

Leverage and the Foreclosure Crisis

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Human Capital and Economic Opportunity: A Global Working Group

Markets Network

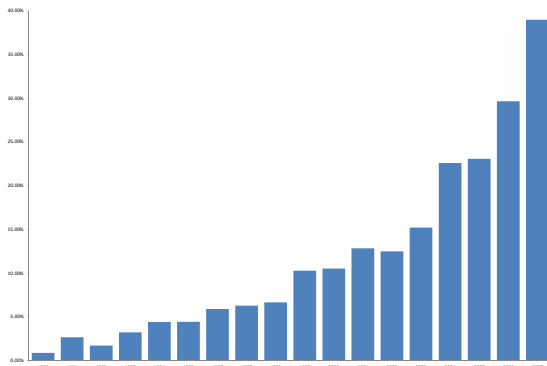
- Area 2: Develop theoretical frameworks for analyzing when/why financial markets do not always extend 'enough' credit to some individuals, and the optimal role of government policies in these situations.

Motivation

- Until 1998, there was a long period where real house prices were relatively constant and the fraction of low downpayment loans in the stock of loans was low.
- From 1999 to the end of 2006, house prices boomed and the fraction of low downpayment loans rose dramatically.
- From 2007, house prices fell by about 30% and foreclosure rates have more than doubled.

Question: How much did changes in the composition of mortgages with respect to leverage contribute to the foreclosure boom?

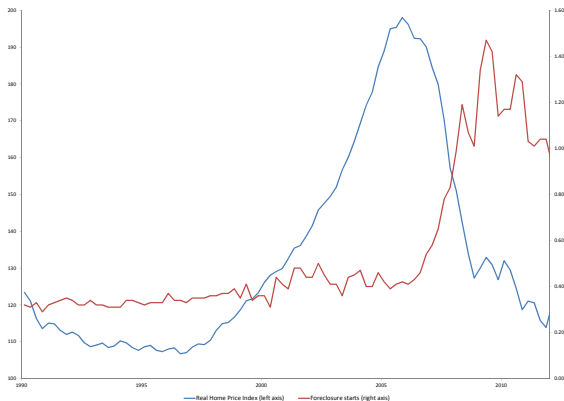
Purchase Loans with $CLTV \geq 97\%$ as a fraction of all loans



Source: Pinto, E. (2010) "Government Housing Policies in the Lead-up to the Financial Crisis: A Forensic Study", mimeo.

► Definition

The Housing Boom and Bust



Sources: Case Shiller, National Delinquency Survey (Mortgage Bankers Association). Quarterly foreclosure rates are the fraction of all loans that enter the foreclosure process in a given quarter.

► Definition

A model of housing

- Heterogeneous agents choose to own or rent, how to finance house purchases, and how to terminate mortgage contracts
- Mortgage holders may default because:
 1. their home equity is negative
 2. they can't afford current payments
- Mortgage terms reflect default risk, hence vary with initial income/asset position, as well as loan size priced in competitive market.
- Changes in house price and approval standards induce important variation in contract selection.

Quantitative experiment

Stage 1: Long period of “normal” aggregate house prices and mortgage approval standards (pre-1998);

Stage 2: House price boom and relaxed approval standards (1999-2006);

Stage 3: House price bust (post-2007).

- All parameters are calibrated to stage 1 only.
- Model can explain 98% of the rise in foreclosures in the data between 2007-2009.
- In a counterfactual where approval standards are not relaxed, the same price shock accounts for 35% of the increase in foreclosures.
- Thus, changes in approval standards can account for 63% of the rise in foreclosures.

Some Literature

1. Empirical: Gerardi et. al. (2009):

- Documents that subprime loans have high CLTV
- Negative net equity is in general necessary but not sufficient for foreclosure.

▶ More on empirical approaches

2. Structural:

- Campbell and Cocco (2011) - Mortgage decision problem with multiple sources of uncertainty (e.g. earnings, house prices, etc.) and default.
- Chatterjee and Eyigungor (2011) - Infinite maturity IOM mortgages.
- Garriga and Schlagenhaut (2009) - Pooling within mortgage types so cannot separate prime vs subprime within a contract.
- Mitman (2011) - One period mortgages with costless refinance.

Outline

- a) Environment
- b) Equilibrium
- c) Parameterization and Cross-section “tests”
- d) Long Run Results
 - Contract Selection
 - Default Hazards across Contracts
 - Distribution of Interest Rates
 - Antideficiency Policies
- e) Boom-Bust Transition Results

Environment

- Time is discrete and infinite.
- Continuum of agents.
- Young agents become mid-aged with probability ρ^M , mid-aged agents become old with probability ρ^O , old agents die with probability ρ^D .
- Young or mid-aged agents earn stochastic income y_t drawn from a n -state $\{y_1^\eta, \dots, y_n^\eta\}$ Markov process with transition matrix P^η where $\eta \in \{Y, M\}$.
- Old agents earn y^O with certainty.
- Agents are born with no assets and with an income level drawn from P^Y .

- Agents value consumption and housing services according to:

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, h_t)$$

where $c_t \geq 0$, $h_t \in \{h^1, h^2, h^3\}$, and

$$u(c, h) \equiv \log c + \log[h \times \theta(h)]$$

with

$$\theta(h^3) = \theta(h^2) > 1 = \theta(h^1)$$

so that homeowners $h_t \in \{h^2, h^3\}$ enjoy a proportional utility premium θ over renters $h_t = h^1$.

- Agents can save at gross rate $1 + r$ in youth and mid-age, and in annuities that pay off $(1 + r)/(1 - \rho_D)$ in old age if alive.

Housing

- Agents can rent quantity h^1 of housing capital at rate R_t .
- When agents become mid-aged they can purchase a house for unit price q_t where $h^3 > h^2 > h^1$.
- House prices follow an exogenous Markov process $q_t \in \{q_L, q_N, q_H\}$ with transition matrix P^q .
- Homeowners face uninsurable idiosyncratic shocks (e.g. neighborhood effects) that follow a Markov process $\epsilon_t \in \{\epsilon_b, 1, \epsilon_g\}$ with transition matrix P^ϵ .
- Housing capital depreciates at rate δ .
- Agents can sell/foreclose on their house in any period, but are then constrained to be renters for at least one period then receive exogenous option to buy with prob γ .
- Old agents must sell their house.

Financial Intermediary

- Stores deposits at rate $r \geq 0$, issues mortgages, and rebundles existing housing for new rentals and purchases in competitive markets.
- Mortgages carry administrative cost ϕ .
- Intermediary loses fraction $\chi > 0$ of principal in event of default.

Mortgages

- A hh who wants to buy a house of size h_t at price q_t must finance it with a fixed rate mortgage of maturity T with downpayment fraction (leverage choice) $\nu_t \in \{LD, HD\}$.
- The mortgage contract stipulates an interest rate $r_t^\nu(a_t, y_t, h_t; q_t, \alpha_t)$ that depends on (at time of origination t):
 - household wealth and income characteristics,
 - house size,
 - downpayment,
 - purchase price,
 - mortgage approval standards α .

▶ Mortgage payment function

- Approval standards: PTI requirement

$$\frac{m_t^\nu}{y_t} \leq \alpha_t \quad (1)$$

Timing

1. Youth:

- Receive age shock and signal of income realization.
- Make savings decision.

2. Middle-age:

- Receive age shock and signal of income realization.
- New mid-aged agents make home-buying and mortgage choice decision.
- Existing homeowners may receive a depreciation shock and decide whether to default or sell.
- Make mortgage or rental payments as well as savings decisions.

3. Old:

- Newly old agents sell their house if they own one.
- Receive death shock or income.
- Make (dis)saving decision.

Recursive Competitive Equilibrium Definition

- Given prices (including $r_t^\nu(a_t, y_t, h_t; q_t, \alpha_t)$), hh savings, house purchases/sales, contract choice ($\nu_t \in K(a_t, y_t, h_t; q_t, \alpha_t)$), and default decisions are optimal given mortgage pricing functions.
- Intermediaries behave competitively:
 - $R_t = rq_t + \delta$ (i.e. PDV of rental payments equals price).
 - For each $\nu_t \in K(a_t, y_t, h_t; q_t, \alpha_t)$, $r_t^\nu(a_t, y_t, h_t; q_t, \alpha_t)$ is such that $W_0^\nu(a_t, y_t, h_t; q_t, \alpha_t) - (1 - \nu_t)q_t h_t = 0$ (i.e. EPDV of mortgage payments equals principal using household optimal default decisions). ▶ IP
- The distributions of household states evolve consistent with shock processes and agent decisions. ▶ Dist

Parameterization

- One period = 2 years, $T = 15$ so consider 30 yr. fixed mortgages.
- Stochastic process for aggregate house prices is chosen to match real Case-Shiller index from 1890-present. [▶ Graph](#)
- Stochastic process for idiosyncratic housing price shocks is chosen within the model. Informative moments are
 - standard deviation of reported capital gains on homes purchased in 1996 or 1997 from SCF by households whose head is between 35 and 64 years old,
 - the rate of mortgage terminations caused by default prior to 1998.

Income process

- From the PSID 1997 and 1999
- Split households into quartiles and age groups (20-34 for young, 35-64 for middle-aged).
- Transition matrix for each age group calibrated to match mobility patterns across quartiles between 1997 and 1999.
 - The incomes of mid-aged agents

$y^M \in \{0.1543, 0.7199, 1.3320, 2.8555\}$ with the median normalized to 1. The transition matrix is

$$\begin{bmatrix} 0.7490 & 0.1926 & 0.0393 & 0.0190 \\ 0.1787 & 0.6388 & 0.1559 & 0.0266 \\ 0.0546 & 0.1615 & 0.6394 & 0.1445 \\ 0.0202 & 0.0303 & 0.1573 & 0.7921 \end{bmatrix}$$

- The incomes of young agents
- $y^Y \in \{0.1452, 0.5725, 0.9216, 1.8533\}$ with transition matrix

$$\begin{bmatrix} 0.5920 & 0.2759 & 0.1034 & 0.0287 \\ 0.1292 & 0.5015 & 0.2769 & 0.0923 \\ 0.0512 & 0.1898 & 0.491 & 0.2681 \\ 0.0317 & 0.0762 & 0.1238 & 0.7683 \end{bmatrix}$$

Moments	Data	Model
HO rate	0.66	0.6511
Asset/Income	1.42	1.5295
Expenditure share	0.15	0.1500
Rent/Income	0.45	0.5602
Spending share	0.173	0.1828
HD rate	0.145	0.1481
Foreclosure rate	0.0145	0.0141
Foreclosure discount	0.75	0.6998
Recovery rate	0.5	0.4965
LD fraction (origination)	0.07	0.0721
S.E. of 2 year capital gains	0.22	0.2319

Parameters		Model
χ	Foreclosing costs	0.4992
ϕ	Mortgage service cost	0.0582
λ	Epsilon shock probability	0.2177
$\tilde{\epsilon}$	Epsilon shock magnitude	0.3515
q_N	Relative price of homes	0.8644
h^2	Size of luxury house	1.8788
h^1	Size of regular house	1.2251
β	Discount factor	0.8489
θ	Owner premium	1.7673
α	PTI requirement	0.2001

Untargeted Cross-Sectional Statistics

	1998 survey				2007 survey			
	LTY		High LTV		LTY		High LTV	
	Data	Model	Data	Model	Data	Model	Data	Model
<i>Income</i>								
Quartile 1	1.61 (0.07)	0.93	0.12 (0.02)	0.22	2.48 (0.13)	2.10	0.18 (0.02)	0.58
Quartile 2	0.98 (0.03)	0.81	0.19 (0.02)	0.07	1.47 (0.05)	1.73	0.24 (0.02)	0.54
Quartile 3	0.79 (0.02)	0.46	0.12 (0.02)	0.00	1.12 (0.03)	0.87	0.13 (0.02)	0.24
Quartile 4	0.63 (0.01)	0.45	0.09 (0.01)	0.00	1.01 (0.02)	0.66	0.06 (0.01)	0.00
<i>Asset-to-income</i>								
Quartile 1	1.03 (0.04)	0.60	0.26 (0.03)	0.29	1.44 (0.04)	1.58	0.26 (0.02)	0.91
Quartile 2	0.88 (0.03)	0.77	0.12 (0.02)	0.00	1.52 (0.06)	1.11	0.12 (0.02)	0.27
Quartile 3	0.96 (0.04)	0.53	0.10 (0.01)	0.00	1.41 (0.08)	1.11	0.10 (0.02)	0.18
Quartile 4	0.99 (0.03)	0.76	0.07 (0.01)	0.00	1.54 (0.05)	1.57	0.06 (0.01)	0.00
<i>Age</i>								
Below 35	0.99 (0.03)	0.66	0.17 (0.02)	0.10	1.60 (0.04)	1.33	0.24 (0.02)	0.43
Above 35	0.95 (0.02)	0.68	0.11 (0.01)	0.04	1.37 (0.04)	1.36	0.10 (0.01)	0.20
<i>Loan size</i>								
Below median	0.79 (0.03)	0.54	0.06 (0.02)	0.11	1.08 (0.03)	1.33	0.16 (0.01)	0.29
Above median	1.10 (0.02)	0.74	0.21 (0.01)	0.04	1.79 (0.04)	1.31	0.15 (0.01)	0.39

Summary of Untargeted Cross-Sectional Statistics

- Matches patterns of data from SCF pretty well:
 - LTY falls with income
 - high LTV at bottom of asset distribution

Young agents' problem

- State: $\omega = (a, y)$

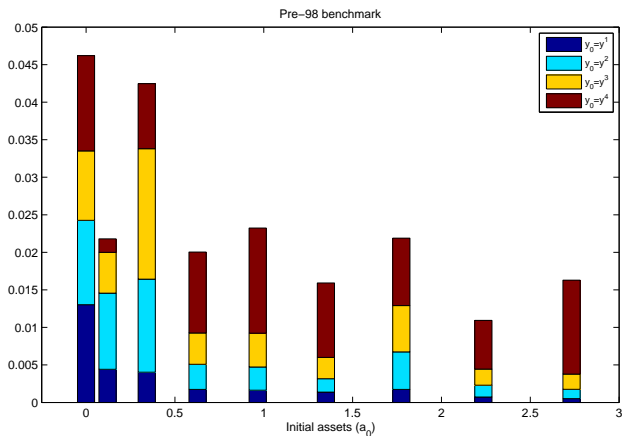
$$V_Y(a, y; q) = \max_{c \geq 0, a' \geq 0} u(c, h^1) + \beta E_{y', q' | y, q} \left[\begin{array}{l} (1 - \rho_M) V_Y(a', y'; q') \\ \rho_M V_M(a', y', n = 0; q') \end{array} \right]$$

$$\text{s.t. } c + a' = y + a(1 + r) - R(q)h^1$$

▶ Mid-aged agents contract choice problem

▶ Mid-aged agents default decision problem

Endog. Distn. of assets upon entering mid-age



Average savings of newly mid-aged hhs fall by 3.75% in boom times relative to normal times (endogenous response to approval standards).

Selection: Contract type by assets and income

Table : Rent-or-own decision rules by asset and income group

Contract House size	Rent	LD		HD	
	h^1	h^2	h^3	h^2	h^3
<i>Low state</i>					
y^1	all a_0	-	-	-	-
y^2	-	$a_0 < 0.34$	-	$0.34 \leq a_0$	-
y^3	-	-	$a_0 < 0.97$	-	$0.97 \leq a_0$
y^4	-	-	$a_0 < 0.34$	-	$0.34 \leq a_0$
<i>Medium state</i>					
y^1	all a_0	-	-	-	-
y^2	$a_0 < 1.77$	-	-	$1.77 \leq a_0$	-
y^3	-	$a_0 < 0.34$	-	-	$0.34 \leq a_0$
y^4	-	-	$a_0 < 0.34$	-	$0.34 \leq a_0$
<i>High state</i>					
y^1	$a_0 < 1.35$	-	-	$1.35 \leq a_0 < 3.26$	$3.26 \leq a_0$
y^2	-	$a_0 < 0.63$	-	$0.63 \leq a_0 < 1.35$	$1.35 \leq a_0$
y^3	-	-	$a_0 < 0.63$	-	$0.63 \leq a_0$
y^4	-	-	$a_0 < 0.63$	-	$0.63 \leq a_0$

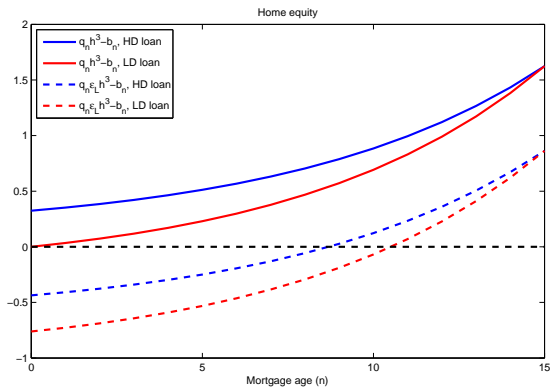
In normal times, low income and low asset hhs rent while in boom times many select small houses and middle class buys bigger houses with LD loans.

Selection: Contract type by average age

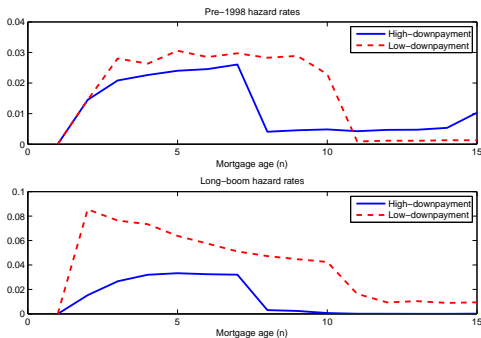
<i>Normal</i>			
Housing decision	Rent	LD	HD
Rental unit	34.52	–	–
Small house	–	30.84	47.38
Large house	–	30.74	36.43
<i>Boom</i>			
Housing decision	Rent	LD	HD
Rental unit	34.29	–	–
Small house	–	32.18	39.68
Large house	–	31.68	38.23

Younger first time home buyers are more likely to choose a low downpayment mortgage.

LDs imply slower home equity accumulation



Default hazard rates by contract type



- Construct hazard rate (fraction of terminations due to default or sale conditional on staying in the home up to date n) from a pseudopanel of 50,000 mortgages drawn from long run distribution of our model economy.
- Default hazards are uniformly higher for LDs than for HDs due to selection and equity effects.

Default frequencies by mortgage type

	Voluntary	Income Shock	Moving Shock	Total
<i>Normal</i>				
HD	0.00	0.28	1.09	1.37
LD	0.41	0.04	1.45	1.90
<i>Boom</i>				
HD	0.10	0.08	0.51	0.69
LD	0.66	2.66	1.04	4.36
<i>Bust</i>				
HD	0.28	0.12	2.10	2.50
LD	2.59	3.28	4.91	10.78

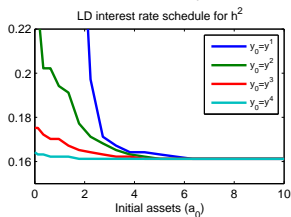
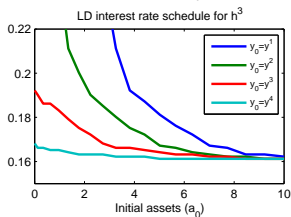
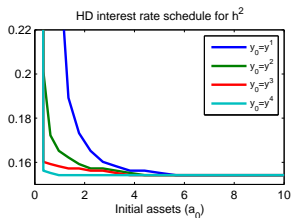
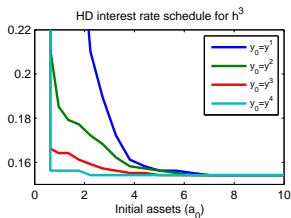
Default rates are much higher on LDs than on HDs.

▶ [Definition of default](#)

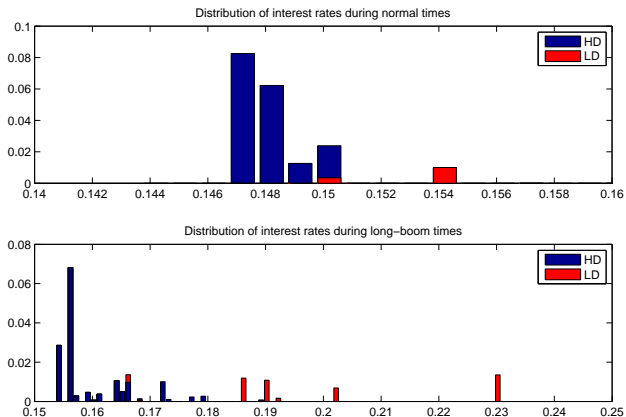
Determinants of foreclosure

- 85.7% of defaults involve negative equity.
- However, 92.8% of agents with negative equity choose to continue meeting payments.
- Thus, negative equity alone is not sufficient for foreclosure in our model. Default occurs with another event like a negative income shock.

Interest rate offerings



Equilibrium distribution of interest rates



- Define a high-priced (subprime) loan as one which is 300 basis points above a prime (best-priced) loan.
- In the boom equilibrium, the fraction of subprime rises from 0 to 31% with LD loans accounting for 88% of that fraction.

Policy: recourse imposes harsher punishment

- Anti-deficiency (Non-recourse) laws: borrower is not responsible for any deficiency. Banks cannot attach to the household's assets.
- Some states have them (AZ,CA,FL . . .), others don't.
- What if all states had recourse?

	Intermediary	Hhs
Non-recourse	$\min\{(1 - \chi)qh, b\}$	$a + \max\{(1 - \chi)qh - b, 0\}$
Recourse	$\min\{(1 - \chi)qh + a, b\}$	$\max\{(1 - \chi)qh + a - b, 0\}$

- Harsher punishment lowers extensive default margin.
- Higher repayment lowers intensive loss incidence.

Long Run Role of Recourse

Moments	Benchmark	Recourse
HO rate	0.6511	0.7605
Asset/Income	1.5295	1.4958
Expenditure share	0.1500	0.1518
Rent/Income	0.5602	0.5602
Spending share	0.1828	0.1840
HD rate	0.1481	0.1408
Foreclosure rate	0.0141	0.0135
Foreclosure discount	0.6998	0.6930
Recovery rate	0.4965	0.8826
LD fraction (origination)	0.0721	0.0383
S.E. of 2 year capital gains	0.2319	0.2319

- Foreclosure rates are 4.2% lower and HO rates are 16.8% higher with recourse.
- Ghent and Kudlyak (2009) estimate that at average borrower characteristics, the likelihood of default is 20% lower with recourse.

Main experiment

Stage 1: Long period of “normal” aggregate house prices and mortgage approval standards (pre-1998);

Stage 2: House price boom and relaxed approval standards (1999-2006);

Stage 3: House price bust (post-2006).

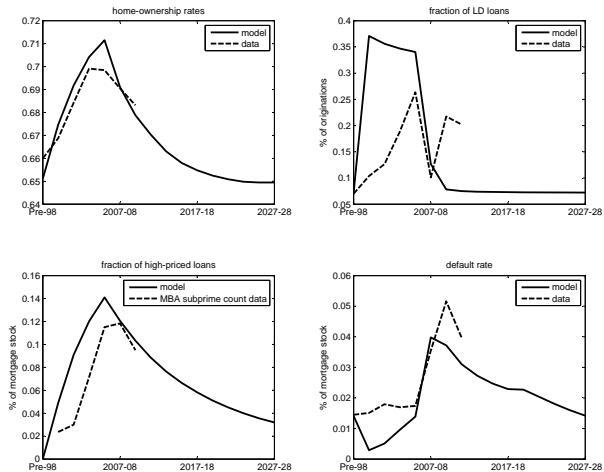
- Intermediary losses following unexpected aggregate shock are paid for through lump sum taxes.

Summary of transition results

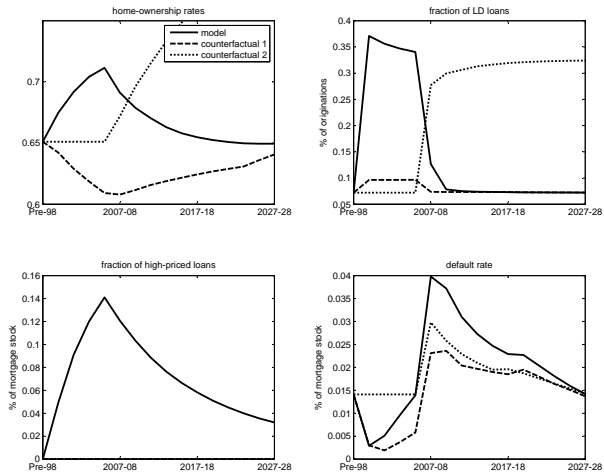
	Data	Benchmark	Counterfactual
Frac of LDs in last period of boom	34%	34.0%	9.7%
Increase in foreclosures 2007Q1-2009Q1	185%	182.3%	63.8%

- Model can explain 98% of the rise of foreclosures in the data between 2007Q1 and 2009Q1.
- In the counterfactual where the PTI requirement remains the same in the boom as during normal times, the price shock alone accounts for 35% of the increase in foreclosures.
- Thus, the relaxation of the PTI requirement with the price shock can explain 63% of the rise in foreclosures.

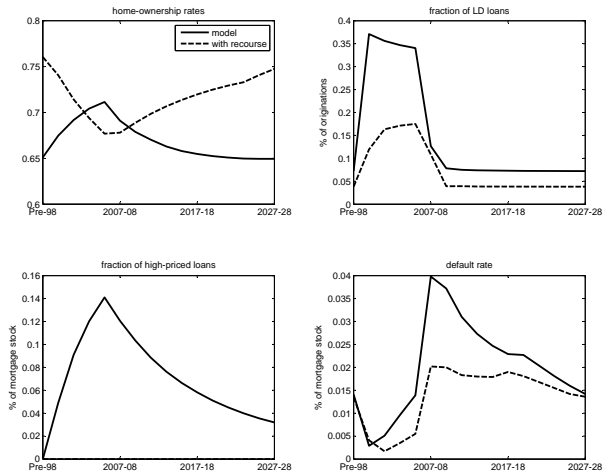
The boom-bust



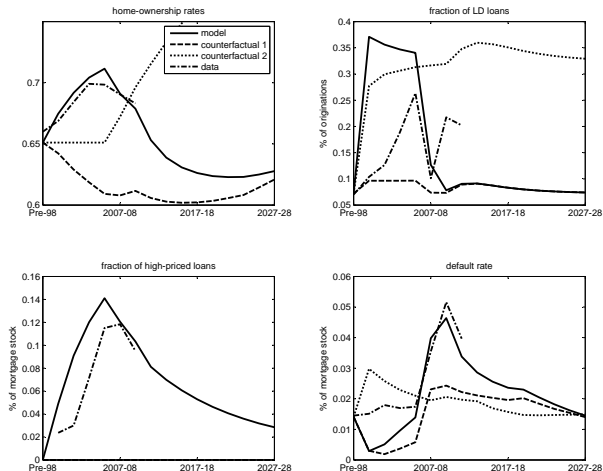
Leverage counterfactuals



Broader recourse mitigates the crisis



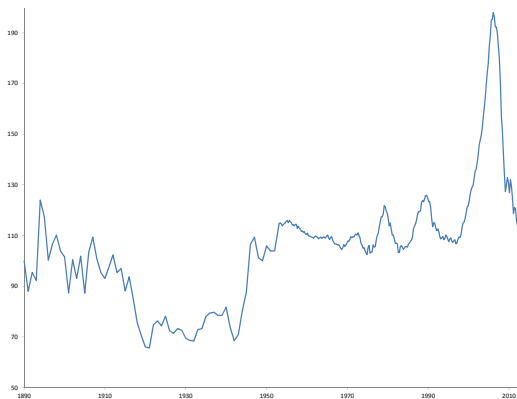
Model with aggregate income shock in second period of the crisis



Summary

- Question: How much did relaxed mortgage approval standards contribute to the foreclosure boom?
- Answer: By nearly $\frac{2}{3}$.
- Large effects not a consequence of “mispricing”.
- Foreclosure rates would have been 50% lower with recourse in the early stages of the transition.

Real home values (CS) in the long-run



The experiment

1. Calibrate price process to match long-term data
2. Calibrate parameters so that, following a long period of $q = q_N$ and using PTI limits of 25% (as they are in the data), the use of low-downpayment mortgages is around 5%
3. Relax underwriting standards for 4 model periods with $q = q_H$
4. Then q_H and underwriting standards return to pre-1998 values

Preliminary results:

- The model captures the rise in low-downpayment after 98, the rise in HO rates, and the the foreclosure boom
- Counterfactual 1: PTI standards not relaxed after 98
- Counterfactual 2: No middle stage, price falls from q_N to q_L
- Foreclosure rates peak 30% to 50% below benchmark

Housing Market Clearing Condition

The market for housing capital clears provided

$$\int_{\Omega_M} h 1_{\{H'=1, h(\omega)=h\}} d\mu_M - \int_{\Omega_M} h' 1_{\{H'=1\}} P(h'|\omega) d\mu_M = Ak$$

- In equilibrium the production of new housing capital must equal the housing capital lost to devaluation.
- Both the rental and owner-occupied markets clear since the intermediary is willing to accommodate any allocation of total housing capital by the arbitrage condition.

Mortgage payment function

- Fixed-rate mortgages (HDs)

$$m_t^\nu(a_t, y_t, h_t; q_t, \alpha_t) = \frac{r_t^\nu(a_t, y_t, h_t; q_t, \alpha_t)}{1 - (1 + r_t^\nu(a_t, y_t, h_t; q_t, \alpha_t))^{-T}} (1 - \nu_t) h_t q_t, \quad \forall n \in \{0, T - 1\}$$

▶ Back

Intermediary's problem

- The intermediary's value function after $n \in \{1, \dots, T - 1\}$ periods of the mortgage contract initiated in state κ is given by

$$\begin{aligned}
 W_n^{(\nu, \kappa)}(a, y, \epsilon; q, \alpha) = & \\
 & \mathbf{1}_{\{h^{(\nu, \kappa)}(a, y, \epsilon, n; q, \alpha) = \hat{h}\}} \min\{(1 - D^{(\nu, \kappa)}(a, y, \epsilon, n; q, \alpha)\chi)q\epsilon\hat{h}, b_n^\nu(\kappa)\} \\
 & + \mathbf{1}_{\{h^{(\nu, \kappa)}(a, y, \epsilon, n; q, \alpha) = \hat{h}\}} \left(\frac{m^\nu(\kappa)}{1 + r + \phi} + E_{y', \epsilon', q' | y, \epsilon, q} \left[\frac{W_{n+1}^{(\nu, \kappa)}(a', y', \epsilon'; q', \alpha')}{1 + r + \phi} \right] \right).
 \end{aligned}$$

► Back to SS def

- If the household does meet both qualification constraints, then:

$$W_0^{(\nu, \kappa)}(\widehat{a}, \widehat{y}, 1; \widehat{q}, \widehat{\alpha}) = \frac{m^\nu(\kappa)}{1 + r + \phi} + E_{y', \varepsilon', q' | y, \varepsilon, q} \left[\frac{W_1^{(\nu, \kappa)}(a', y', \varepsilon'; q', \alpha')}{1 + r + \phi} \right].$$

▶ [Back to SS def](#)

Truncated Rates

- The rate is truncated since the household default probability is too high for the bank to break-even at any mortgage rate below the rate at which the mortgage payment in the first period is so high that the budget set is empty.
- The left truncation can be thought of as an endogenous borrowing constraint associated with different borrower characteristics.
- In that period (i.e. when $n = 0$), the budget set is empty when $c = a' = 0$ and

$$m(0; \zeta, r^\zeta) > y_0 + (a_0 + \iota - vq\bar{h} \cdot 1_{\{\zeta=HD\}})(1 + r).$$

Since $m(0; \zeta, r^\zeta, h_0)$ is strictly increasing in r^ζ , we know there is an interest rate \bar{r}^ζ that depends on y_0 and a_0 such that for any $r > \bar{r}^\zeta$ the bank cannot break even.

Newly middle-aged agents $n = 0$

$$V_M(a, y, n = 0; q, \alpha) = \max_{c, a', h, \nu \in K} u(c, h) + \beta \rho_0 E_{q'|q} \left[V_O(a' + \mathbf{1}_{h \in \{h^2, h^3\}} S_{n=1}^{(\nu, \kappa)}(q', \varepsilon'); q') \right] \\ + \beta(1 - \rho_0) E_{y', q'|y, q} \left[\begin{array}{l} \iota_{\{h=h^1\}} \left(\begin{array}{l} (1 - \gamma) V_M^R(a', y'; q') \\ + \gamma V_M(a', y', n = 0; q') \end{array} \right) \\ + \iota_{\{h \neq h^1\}} \left(V_M^{(\nu, \kappa)}(a', y', \varepsilon', n = 1; q') \right) \end{array} \right]$$

where if $h = h^1$, then

$$\text{s.t. } c + a' = y + a(1 + r) - R(q)h^1$$

and if $h = \{h^2, h^3\}$, then

$$c + a' = y + (1 + r)[a - \nu qh] - m^\nu(\kappa) - \delta h \\ a \geq \nu qh \tag{2}$$

$$\frac{m^\nu(\kappa)}{y} \leq \alpha \tag{3}$$

Value function for a mid-aged agents with mortgage

$$V_M^{(\nu, \kappa)}(a, y, \epsilon, n; q) = \max_{c, a', h} u(c, h) + \beta \rho O E_{q'|q} \left[V_O(a' + 1_{\{h=\hat{h}\}} S_{n+1}^{(\nu, \kappa)}(q', \epsilon'); q') \right] \\ + \beta (1 - \rho O) E_{y', q'|y, q} \left[\begin{array}{l} 1_{\{h=h^1\}} V_M^R(a', y'; q') \\ + 1_{\{h=\hat{h}\}} V_M^{(\nu, \kappa)}(a', y', \epsilon', n+1; q') \end{array} \right]$$

where if $h = \hat{h}$, then

$$s.t. c + a' = y + a(1+r) - m^\nu(\kappa) - \delta h$$

and if $h = h^1$, then

$$c + a' = y + (1+r) \left[a + S_n^{(\nu, \kappa)}(q, \epsilon) \right] - R(q)h^1$$

$$S_n^{(\nu, \kappa)}(q, \epsilon) = \max \left\{ (1 - D^{(\nu, \kappa)}(a, y, \epsilon, n; q)) \chi q \epsilon \hat{h} - b_n^\nu(\kappa), 0 \right\}$$

$$D^{(\nu, \kappa)}(a, y, \epsilon, n; q) = 1 \text{ if } y + a(1+r) - m^\nu(\kappa) - \delta h < 0 \text{ or } q \epsilon \hat{h} - b_n^\nu(\kappa) < 0.$$

Definition of default

1. Involuntary default $D^I(\omega) = 1$

$$\begin{cases} H = 1 \\ y + (a + \iota)(1 + r) - m(n; \kappa) - \delta h < 0 \end{cases}$$

2. Voluntary default $D^V(\omega) = 1$

$$\begin{cases} H = 1 \\ y + (a + \iota)(1 + r) - m(n; \kappa) - \delta h \geq 0 \\ qh - b(n; \kappa) < 0 \\ H' = 0 \end{cases}$$

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Distribution of young agents

Let (n_L, n_M, n_H) be the invariant income distribution implied by the income process. The invariant distribution μ_Y on Ω_Y solves, for all $y \in \{y_L, y_M, y_H\}$ and $A \subset \mathbb{R}^+$:

$$\mu_Y(A, y) = \mu_0 \mathbf{1}_{\{0 \in A, y = y_j\}} n_j + (1 - \rho_M) \int_{\omega \in \Omega_Y} \mathbf{1}_{\{a'_Y(\omega) \in A\}} \Pi(y|\omega) d\mu_Y(\omega)$$

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Middle-aged agents

$$\begin{aligned}
 \mu_M(A, y, H, h, n; \kappa) &= \rho_M \int_{\Omega_Y} \mathbf{1}_{\{(H, h, n) = (0, h^1, 0)\}} \mathbf{1}_{\{a'_Y(\omega) \in A\}} \Pi(y|\omega) d\mu_Y(\omega) \\
 &+ (1 - \rho_0) \int_{\Omega_M} \mathbf{1}_{\{(H'(\omega) = H, n(\omega) = n-1, a'_M(\omega) \in A\}} \Pi(y|\omega) P(h|\omega) d\mu_M(\omega) \\
 &\times \left\{ \mathbf{1}_{\{n(\omega) = 0, \Xi(\omega) = \kappa\}} + \mathbf{1}_{\{n(\omega) > 0, \kappa = \kappa(\omega)\}} \right\}
 \end{aligned}$$

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Old agents

$$\begin{aligned}\mu_O(A) &= (1 - \rho_D) \int_{\Omega_O} \mathbf{1}_{\{a'_O(\omega) \in A\}} d\mu_O(\omega) \\ &\quad + \rho_O \int_{\Omega_M} \mathbf{1}_{\{a'_M(\omega) + \max\{H'(\omega)[qh(\omega) - b(n+1, \kappa)], 0\} \in A\}} d\mu_M(\omega)\end{aligned}$$

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On calibrating to HDs only before 2003

- In Figure 1, we can see the fraction of non-HDs accounts for about 15 percent of all mortgages before 2003.
- However, 2/3 of that fraction of non-HDs were standard nominally indexed ARM, which look more like traditional mortgages than LDs, until 2002.

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Some Steady State Accounting

$$C + H \cdot (R + \delta) = Y + r \cdot S + H \cdot R + X$$

where

- C is goods consumption
- $R \cdot H$ is housing services consumption
- $\delta \cdot H$ is investment
- Y is the aggregate endowment
- $r \cdot S$ is return to storage (or interest payments abroad if $S < 0$)
- $R \cdot H + X$ is imputed rents plus “rental income of persons” (i.e. X is the difference between imputed rents and what people actually pay for their housing consumption like mortgage payments plus maintenance for owners)

Gerardi et. al.'s approach

1. Estimate a default/refi competing hazard model with panel mortgage data that includes a proxy for home values (home equity) as an explanatory variable
2. Ask: if 2002 vintage of loans had experienced the same average price shock as 2005 vintage, at what average rate would they have defaulted?
3. Idea: 2002 vintage was written under more typical/stringent leverage and income tests standards
4. Answer: 2002 loans would have defaulted at about half the rate 2005 loans did

How our approach differs from and complements the econometric approach

- These numbers are predicated on
 1. a specific econometric model,
 2. the quality of controls (zip-codes vs actual home values), and
 3. the assumption that the 2002 borrower pool is what the 2005 pool would have been with 2002 underwriting standards (no sample selection effects)
- Our calculations do not require these assumptions but, of course, are conditional on our modeling choices
- Further, our model can be used to simulate the role of policy, such as recourse statutes

Definition of high-CLTV fraction

$$\text{Fraction of loans with CLTV} \geq 97\% = \frac{\text{Volume of loans with CLTV} \geq 97\%}{\text{Total volume of loans}}$$

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National Delinquency Survey definitions

Fraction of subprime mortgages is the stock of loans lenders report as subprime in NDS divided by the total stock of loans

The foreclosure rate is the number of foreclosure starts in the course of a given quarter divided by the total stock of mortgages at the start of the quarter

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