with the asset-to-loan ratio. Income and home equity have a negative impact on both hazards. After controlling for all these characteristics, the loan type dummy has no significant effect on default which confirms that loan selection affects survival prospects mainly through its consequences on equity accumulation and because different borrower pools have different income distributions.

Table 5: Determinants of mortgage termination

Covariate	Default		Sale	
LIP indicator	-0.0661	(0.0417)	-0.4005^{***}	(0.0341)
Loan to Income Ratio	0.3690^{***}	(0.0186)	0.8782^{***}	(0.0199)
Assets to Loan Ratio	-0.5938^{***}	(0.0472)	-0.8792^{***}	(0.3320)
Income	-1.0817^{***}	(0.0219)	-4.2591^{***}	(0.0197)
Home equity	-4.5963^{***}	(0.0541)	-0.0048^{**}	(0.0215)

Notes: Bootstrapped standard errors (based on 50 replications) are in parenthesis; Log Likelihood :-285818.42*; *** significant at 1% level; ** significant at 5% level.*

In both the benchmark economy and the boom economy, the vast majority (86% in the pre-98-benchmark, 78% in the Long Boom) of defaults involve negative equity. The key reason for this is that agents who have positive equity in their house can always sell unless they can't make their mortgage payment (i.e. their budget set happens to be empty). Following an aggregate price correction (i.e. in the first period of the bust in the transition experiment we describe in the net section), almost all default (over 99%) of defaults involve negative equity.

While most foreclosures involve negative home equity, most households (roughly 93% in the pre-98 benchmark, 90% in the Long Boom) with negative home equity choose to keep their home and continue meeting their mortgage obligations. In the benchmark equilibrium,

It is difficult to compare our results directly to the outcomes of these studies because they usually control for covariates that have no clear counterpart in our model, and usually lack detailed information on borrower assets. Gerardi et al., 2009, for instance, include proxies for county-level economic and state-wide house price conditions, but do not control for assets at origination. Still, theirs and most other papers find as we do that high LTVs at origination and high debt-to-income ratios have a significant effect on default rates as do proxies for home equity and borrower income. We estimate the model using method A in Lunn and McNeil, 1995.

default decisions that are voluntary in the sense that agents could choose to make their current mortgage payment and do not experience an aging shock that would constrain them to sell account account for barely 2% of the overall default rate. In the long boom equilibrium, that share rises to about 15% but remains low. The aging shock is involved in 80% of defaults on both types loans in the benchmark since most borrowers begin with comparatively high income or assets before the change in approval standards. In the long boom however, income shocks play a much more significant role, accounting for 35% of all defaults. On LD loans in particular, three quarters of default entail an income shock in that equilibrium.

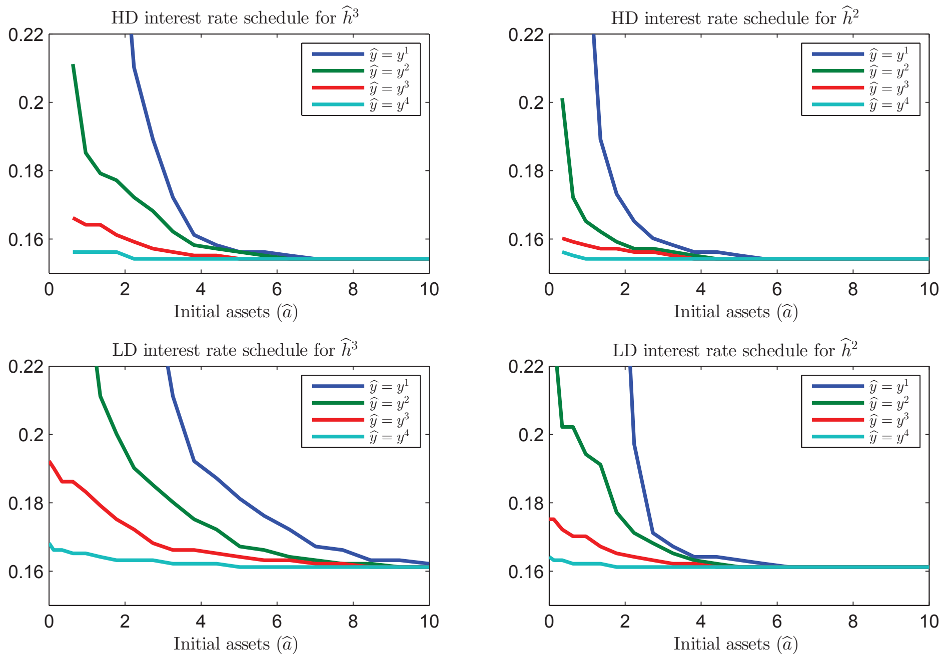
These model predictions are consistent with the empirical literature on the determinants of foreclosure (see, e.g., Gerardi et al., 2009). Available data suggest that most foreclosures involve negative equity but that, at the same time, most households with negative equity choose not to foreclose. Our model, in this sense, captures the fact that most foreclosures involve a combination of negative equity and other adverse shocks.

5.3 Interest Rates

A distinguishing feature of our model is that mortgage terms depend not only on downpayment choices but also on the initial asset and income position of borrowers as well as the size of the loan. Figure 7 plots the menu of equilibrium interest rate offerings conditional on the house size they opt for and their asset and income position at origination in the aggregate boom state.

Some schedules in Figure 7 are left-truncated because agents whose income and assets are too low do not get a mortgage in equilibrium. The left truncation can be thought of as an endogenous borrowing constraint associated with different borrower characteristics. Some agents become mid-aged with asset and income characteristics such that the intermediary could not break even on a mortgage issued to them, which may occur even when the agents have the means to finance the initial downpayment. Among agents who do receive a mortgage offer, rates fall both with assets and income.

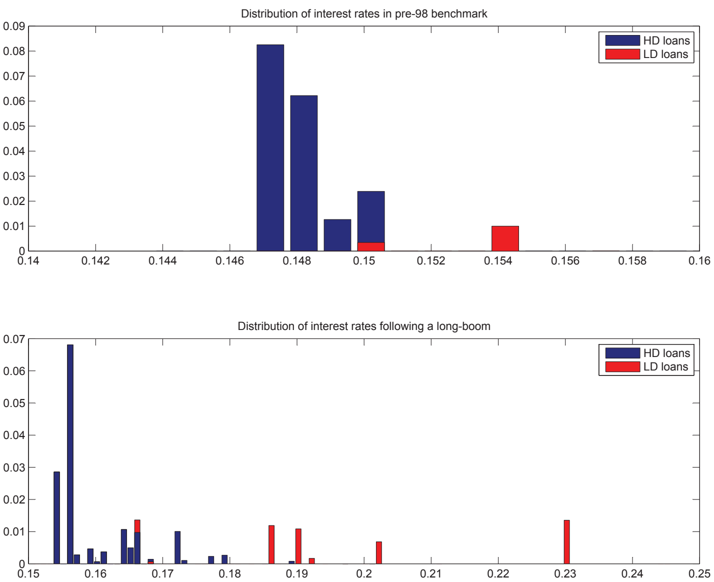
Figure 7: Equilibrium interest rate schedules in the boom state



While Figure 7 graphs equilibrium offers, Figure 8 graphs the equilibrium distribution of interest rates chosen by mortgage type following a long boom period and in the pre-98 benchmark. The distribution of rates on low-downpayment loans displays a large amount of dispersion because, as we discussed earlier, these loans are taken up both by high-risk and low-risk borrowers. The dispersion of interest rates becomes much higher during the boom than in the pre-98 benchmark since only relatively safe borrowers are able to borrow until approval standards are relaxed.

Table 6 shows that much of the equilibrium variation in yields can be accounted for by loan and borrower characteristics at origination in the long boom economy. Once again using our representative sample of 50,000 mortgages to regress log mortgage rates on assets, income and loan size at origination yields an R^2 of nearly three-quarters. Furthermore, higher assets and income are associated with lower yields since they reduce the likelihood of default. A higher

Figure 8: Distribution of equilibrium interest rates



loan size at origination, however, is associated with a lower rate because large loans tend to be selected by agents with high assets and high income. In our calibration, this selection effect turns out to more than offsets the fact that large loans obviously involve larger payments.[28]

One of the most publicized features of the recent boom period in the US is the explosion of "subprime" originations. Also well known is the fact that subprime is a difficult notion to define precisely. In empirical work, loans are classified as subprime either because they are reported as such by lenders or, more compellingly, because they carry a significant interest rate premium over the best-priced ("prime") loans at origination. Under either definition, subprime originations rose considerably in the few years preceding the crisis.

[28]By way of comparison, consider the 2007 sample of recent home-buyers in the SCF we used to produce table 3. Among those buyers, interest rates on first mortgages are negatively correlated with reported income, with loan size and with net worth, with the first two correlations being statistically significant. Running the regression displayed in table yields coefficients with the right (negative) sign but only income is significant at conventional levels. Furthermore and not surprisingly given the heterogeneity among SCF borrowers and loans which our model does not contain, the R^2 in that regression is low.

Table 6: Determinants of log mortgage rates

Covariate	Coefficient	
Constant	-1.4880***	(0.0017)
Assets at origination	-0.0724***	(0.0004)
Income at origination	-0.0735***	(0.0005)
Loan size	-0.0300***	(0.0009)

Notes: Standard errors are in parentheses; $R^2 = 0.72$*; *** significant at 1% level*

Our model correctly predicts that the frequency of high-priced loans should explode when approval standards are relaxed. To see this, define a loan to be high-priced in our simulations if it carries a rate at origination 300 basis points above the best-priced loan. In normal times, the fraction of such loans is zero due to strict approval standards. In the boom equilibrium, that fraction rises to 31%. LD loans, not surprisingly, account for the majority of those loans (88%) but, importantly, not all LD loans are high-priced since they are used by some high-income borrowers to buy bigger homes. In the Long Boom equilibrium, as many as 18% of LD loans are priced within 300 basis points of the best rate available hence by that definition would be considered prime loans. On the flip side, while most (94%) HD loans are prime in that sense, the rest are originated to agents whose assets are sufficient to meet the down-payment requirement but whose income is low, and as a result are high-priced.

In our model like in the data then, leverage and "subprime" loan pricing are correlated but distinct notions. Some high-income agents employ leverage to buy bigger homes or improve their consumption profile without becoming risky borrowers. Some very low income agents have to put down a positive downpayment to keep payments low or be offered a mortgage in the first place.

5.4 The Long-run Effects of Recourse

So far we have maintained the assumption that, in the event of default, the borrower's liability is limited to their home. In several states – known as anti-deficiency or non-recourse states –

the law does in fact make it difficult for mortgage lenders to pursue deficiency judgments. The list of such states varies but generally includes Arizona, California, Florida (and sometimes Texas).[29] There are other states, known as "one-action" states, that allow the holder of the claim against the household to only file one lawsuit to either obtain the foreclosed property or to sue to collect funds. The list of such "nearly non-recourse" states includes Nevada and New York. Even in states where deficiency judgments are legal, conventional wisdom is that the costs associated with these judgments are so high, and the expected returns are so low, that recourse is seldom used.

Some empirical studies (see Ghent and Kudlyak , 2009) find that recourse decreases the probability of default when there is a substantial likelihood that a borrower has negative home equity. In this subsection we quantify the role of the recourse for long-run equilibrium statistics. Table 7 compares long-run statistics in our pre-98 benchmark economy (which assumes no recourse) to their counterparts in the same economy with recourse. Specifically, in the event of default by a borrower with assets $a \geq 0$ and house size h, the intermediary collects $\min\{(1-\chi)q\epsilon h + a, b\}$ with recourse (as opposed to $\min\{(1-\chi)q\epsilon h, b\}$ without recourse), while the household retains $\max\{(1-\chi)q\epsilon h + a - b, 0\}$ with recourse (as opposed to $\max\{(1-\chi)q\epsilon h - b, 0\}$ without recourse). In other words, in the recourse economy, any asset the household owns at the time of default can be claimed as collateral by the lender. Recourse imposes a harsher punishment on borrowers, thus lowering the extensive default margin, and the higher repayment by borrowers lowers the intensive loss incidence margin to the lender. Both effects on these margins lead to lower interest rates.

As the table shows, this change in the environment greatly raises average recovery rates (from 50% to 88%) for obvious reasons. Conditional on a given set of characteristics at origination, the fact that default is a more costly option and the fact that recoveries are greater when default does happen make lenders willing to originate loans with a lower risk premium. With falling payments, it turns out that more agents are able and willing to

[29]See, for instance, http://www.helocbasics.com/list-of-non-recourse-mortgage-states-and-anti-deficiency-statutes.

Table 7: The long-run effects of recourse

	Pre-98 benchmark	Pre-98 benchmark with recourse
Home-ownership rate	0.65	0.76
Interest rate on HD loans	0.148	0.141
Interest rate on LD loans	0.153	0.142
Foreclosure discount	0.70	0.69
Recovery rate	0.50	0.88
Fraction of LD loans	0.07	0.04
Foreclosure rate	1.41	1.35

enter mortgage markets, and the home-ownership rate rises. While each loan is safer due to recourse, the pool of borrowers changes to allow more low income and low asset agents to enter. The net result on default rates is a rather small decrease.[30]

In summary, the long-run consequences of recourse on default rates are quantitatively small in our economy. One could be tempted to infer from this result that broader recourse would have done little to prevent the flare-up in default rates that followed the 2007 home price correction in the US. That intuition is wrong as the next section will show, and table 7 contains a preview of why it so: LD loans become less popular when recourse broadens because they make default more likely, and default is now very costly. As the next section shows and for the same reason, our model predicts that the frequency of LD loans would have been limited in the few years preceding the crisis had recourse been tougher. Since the next section will also show that the rise of leverage made the foreclosure crisis twice as large as it would have been otherwise, tougher recourse would have greatly muted the rise in default rates.

[30]Quintin (2012) discusses the interaction of the direct effect and composition effect of recourse in details. While broader recourse causes rates on loans to fall conditional on borrower characteristics at origination, the equilibrium effects on default are fundamentally ambiguous because broader recourse can enable riskier borrowers to obtain a mortgage. This is exactly what transpires in this experiment.

6 The Crisis: Transitional Consequences of Leverage

The previous section shows that relaxing approval standards has significant long-run effects on foreclosure rates by making high-leverage loans much more common and allowing high-risk borrowers to enter the mortgage market. This section describes an experiment designed to answer the question "How much of the recent rise in foreclosures can be explained by the large number of high-leverage mortgage contracts originated during the housing boom?" No parameters were chosen to match data in this section, so all experiments can also be seen as a "test" of the model.

6.1 A Boom-bust Experiment

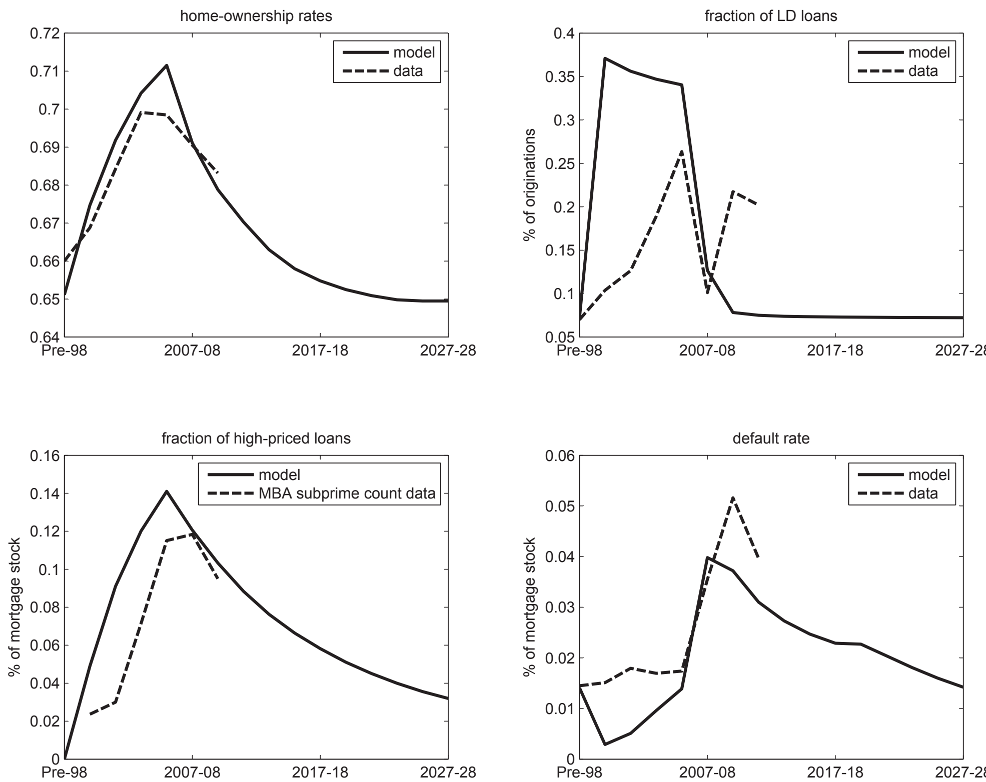
We will think of the recent history of housing markets in the United States as follows. We assume that in 1998 the US economy is near the long-run distribution that would prevail following a long period of normal home values and normal approval standards. We take the following 8 years (1999-2006) as 4 model periods of boom (that is, 4 periods of high home prices and relaxed approval standards). The economy then returns to the normal state. Data counterparts are averages over two-year periods for each period of the transition. The fifth period of the transition, for instance, corresponds to 2007-08, and we think of it as the first two years of the crisis.[31]

Figure 9 plots the outcome of the experiment.[32] The first panel of the figure shows that

[31] Following the aggregate price collapse, the intermediary experiences losses for several periods on mortgages priced before the realization of the aggregate shock. The previous version of this paper detailed how to measure the intermediary's losses on a period-by-period basis and imposed lump-sum taxation on households to pay for the losses. In this version of the paper, those losses amount to a small fraction (at most under 3/10 of one percent) of aggregate household earnings and as long as such a bailout is unexpected introducing this taxation does not change our quantitative results. An anticipated bailout, of course, would change mortgage pricing and hence could change mortgage choices as well. To keep the analysis simple, here we assume that the risk neutral, deep pocket investors in the intermediary bear the ex-post loss.

[32] The behavior of the equilibrium variables shown in table 2 but not displayed on the figure is mostly stable during the transition. Housing expenditure shares for homeowners rise somewhat due to the increase in aggregate home prices. This is broadly consistent with the available evidence from the Consumer Expenditure Survey. The fraction spent on shelter by homeowners goes from 17.3% in 1998 as mentioned in the calibration section to 18.7% in 2006.

Figure 9: The boom-bust



Notes: Data on home-ownership rates are for households whose head is between 30 and 55 years old as estimated by the Census Bureau. The fraction of LD loans is from Pinto (2010, see figure 1) until 2006 and roughly estimated as two-thirds of FHA's share of purchase originations for 2007-10. The fraction of high-priced loans is compared to the fraction of subprime mortgages in the stock of mortgages covered by the MBA's National Delinquency Survey. Default rates are computed from that same survey as described in the calibration section.

once the boom begins, home-ownership rates start rising towards a peak of roughly 71% at the onset of the crisis. Even though we did not target that aspect of the data in any fashion, the model tracks the empirical behavior of this variable remarkably closely.

The frequency of low-downpayment loans jumps to over 30% during the boom. Compared to the numbers produced for conventional loans by Pinto (2010, see figure 1) the rise in LD originations is too sharp which is not surprising since we relax PTIs fully in one period rather than progressively. Still, recall from figure 1 that Pinto's LD origination rate estimate peaks at near 40% by 2007 as does our model.[33] Furthermore, Pinto's numbers only include conventional originations hence probably understate the actual share of LD loans during the boom.

In the aftermath of the crisis, the share of FHA-insured loans in purchase originations rose above 25%. Furthermore, roughly two-thirds of FHA loans are issued at LTVs in excess of 95%[34] This suggests that the share of LD loans in origination has remained elevated since the crisis, as shown in the second panel of figure 9. Our experiment does not capture that aspect of the evidence because it assumes a return to pre-98 approval standards following the home value collapse. Different scenarios would require that we augment the model to include a fourth aggregate shock. It should be clear, however, that the post-crisis strength of LD originations in the data cannot influence our counterfactual accounting as long as it was unexpected during the boom period. Indeed, the foreclosure spike is the result of decisions made by agents who already have a mortgage when home values collapse and of the shocks that hit those incumbent borrowers. These borrowers' choice set and decisions are not impacted by the PTIs that new borrowers face. As long as the sudden boom in FHA activity was mostly unforeseen then, the initial behavior of default rates cannot depend on this aspect of the crisis' aftermath.

As we mentioned in the previous section, relaxing approval standards causes the frequency of high-price loans to rise, as displayed in the third panel of the figure. By that metric, the experiment thus generates a significant subprime boom, one that closely tracks estimates of

[33] In other words, the data counterpart for the fraction of LD loans does not rise as much in this figure as it does in figure 1 because the last period of the boom is matched with 2005-06 date, which excludes the 40% peak reached in Pinto's data. The 2006-07 average fraction of LD loan in figure 1 is around 35%, very near the peak our model predicts.

[34] See pages 16 and 17 of the 2012 US Department of Housing and Urban Development Annual Report to Congress, p17, available for download at http://portal.hud.gov/hudportal/documents/huddoc?id=F12MMIFundRepCong111612.pdf.

the share of subprime loans in the stock of US mortgages available from the Mortage Bankers Association (MBA).

The final panel of the figure shows the key outcome of this transition experiment for our purposes: the path of default rates during the boom and following the collapse of home prices. Despite the fact that they are in the process of converging towards the higher long-term level we analyzed in the previous section, default rates are not very different at the onset of the crisis from what they were before the boom began. Because we model home prices starkly using three possible values, the default rate actually falls at the start of the boom as all loans originated at lower aggregate prices receive an equity injection. The effect of that aggregate injection dissipates over time as pre-existing loans exit the stock of mortgages through sale, foreclosure, or simply being paid off.

Default rates increase by 182% in the first period of the crisis relative to their pre-98 baseline. In the National Delinquency Survey data, default rates rise by roughly 185% between the last quarter of 2006 and the last quarter of 2008 so that our transition experiment captures more than 98% of the initial spike in default rates. Again, we emphasize that this was not targeted in our estimation so the fact that the model captures so much of the foreclosure data provides an important test.

However, as the quarterly foreclosure numbers in figure 2 show, while default rates begin briefly retreating in 2008 they spike again in 2009. The two-year averaged data counterpart for our model's default rate correspondingly shows a second peak in the second period (i.e the 2009-10 time period) of the crisis. Since the experiment so far only contains a one-time aggregate shock to home prices, it cannot produce that second spike. A second home-price shock (down to q_L) would generate a second spike, but the data displayed in figure 2 show that the aggregate price shock was mostly complete two years into the collapse. A more likely explanation is that the impact of the housing collapse on the distribution of income caused the second spike. We will take up this possibility below and show that the frequency of high leverage loans magnify the impact of income shocks on default rates just like they magnify the effect of prices drops.

6.2 The Importance of Selection

Table 8 shows default rates by borrower types in the pre-1998 benchmark and in the first period of the crisis. In a pure accounting sense, the contribution of LD loans to the increase in foreclosure rates between 1998 and the peak of the crisis is easy to measure. To see this, let ξ_t^ν be the share of loans of type $\nu \in \{HD, LD\}$ in the stock in period t while D_t^ν is the default rate on those loans. Using this notation and as a matter of accounting, the overall default rate at time t is always the sum of two parts

$$D_t = \xi_t^{LD} D_t^{LD} + \xi_t^{HD} D_t^{HD}.$$

One way to measure the contribution of LD loans to the increase in default rates between 1998 and the start of the crisis is to divide the total increase in D by the increase in the first element of the sum, namely

$$\Delta\left(\xi_t^{LD} D_t^{LD}\right) \approx \Delta\left(\xi_t^{LD}\right) D_t^{LD} + \xi_t^{LD} \Delta\left(D_t^{LD}\right)$$

where Δ is the time difference operator between 2007 and 1998 (e.g. $\Delta\left(\xi_t^{LD}\right) = \xi_{2007}^{LD} - \xi_{1998}^{LD}$).

Thus changes in the pool of borrowers (i.e. $\Delta\left(\xi_t^{LD}\right)$) and changes in default rates of that pool (i.e. $\Delta\left(D_t^{LD}\right)$) matter. The table shows that both the change in the LD share and the change in the default rate are large, which means that the accounting contribution of LD loans to the increase in overall default rates is large as well.

The result of those calculations are difficult to interpret, however, because as this section shows, new entrants and switchers into the LD pool of borrowers account for much of the increase in $\xi_t^{LD} D_t^{LD}$. Restricting leverage would presumably cause many of these borrowers to make different contract choices, exit mortgage markets altogether and/or affect their propensity to default. In particular, whenever $\xi_t^{LD} D_t^{LD}$ changes, $\xi_t^{HD} D_t^{HD}$ is bound to change as well. Most obviously, as different households select into the set of HD borrowers when access to leverage changes, D_t^{HD} is likely to be affected. Our model enables us to measure the effects of access to leverage while taking all endogenous effects into account.

Almost 28% of LD holders in the first period of the crisis are households who took a loan during the boom and, given their asset and income when they were given the option to buy a home, would have rented in the pre-98 benchmark. Table 8 shows that in the first period of the crisis these “new entrants” in the LD sample default at 4 times the pace of “incumbent” LD households do (an incumbent household is defined as one which already had taken an LD loan before the boom started or had characteristics such that if given the opportunity to purchase would have opted for an LD loan pre-98). In addition, roughly 28% of LD holders at the onset of the crisis are “loan-type switchers.” These households took their loan during the boom and, in the pre-98 economy, would have opted for an HD loan by choice or by constraint. These loan-type switchers default at almost twice the pace incumbents do when the crisis strikes. Entry also contributes to the increased default rates on HD loans. Roughly 5% of HD holders are new entrants when the crisis strikes and they default at 3 times the pace HD incumbents do. We find that no households who take an HD loan during the boom would have taken an LD loan pre-98 (which accounts for “NA” in the table).

Given these fundamental changes in the composition of each pool of borrowers during the boom, measuring the effect of different mortgage access scenarios on default rates requires predicting how these scenarios change mortgage housing choices. It also requires predicting the default decisions of households who are constrained to change mortgage choices but remain home-owners. Not only do we need to predict who would exit mortgage markets given different access scenarios, we also need to predict how loan-switchers would default if forced to return to their pre-98 choices. The next section uses our model to run counterfactual experiments that take these endogenous effects into account.

The table also shows that, in our model, default rates are not that different across loan types pre-98. As we discussed in the long-run section, the gap in default propensities increases in the boom state. But it is following the aggregate price correction that the two loan types truly separate in terms of their propensity to default. This parting of default rates is broadly consistent with the evidence presented in Figure 4 in Gerardi et. al. (2009.) However, our

Table 8: Default rates by borrower type

	LD loans	HD loans	All loans
pre-1998			
Fraction of stock	6.92	93.08	100.00
Default rate	1.90	1.37	1.41
First period of the crisis			
Fraction of stock	17.86	81.14	100
Default rate	10.78	2.50	3.98
Default rate among incumbents	5.49	2.28	2.57
Default rate among loan-type switchers	10.28	NA	10.28
Default rate among new entrants	19.90	7.54	14.70

Notes: All entries are model predictions. New entrants are borrowers who took their loan during the boom and, given their characteristics at origination, would have been renters in the pre-98 economy. Loan-type switchers are boom-stage borrowers who would have opted for a different loan type in the pre-98 economy. Incumbents are all other home-owners in the first period of the crisis.

default rates begin drifting significantly apart starting in period 2 of the boom period (i.e in 2001-2002) which is somewhat earlier than the evidence in Gerardi et. al. (2009) would seem to suggest.

6.3 Counterfactuals

To quantify the role of leverage in the crisis using our model, we run two distinct counterfactuals. In counterfactual 1, we leave the price path as in the baseline experiment but make approval standards (PTIs on both types of loans) during the boom the same as during normal times. Since the boom period now features more costly homes without any offsetting changes in access to mortgages, home-ownership rates fall and the use of high leverage loans remains virtually unchanged, as figure 10 displays. Default rates actually fall prior to the boom because only the safest (high-income and high-asset) borrowers remain home-owners. Default rates do spike when aggregate home prices collapse but only rise 64% above their pre-boom level in the first two years of the crisis, which is roughly a third of the baseline increase in default rates. This first counterfactual thus suggests that relaxed approval standards and the resulting greater availability of LD loans can account for almost two thirds $\left(1 - \frac{64}{182} \approx 65\%\right)$ of the rise in foreclosure rates in the first two years of the crisis.

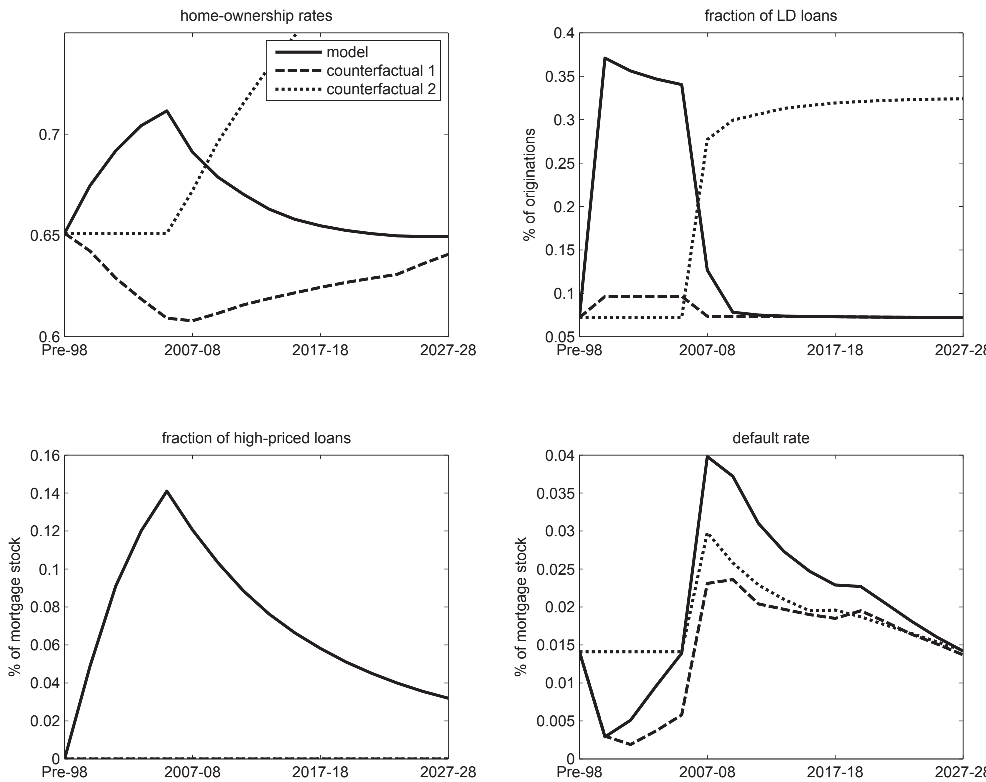
Counterfactual 2 assumes no changes in approval standards and home prices during the boom so that the boom period is now nothing but another four periods of normal times. At the onset of the crisis then, the distribution of borrowers is exactly what it was in 1998, before approval standards change. The importance of high-leverage loans, in particular, remains at 7% until the crisis strikes. In late 2006, we subject those borrowers to a fall in home values from q_N to q_L (which is a fall of roughly 30% in aggregate values, much like in the baseline experiment.) In that case, foreclosure rates rise by 111%, so that the overall increase in default rates is about 39% $\left(\approx 1 - \frac{111}{182}\right)$ lower than in our baseline experiment.[35]

Both these counterfactuals ask the same fundamental question: how much lower would the default rate spike have been had approval standards not changed post 1998? The first one does so assuming that the path of home values would have been the same regardless of these standards. The second one, in essence, makes the exact opposite assumption: prices would not have risen during the boom without the change in standards. These two polar experiments suggest that the contribution of high leverage loans to the initial spike in default was somewhere between 40% and 65%.

Why does the second counterfactual suggest a role for LD loans that is significantly lower than what the first counterfactual suggests? Once again, the answer is selection. Keeping approval standards the same, the increase in price causes home-ownership rates to drop. Intuitively, agents who became home-owners during the boom in the benchmark experiment but choose to remain renters in counterfactual 1 tend to have lower income and assets than

[35] Gerardi et. al (2009) run a very similar counterfactual using their econometric model. They subject borrowers who took a loan in 2002 to the same price history as their 2006 counterparts and find (see figure 10) that 2002 loans would have defaulted at approximately half the rate of their 2005 counterparts. We view these two approaches as complementary and find it reassuring that they yield similar quantitative answers. The empirical approach is conditional on the econometric structure used to estimate the elasticity of default decisions to price shocks. The outcome of our counterfactual experiment likewise depends on the modeling assumptions we make, but, conditional on that economic structure, the effect of different price and underwriting scenarios on default can be measured exactly.

Figure 10: Leverage counterfactuals



those households who buy regardless of approval standards. In countefactual 1 therefore, only the safest households remain in mortgage markets and those households, as we have repeatedly discussed in this paper, are more likely to opt for HD loans and are less prone to payment difficulties.

6.4 Broader Recourse, Smaller Crisis?

Could the foreclosure crisis have been avoided had tougher recourse statutes been in place in the United States?[36] Figure 11 compares the baseline path of default rates during the transition to what the path would have been with recourse broadened to include ex-housing assets. With recourse access is already broad in the pre-98 benchmark hence the relaxation of approval standards has less impact. The increase in home prices becomes the dominant impact and home-ownership rates actually fall in the second stage. The increase in LD use is muted, as the second panel of the figures shows, and there is no pick up in the frequency of high-priced, risky loans. Not surprisingly then, the collapse in aggregate home values only causes default rates to peak at 2%, 50% lower than in the baseline experiment. Even though recourse has little long term effect on default rates in our model economy as shown in Subsection 5.4, its presence greatly limits the use of LD loans even when they are broadly available, and all but eliminates the crisis.

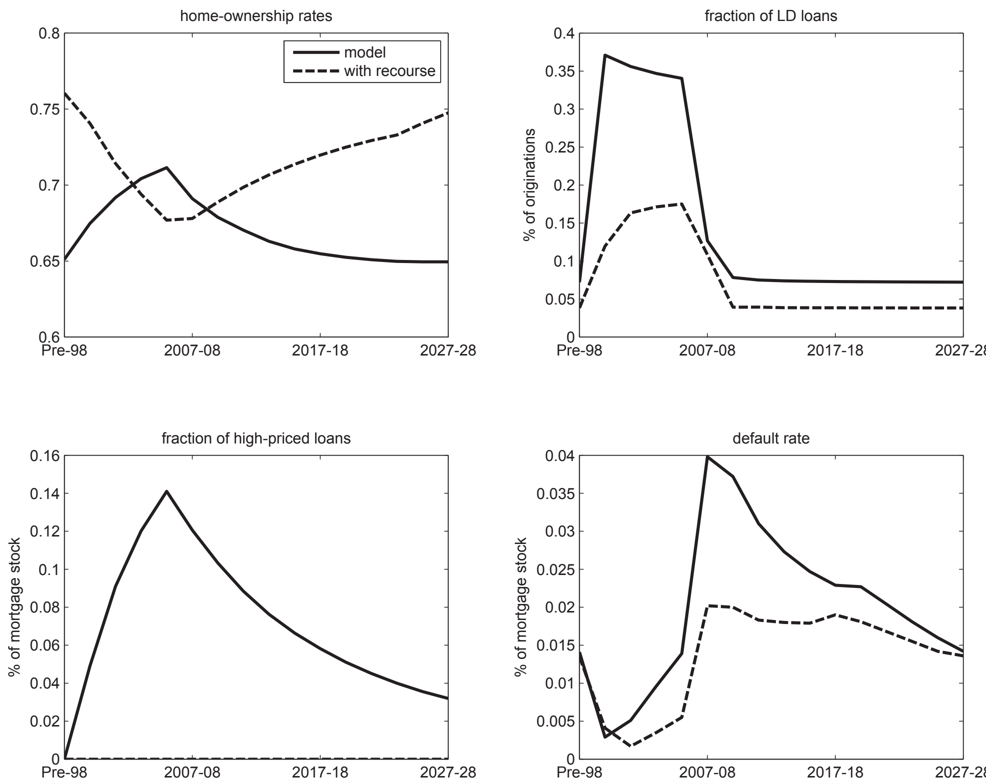
We should emphasize that this result is obtained under the strong assumption that lenders can claim ex-housing assets fully and at no cost. In practice, even when households in foreclosure have other assets to claim, they also have opportunities to protect or dispose of them before they become subject to deficiency judgments. Our experiment should only be interpreted as saying that economies where recourse is broad and cost-effective in practice may in fact be less sensitive to aggregate home price shocks.

6.5 The Second Spike

After declining for two quarters in late 2008, default rates flared up again to reach a new peak in the last quarter of 2010. Our experiment as it stands cannot generate this second peak because the only shock occurs in the first two years (the first model period) of the crisis.

[36] In a 2008 Wall Street Journal editorial, Martin Feldstein writes: "The no-recourse mortgage is virtually unique to the United States. That's why falling house prices in Europe do not trigger defaults. The creditors' ability to go beyond the house to other assets or even future salary is a deterrent." See http://online.wsj.com/article/SB122697004441035727.html.

Figure 11: Broader recourse mitigates the crisis



One explanation for the second peak is the fact that the housing crisis was followed by a deterioration of household income. To evaluate this possibility while retaining tractability, we consider the effect of a one-time, fully unexpected shock to the income distribution at the start of the second period of the bust.[37] Specifically, we consider a first-order degradation of the income distribution indexed by $\zeta \in (0,1)$ among young mid-aged agents. For any $y \in Y^{\mathcal{Y}}$ or $y \in Y^{\mathcal{M}}$, an agent whose income is currently y remains at that income level with probability $1-\zeta$ but, in the complementary event, sees their income fall with equal

[37] The fact that the change in the distribution is unexpected means we do not need to add more aggregate states.

probability to one of the lower income levels.

In other words, while agents make decisions thinking that the transition probability matrix between the first and the second period of the bust will be $P^{\mathcal{Y}}$ or $P^{\mathcal{M}}$ as expected, the actual transition matrix is the expected one convoluted with:

$$P^{\zeta} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ \zeta & 1-\zeta & 0 & 0 \\ \frac{\zeta}{2} & \frac{\zeta}{2} & 1-\zeta & 0 \\ \frac{\zeta}{3} & \frac{\zeta}{3} & \frac{\zeta}{3} & 1-\zeta \end{bmatrix}.$$

We set $\zeta = 0.2126$ so that average household income $(y + ar)$ at the start of the second period of the bust is 15% below what it was at the start of the first period. According to Saez (2013), average real income declined by 17.4% between 2007 and 2009.

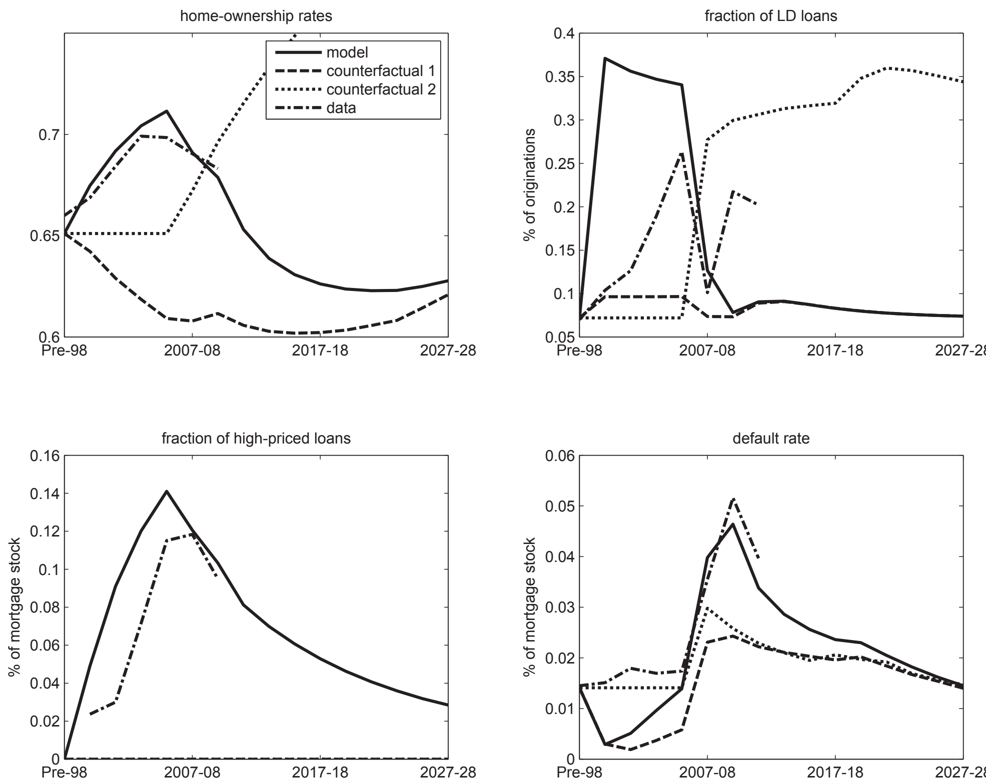
Figure 12 shows the outcome of the experiment. The model now predicts a second spike in the second period of the crisis of a magnitude very similar to the data. In either counterfactual experiment and in sharp contrast, the income shock has little impact on the path of default rates since there are much fewer negative equity loans in the stock when the income shock hits. Put another way, the frequency of high leverage loans magnifies the effect on default rates of the aggregate income shock much like they magnify the effect of the aggregate price shock.

7 Conclusion

The calculations we present in this paper suggest that the rise in high leverage loans between 1999 and 2006 accounted for somewhere between 40% and 65% of the magnitude of the foreclosure crisis. We also find a quantitatively significant role for recourse in the short run.

Our findings raise at least two natural questions. First, did relaxed approval standards and higher leverage mortgages directly contribute to the run-up and eventual collapse in home-

Figure 12: Model with aggregate income shock in second period of the crisis



prices? Second, what caused the marked change in mortgage practices starting in 1999?

Our model takes the path of home prices as given. If high leverage mortgages contributed to the price collapse directly as well, their contribution to the crisis may be even greater than our numbers suggest. For instance, the availability of these mortgages may have led to some form of overbuilding as in Chatterjee and Eyigungor (2009). Their presence may also have contributed to the fragility and eventual freeze of the financial system, leading to a collapse of demand for housing, hence of housing prices. Formalizing and quantifying these ideas are promising avenues for future work, and should reinforce our main message: the rise of leverage played a significant role in the recent foreclosure boom.

As for what caused the change in the composition of mortgage originations starting in the late 1990s, or alternatively, the relaxation of approval standards, several explanations have been proposed. For instance, some view it as the natural consequence of the US government's effort to promote home-ownership over the past two decades, which included a loosening of the restrictions on what mortgages government-sponsored agencies could back or purchase. Our experiments are consistent with the idea that greater access to mortgage markets – whatever its cause – was a driving force behind the rise of home-ownership during the boom. Further research into endogenizing the timing and reasons for changes in approval standards will greatly enhance our understanding of the current foreclosure crisis.

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Computational Appendix (not intended for publication)

A Long Run Equilibrium

1. In the model, except for assets, all other state variables are elements of finite sets. We discretize the asset space into twenty grid points between 0 and 10 times the median income level. The last value of the asset grid is chosen so that no households are constrained at the highest asset grid point. The asset grid is unevenly spaced where every point of an equally spaced grid between 0 and $10^{2/3}$ is raised to the 3/2 power. This type of grid contains more points close to zero and offers better numerical performance.

2. Use value function iteration to find $V_O(a; s)$ from which we obtain savings decision rules for old agents.

3. Assuming $\gamma = 0$, obtain a candidate value function $V_M^R(a, y; s)$ from which we obtain savings decision rules for mid-aged renters.

4. Find the value function for mid-aged homeowners who have paid off their mortgage $V_M^{(\nu,\kappa)}(a, y, \epsilon, n \geq T; s)$ from which we obtain saving and home sales decision rules.

5. Calculate homeowners value functions where $n \in \{0, ..., T-1\}$.

   (a) To avoid calculating value functions for contracts which are not feasible for a given ν (say ν_0) and house size $\widehat{h}$ (say h_0) with $\kappa = (\widehat{a}, \widehat{y}, h_0; \widehat{s})$ (i.e. to ensure $\nu_0 \in K$), check if the household in that state has enough assets to make a downpayment (i.e. constraint (3.1) is satisfied) and that if the mortgage rate is given by $r_i^{\nu_0}(\kappa)$, the implied mortgage payment meets the PTI requirement (i.e. constraint (3.2) is satisfied). As an initial guess, let $r_0^{\nu_0}(\kappa) = r + \phi$ (i.e. the risk free rate).

   (b) For each feasible ν_0 and h_0, use backward induction starting from $V_M^{(\nu,\kappa)}(a, y, \epsilon, h_0, n = T-1; s)$ to an initial value function $V_M^{(\nu_0,\kappa)}(\widehat{a}, \widehat{y}, 1, h_0, n = 0; \widehat{s})$ with the risk-free rate as the initial guess for the mortgage interest rate. The former value functions yield saving, default, and sale decision rules for $n \in \{1, ..., T-1\}$. The final value function $V_M(\widehat{a}, \widehat{y}, n = 0; \widehat{s}) = \max_{\nu_0 \in K, h_0 \in \{h^2, h^3\}} \{V_M^{(\nu_0,\kappa)}(\widehat{a}, \widehat{y}, 1, h_0, n = 0; \widehat{s})\}$ yields the optimal mortgage choice.

   (c) Calculate $W_0^{(\nu,\kappa)}(\widehat{a}, \widehat{y}, \widehat{\epsilon} = 1; \widehat{s})$ according to the resulting decision rules for all possible paths of (y, ϵ, s).

   (d) If the present value of $W_0^{(\nu,\kappa)}(\widehat{a}, \widehat{y}, \widehat{\epsilon} = 1; \widehat{s})$ is less than the initial loan size, increase the interest rate by a small amount (i.e. $r_{i+1}^{\nu_0}(\kappa) > r_i^{\nu_0}(\kappa)$) and repeat this step. Otherwise, the equilibrium interest rate is found.

6. Once we obtain the value functions for homeowners, we can update the values of mid-aged renters by using the true γ. Repeat from step 4 and iterate until the value function for mid-aged renters converges.

7. Find $V_Y(a, y; s)$ from which we obtain asset decision rules for young agents. Because of a potential discontinuity caused by the downpayment requirements, the value functions for young agents are solved for by grid search (all others use interpolation).

8. Compute a long run cross-sectional distribution for any s by starting at normal times, with zero assets, using the invariant distribution of income for the young and find the sequence of distributions given in subsection 3.3.

B Transition Dynamics

1. Begin with the benchmark equilibrium distribution generated with a long sequence $s_t = N$ such that the distribution difference from another N draw of the aggregate shock is below double precision.

2. Use the operators in Section 3.3 to compute $\mu_{\mathcal{Y}}^{t+1}(A', y'|s^t)$, $\mu_{\mathcal{M},R}^{t+1}(A', y'|s^t)$, $\mu_{\mathcal{M},n=0}^{t+1}(A', y'|s^t)$, $\mu_{\mathcal{M},n\geq 1}^{t+1}(A', y', \epsilon', n', \widehat{\nu}, \widehat{\kappa}|s^t)$, and $\mu_{\mathcal{O}}^{t+1}(A'|s^t)$ with aggregate shock $s_{t+1} = H$.

3. Repeat step 2 for $s_{t+2} = s_{t+3} = s_{t+4} = H$ and $s_\tau = N$ for $\tau > t + 4$.