Online Appendix for Price Discrimination and Public Policy in the U.S. College Market

Appendix A Data Sources and Variable Creation

A.1 National Postsecondary Student Aid Study 2007–2008 (NPSAS)

The National Postsecondary Student Aid Study (NPSAS) is a nationally representative cross-section of college students in the United States. The survey collects data from students on many aspects of the college experience, with a particular focus on understanding how students pay for college. A new wave of NPSAS is collected every three to four years. I use the 2008 wave which surveyed students during the 2007–2008 school year. NPSAS collects information at the student level from several different sources: government records, college administrative records, third-party organizations (e.g. ACT and the College Board) and a student interview.

A.1.1 Raw Data

NPSAS is a restricted-use dataset obtained from the National Center for Education Statistics (NCES). I obtained access to the data through a data use agreement which prohibits publishing the data. The data comes in the form of several flat text files along with an electronic codebook. The electronic codebook contains the data documentation and allows the user to select desired variables. Once the user has selected the desired variables, the electronic codebook will produce an SPSS script that will read in and format the data. From SPSS, the data can be saved in several formats including Stata (.dta) format.

A.1.2 Variable Creation

Not all students attend college full-time for the full year. I define the number of full-time equivalent months to be equal to the number of full-time months plus half of the number of half-time months plus one-quarter of the number of less-than-half-time months.

The college selectivity variable used throughout the paper is based on a classification developed by the National Center for Education Statistics. The methodology is described in Appendix E of Cunningham (2005). Colleges are assigned to selectivity categories based on a few characteristics. Two-year colleges are put into their own category as are open admission four-year colleges.

> For non-open admission institutions, an index was created from two variables: 1) the centile distribution of the percentage of students who were admitted (of those who applied); and 2) the centile distribution of the midpoint between the 25th and 75th percentile SAT/ACT combined scores reported by each institution (ACT scores were

converted into SAT equivalents). The two variables were given equal weight for those non-open admission institutions that had data for both, and the combined centile variable was divided into selectivity categories—very selective, moderately selective, and minimally selective—based on breaks in the distribution. Institutions that did not have test score data (about 10 percent of non-open admission institutions) were assigned to the selectivity categories using a combination of percent admitted and whether they required test scores; institutions that did not require test scores were assigned to the "minimally selective" category, while the remainder were assigned according to the range of centiles of "percent admitted" in which they fell. (page E-1)

Table E-1. Selected 4-year institutions in the study universe, by institutional selectivity

	Very Selective	Moderately selective	Minimally selective	Open admission
Public institutions	Cornell University	Ball State University	Black Hills State University	Northern Kentucky University
	SUNY-Binghamton	Ohio State University	University of Northern Alabama	Texas Southern University
	University of Virginia	University of Oregon	Winston-Salem State University	University of Toledo
Private not-for-profit institutions	Duke University	DePaul University	Cabrini College	University of Rio Grande
	Princeton University	Mary Baldwin College	Wayland Baptist University	Pikeville College
	Williams College	University of San Francisco	Albertus Magnus College	Rochester College

NOTE: Selected institutions are in no particular order.
SOURCE: U.S. Department of Education, National Center for Education Statistics, Integrated Postsecondary Education Data System, Institutional Characteristics, Collection Year 2002.

Table 13: Selected 4-year Institutions, by Institutional Selectivity

Table E-1 from Cunningham (2005), reproduced here for reference, provides examples of the types of colleges that fall into each selectivity category. I collapse the "Minimally Selective" and "Open Admission" categories into a single category I label "Not Selective."

I use the NPSAS variable TUITION2 as my measure of gross tuition or sticker price. Discounts come from the institutional grants variable INGRTAMA. Transaction prices are simply the difference between the sticker price and discount. I adjust all prices and discounts to be in terms of 9 full-time equivalent months (those with less than 9 months were dropped, see below).

The number of additional colleges listed on the FAFSA comes from the NPSAS variables C08100, C08102, C08104, C08106, C08108, C08110. For each student, I added up the number of non-missing entries in these variables and subtracted one. The result was a number between 0 and 5. The variable was set to zero for students who did not complete the FAFSA. The variable was set to missing for those who did complete the FAFSA but had no colleges listed (as a result, these students were excluded from the analysis).

High school GPA was reported in ranges in the NPSAS variable HSGPA. Therefore I assigned each student a high school GPA equal to the minimum of the range which she reported.

I collapsed the NPSAS variable RACE into five categories "white," "black", "Hispanic", "Asian", and "other/multiple."

A.1.3 Sample Selection

I applied the following sample selection criteria

- Undergraduates only
- Exclude students on athletic scholarship
- Exclude foreign students and those from Puerto Rico
- Exclude students who attended multiple colleges during the 2007–2008 school year
- Exclude students not enrolled in a degree program
- Exclude students who have already earned a bachelor's degree
- Exclude students who received tuition waivers because of their parent's employment at the college
- Exclude students who had imputed tuition data from IPEDS
- Exclude students who completed a FAFSA but were missing FAFSA information
- Exclude students over age 30
- Exclude students who attended less than 9 full-time equivalent months
- Exclude colleges outside the 50 states plus Washington, D.C.
- Exclude for-profit colleges
- Exclude less-than-2-year colleges
- Exclude specialized colleges (but keep engineering and business schools)
- Exclude "special use" two-year colleges

In addition, there appears to be some misreporting of tuition prices. It seems that some colleges reported net tuition when they were supposed to report gross tuition. I drop students at colleges where the average transaction price (reported gross tuition minus institutional grants) is not positive.

For most of the reduced form analysis, I further restrict the sample to dependent freshmen. In some cases, I also restrict the analysis to students at elite or nonelite colleges.

	Freshmen Sample		Elite Sample	
	Colleges	Students	Colleges	Students
All Colleges	**1,110**	**6,780**	**490**	**2,300**
Very Selective	180	1,120	180	1,120
Moderately Selective	440	2,570	240	960
Not Selective	160	920	60	220
Two-Year	330	2,170	0	0
Private	**430**	**1,830**	**420**	**1,760**
Very Selective	120	580	120	580
Moderately Selective	240	960	240	960
Not Selective	60	220	60	220
Two-Year	10	70	0	0
Public	**690**	**4,960**	**70**	**540**
Very Selective	70	540	70	540
Moderately Selective	190	1,610	0	0
Not Selective	100	710	0	0
Two-Year	320	2,100	0	0
Public, In-State	**---**	**4,580**	**---**	**490**
Very Selective	---	490	---	490
Moderately Selective	---	1,440	---	0
Not Selective	---	650	---	0
Two-Year	---	2,010	---	0
Public, Out-Of-State	**---**	**370**	**---**	**50**
Very Selective	---	50	---	50
Moderately Selective	---	170	---	0
Not Selective	---	60	---	0
Two-Year	---	90	---	0

The elite sample consists of freshmen at private and very selective public four-year colleges. Each cell contains the raw count of the number of colleges and students in the sample. Per NCES requirements, counts have been rounded to the nearest ten.

SOURCE: U.S. Department of Education, National Center for Education Statistics, National Postsecondary Student Aid Study, 2008, and Beginning Postsecondary Students, 2004.

Table 14: Cell Counts

A.2 Beginning Postsecondary Students 2003-2009 (BPS)

Beginning Postsecondary Students 2003–2009 is a panel dataset that follows first-time freshmen that were sampled as part of the 2003 wave of NPSAS. Because BPS is derived from NPSAS, it contains much of the same information. However, because it is focused on freshmen, it also specifically asks about the number of colleges applied to (BPS variable APPS04) which I top-coded at 10. The raw data, variable creation, and sample selection for BPS were identical to that for NPSAS.

Appendix B Assumption on the Distribution of Match Surpluses

In Assumption 3.1, I assume that the cdf of the largest match surplus among a college's competitors $\mathcal{G}(\cdot)$ satisfies

$$\mathcal{G}(s)^2 > \mathcal{G}'(s) \int_0^s \mathcal{G}(y)dy \quad \forall s \in \boldsymbol{S}$$

Although this assumption is difficult to interpret on its own, in this section I offer two ways to understand the restrictions that Assumption 3.1 imposes.

First, recall that $\beta'(s|X) = (s - \beta(s|X))\frac{\mathcal{G}'(s|X)}{\mathcal{G}(s|X)}$. Thus, Assumption 3.1 implies that

$$\begin{aligned} & 1 > (s - \beta(s|X))\frac{\mathcal{G}'(s|X)}{\mathcal{G}(s|X)} \\ \Rightarrow \quad & 1 > \frac{s - \beta(s|X)}{s} \times \frac{\partial \log(\mathcal{G}(s|X))}{\partial \log(s)} \end{aligned} \tag{13}$$

The function $\mathcal{G}(s)$ gives the probability, in equilibrium, that a college with match surplus s wins the auction. Equation (13) says that, in equilibrium, a bidder will respond to a marginal increase in its match surplus by increasing its bid by the product of 1) its share of the match surplus, and 2) the elasticity of its probability of winning. The former is clearly less than one, but the latter could be greater than one if the density of $\mathcal{G}$ suddenly spiked.[1]

Second, recall that in an independent private values setting the equilibrium bid function is given by

$$\beta(s) = \mathbb{E}[Y|Y < s]$$

where Y denotes the largest match surplus among the college's competitors. That is, college j bids the expected match surplus of the student's next best option, conditional on j being the winning bidder. So Assumption 3.1 is equivalent to requiring that

$$\frac{\partial\, \mathbb{E}[Y|Y < s]}{\partial s} < 1$$

[1] A sudden spike in the density would cause $\mathcal{G}(s)$ to increase very quickly. But notice that β' can only stay above 1 for a small region of the support of S, because once $\beta(s)$ starts to approach s, the first term of (13) will approach zero and the bid function will flatten out.

In other words, the expected value of the truncated distribution of Y (truncated from the right at s) must not increase more quickly than the truncation point. This could be violated if the truncation point were just below a big spike in the density of Y. Then a one unit increase in s could cause a greater than one unit increase in $\mathbb{E}[Y|Y < s]$.[2] But in order for this to happen, the density would need to spike rather quickly at s. A gradual increase in the density would not be enough.

How restrictive is Assumption 3.1? That is, what type of distribution would violate this assumption? By way of illustration, consider the beta distribution with support $[0, 1]$. If $X \sim Beta(\alpha, \beta)$ and $\alpha = \beta$, then the distribution of X is symmetric around 0.5. As $\alpha = \beta \to \infty$, $f_X(0.5) \to \infty$. That is, we can make the density at 0.5 arbitrarily large by increasing α and β. One might think that, with a large enough value for α and β, we could cause the distribution to violate Assumption 3.1. This turns out not to be the case. Rather, $\lim_{\alpha=\beta\to\infty} \frac{\mathcal{G}'(s)}{\mathcal{G}(s)^2} \int_0^s \mathcal{G}(y)dy \approx 0.64$. This example illustrates that, in order to violate Assumption 3.1, a large value for the density is insufficient. We could cause a Beta distribution to violate Assumption 3.1 if we chose α and β so that the pdf had an asymptote at $y = 1$. For instance, setting $\alpha = 5$ and $\beta = 0.5$ gives a distribution that violates Assumption 3.1 for values close to one (although the slope of the bid function is less than one for nearly the entire support).

Appendix C Estimating Conditional Distributions

I employ the following method, described in detail in Fillmore (2017), to estimate conditional distributions. The estimator is able to handle a large number of covariates while still allowing for $F_{B|X}$ and $f_{B|X}$ to depend on X in a flexible way. I describe this method, quantile density estimation, below.

To illustrate how quantile density estimation works in practice, suppose we want to estimate the cdf F_Y and pdf f_Y of the random variable Y. Further, suppose that the support of Y is the interval $[\underline{y}, \overline{y}]$ with $f_Y > 0$ at all points in the support. Let $\{y_i\}_{i=1}^N$ denote a sample of size N from the distribution. Consider a grid of points $\{p_j\}$ on the interval $[0, 1]$ and define $\{q_j\}$ to be the empirical quantiles of the sample that correspond to the percentiles $\{p_j\}$. Now suppose we plotted the q_j on the horizontal axis and the p_j on the vertical axis. We would simply be plotting points from the empirical distribution function, and if the p_j were dense enough we would trace out the full step function.

The empirical cdf is an unbiased estimator of F_Y, but it is not smooth and, at least in its raw form, is unsuitable for density estimation. The quantile density estimator fits a smooth (twice continuously differentiable) monotone function $\widehat{F}_Y$ through the points $\{q_j, p_j\}$ subject to the constraints that $\widehat{F}_Y(\underline{y}) = 0$ and $\widehat{F}_Y(\overline{y}) = 1$. These two constraints, combined with monotonicity, guarantee that $\widehat{F}_Y$ is a proper cdf and that its derivative, $\widehat{f}_Y$, is a proper pdf.

[2]Such a spike would cause $\mathbb{E}[Y|Y < s]$ to increase quickly. But again, this can only be true for a small region of the support of Y, because once $\mathbb{E}[Y|Y < s]$ starts to approach s, it must slow down.

The quantile density estimator is calculated in two steps. First, a grid of percentiles is chosen and for each one the conditional quantile is modeled as

$$Q_Y(p|\boldsymbol{X}) = \boldsymbol{X} \cdot \beta^p \qquad p \in (0,1)$$

and estimated via quantile regression. The function $Q_Y(p|\boldsymbol{X})$ gives the pth quantile of the random variable Y, conditional on the values of the covariates $\boldsymbol{X}$. Note that the coefficients β^p are indexed by p because a separate quantile regression is run for each value of p. Let $\widehat{Q}_Y(p|\boldsymbol{X})$ denote the fitted values for a given value of the covariates. Plotting the points $\left(\widehat{Q}_Y(p|\boldsymbol{X}), p\right)$ would trace out an estimate of the conditional cdf of Y. In the next step, the estimator fits a smooth monotone curve through these points.

Ramsay (1998) considers the class of functions where $\log(F')$ is differentiable and $\{\log(F')\}' = \frac{F''}{F'}$ is Lebesgue square integrable, thus guaranteeing that F is monotone and that F' is smooth and bounded almost everywhere. All functions in this class can be expressed as the solution to the second-order differential equation

$$F'' = wF'$$

where w is a Lebesgue square integrable function. The solution to this ODE is simply

$$F(y) = C_0 + C_1 \int_{\underline{y}}^{y} \exp\left\{\int_{\underline{y}}^{t} w(s)ds\right\} dt \qquad y \in [\underline{y}, \overline{y}]$$

where C_0 and C_1 are constants. Since we are interested in functions that are valid cdf's, these constants must satisfy $C_0 = 0$ and $C_1 = \left[\int_{\underline{y}}^{\overline{y}} \exp\left\{\int_{\underline{y}}^{t} w(s)ds\right\} dt\right]^{-1}$. For convenience define $W(t) = \int_{\underline{y}}^{t} w(s)ds$. Thus, any cdf in this class of functions can be expressed in the form

$$F(y) = \begin{cases} 0 & \text{if } y < \underline{y} \\ \frac{\int_{\underline{y}}^{y} \exp\{W(t)\}dt}{\int_{\underline{y}}^{\overline{y}} \exp\{W(t)\}dt} & \text{if } y \in [\underline{y}, \overline{y}] \\ 1 & \text{if } y > \overline{y} \end{cases} \tag{14}$$

with density

$$f(y) = \begin{cases} 0 & \text{if } y < \underline{y} \\ \frac{\exp\{W(y)\}}{\int_{\underline{y}}^{\overline{y}} \exp\{W(t)\}dt} & \text{if } y \in [\underline{y}, \overline{y}] \\ 0 & \text{if } y > \overline{y} \end{cases} \tag{15}$$

where $W(t)$ is continuous and differentiable almost everywhere and $W(\underline{y}) = 0$.

The econometrician has a fair amount of freedom in specifying $W(t)$. In the paper, I use a cubic spline for the function $W(t)$. In practice, specifying $W(t)$ as a cubic spline seems to provide a nice balance between flexibility and parsimony. The practical consequence of choosing a cubic spline is to impose some additional smoothness on the pdf $f(y)$. Other specifications for $W(t)$

are possible, of course, as long as $W(t)$ is continuous and differentiable almost everywhere.[3]

The method outlined here is semiparametric. The second step, fitting the smooth monotone curve, is flexible enough to fit any monotone twice differentiable distribution function (Ramsay 1998). The first step, fitting the quantile regressions, can also be quite flexible, depending on how one specifies the regressions. Thus, the flexibility of the method as a whole is really driven by the flexibility of the quantile regression specification. However, note that even if we choose a linear specification, as I do in the paper, the method still allows the vector of coefficients β^p to differ for each value of p.

Appendix D Potential Effects on Students' Application Behavior

When simulating the counterfactuals, I hold students' application behavior constant. However, the estimates indicate that students would receive a larger share of the match surplus in each counterfactual, which would strengthen their incentive to apply to more colleges. The expected value of applying to college j' if the student's current best option is j can be written as

$$\mathbb{E}[\text{Value of Applying to } j'|j] = \mathbb{P}[s_{ij'} > s_{ij}] \times \mathbb{E}[\beta(s_{ij'}) - \beta(s_{ij})|s_{ij'} > s_{ij}].$$

The first term, $\mathbb{P}[s_{ij'} > s_{ij}]$, represents the probability that j' beats j, while the second term, $\mathbb{E}[\beta(s_{ij'}) - \beta(s_{ij})|s_{ij'} > s_{ij}]$, represents the expected marginal gain to the student in the event that j' beats j. Applying to one additional college only pays off if the college beats out the student's current best match. And if it does, the student will only receive a fraction of the additional match surplus because the new college will extract much of it through price discrimination. Although I do not directly model students' application decisions, my estimates can speak to the incentives students face when choosing how many applications to send out. In Table 15, I calculate the expected return from applying to an additional college. In this calculation I make the (extreme) assumption that student applications are independent of the match surplus. Thus, for each student I assume $\mathbb{P}[s_{ij'} > s_{ij}] = 1 - F_{S|X_i}(s_{ij})$. Then I multiply this probability by the conditional expected utility bid increase (conditional on $s_{ij'}$ exceeding s_{ij}). This calculation indicates that the expected return to applying to an additional college at random is \$673. However, if, as seems likely, students tend to apply to colleges that are a better match first, then $\mathbb{P}[s_{ij'} > s_{ij}] < 1 - F_{S|X_i}(s_{ij})$ and the estimates in Table 15 will overstate the returns of an additional application. I also recalculate the expected return from applying to an additional college (again assuming applications are independent of match surplus) in each counterfactual. Because colleges are bidding more aggressively, relative to baseline, the return to applying rises by between \$93 and \$316, depending on the counterfactual. If students respond to these incentives by applying to more colleges, then the increased competition will further lower prices as colleges are forced to bid more aggressively for students. This suggests that the estimates in the paper might be

[3] Alternatives to the cubic spline include piecewise linear functions or polynomials. Ramsay (1998) chooses $w(t)$ to be a step function, implying that $W(t)$ is a piecewise linear function.

understating the full effects of restricting colleges' use of FAFSA information.

	Baseline	Counterfactual 1	2	3	4	5
Avg return from an additional application	$673	$821	$868	$766	$900	$989
FAFSA Information Available						
Parent income	Yes	Poverty Dum	No	Yes	Poverty Dum	No
Number of schools listed on FAFSA	Yes	Yes	Yes	No	No	No
Whether completed FAFSA	Yes	Yes	Yes	Yes	No	No

Each cell reports the average return from applying to an additional college under each counterfactual. Returns represent the expected utility gain (in dollars) if the new college beats the student's current best option multiplied by the probability that this occurs. The returns were calculated under the assumptions that a) the number of colleges listed on the FAFSA remains fixed and b) students apply to colleges randomly. If, as seems likely, students tend to apply to colleges that are a better match first, these estimates provide an upper bound on the average returns to applying to an additional college. No sample weights were used.

SOURCE: U.S. Department of Education, National Center for Education Statistics, National Postsecondary Student Aid Study, 2008, and Beginning Postsecondary Students, 2004.

Table 15: Average Return From Applying to an Additional College

Appendix E Simulating the Counterfactuals

In order to simulate the counterfactuals, I must estimate the forecast error terms e_i and ξ_i. I use the following procedure:

1. Regress $\widehat{v}_{ij}$ on student covariates X_i and store the fitted values $\widehat{v}^1_{ij}$. Then regress $\widehat{v}_{ij}$ on $\widetilde{X}_i$, the reduced set of student covariates, and store the fitted values $\widehat{v}^2_{ij}$. Calculate $e_i = \widehat{v}^1_{ij} - \widehat{v}^2_{ij}$.

2. Repeat step 1 for $\widehat{w}_{ij}$ to estimate $\xi_i = \widehat{w}^1_{ij} - \widehat{w}^2_{ij}$.

3. Calculate $\widetilde{s}_{ij} = \widehat{s}_{ij} - e_i - \xi_i$.

4. Estimate F_e and f_e from the e_i using the same method described in section C.

5. Estimate $G_{\widetilde{S}|\widetilde{X}_i}$ and $g_{\widetilde{S}|\widetilde{X}_i}$ from the $\widetilde{s}_{ij}$ using the method from section C. Remember that this is the distribution of *winning* (counterfactual) match surpluses.

6. Solve for $F_{\widetilde{S}|\widetilde{X}_i}$ and $f_{\widetilde{S}|\widetilde{X}_i}$, the parent distribution of (counterfactual) match surpluses.

7. Solve for the counterfactual equilibrium bidding function, $\widetilde{\beta}(\cdot|\widetilde{X}_i)$, using Equation (12) in the paper.

8. Calculate the observed (to the college) winning bid, $\widetilde{u}_{ij} = \widetilde{\beta}(\widetilde{s}_{ij}|\widetilde{X}_i)$.

9. Calculate the true winning bid, $\widetilde{u}_{ij} + e_i$. If $\widetilde{u}_{ij} + e_i < 0$, then the student switches to a nonelite college and receives zero utility. Otherwise, the student remains at her college and pays a price equal to $v_{ij} - (\widetilde{u}_{ij} + e_i)$.

E.1 Using Lasso to choose a forecast model specification

In steps one and two above, I project student valuations on student covariates. In the paper, I do this using OLS. In order to explore other regression specifications, I also used a Lasso approach. The results are qualitatively the same with modest quantitative changes relative to the main results in the paper.

	Baseline		Counterfactual 1		2		3		4		5	
	(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)
Panel A. Levels												
Consumer (student) surplus per student	$5,023	$5,023	$5,645	$5,567	$5,697	$5,596	$5,504	$5,442	$5,778	$5,735	$5,851	$5,779
Total surplus per student	$15,417	$15,417	$15,339	$15,348	$15,322	$15,330	$15,317	$15,354	$15,279	$15,308	$15,183	$15,264
Of those who remain at elite colleges:												
Mean student share of surplus	30.1%	30.1%	37.5%	37.3%	37.7%	37.5%	36.7%	36.2%	38.3%	38.1%	38.4%	38.1%
Mean transaction price	$13,158	$13,158	$13,066	$13,156	$13,024	$13,129	$13,305	$13,313	$13,040	$13,045	$13,088	$13,069
Panel B. Changes Relative to Baseline												
Consumer (student) surplus per student	$0		$622	$544	$673	$573	$481	$419	$755	$711	$827	$755
Total surplus per student	$0		-$78	-$69	-$95	-$87	-$100	-$63	-$138	-$109	-$234	-$153
Percent of students who inefficiently choose a non-elite college	0.0%		8.0%	6.9%	8.5%	7.5%	9.0%	7.5%	10.4%	9.3%	12.7%	10.5%
Of those who remain at elite colleges:												
Mean change in student share of surplus	0.0%		5.8%	5.8%	5.8%	5.8%	4.7%	4.7%	6.1%	6.2%	5.9%	6.0%
Mean change in transaction price	$0		-$684	-$594	-$748	-$633	-$540	-$462	-$860	-$800	-$986	-$869
Percent of students with price drop	0.0%		71.6%	71.8%	71.3%	70.9%	80.1%	72.6%	75.6%	74.4%	68.2%	71.1%
Within-college variance in price (in millions)	38.81		-15.0%	-17.0%	-16.2%	-17.2%	-9.6%	-11.9%	-16.6%	-18.6%	-22.2%	-23.5%
FAFSA Information Available												
Parent income	Yes		Poverty Dummy		No		Yes		Poverty Dummy		No	
Number of schools listed on FAFSA	Yes		Yes		Yes		No		No		No	
Whether completed FAFSA	Yes		Yes		Yes		Yes		No		No	

Subcolumn 1 contains the main estimates reported in Table 11. Subcolumn 2 repeats the analysis in subcolumn 1 using a Lasso estimator (rather than OLS) to estimate e and ξ for the counterfactuals.

SOURCE: U.S. Department of Education, National Center for Education Statistics, National Postsecondary Student Aid Study, 2008, and Beginning Postsecondary Students, 2004.

Table 16: Comparing Counterfactual Estimates When Using Lasso to Estimate Forecast Errors e and ξ

Appendix F Extensions of the Model

F.1 Conditionally Independent Private Values

The independent private values (IPV) paradigm is very popular in the empirical auctions literature, particularly in the case of first-price auctions. In a seminal contribution, Guerre et al. (2000) showed, for first-price auctions in the IPV paradigm, how to derive a mapping between bids and bidder valuations that not only provided a transparent proof of nonparametric identification, but also suggested a simple and straightforward estimator. A productive line of research since then has extended the approach of Guerre et al. (2000) to more general informational settings. I think it is useful to quickly review how the IPV paradigm fits within the broader conditionally independent private values (CIPV) paradigm.

The CIPV paradigm can be described in the following way. There are N bidders. Bidder i has a private valuation s_i for the good.[4] Product quality is captured by the random variable $V \sim F_V$. The joint pdf of the private valuations and product quality is given by

$$f_{\mathbf{S},V}(s_1, \ldots, s_N, v) = f_V(v) \prod_{i=1}^{N} f_{S|V}(s_i|v)$$

Conditional on product quality V, the bidder valuations are drawn independently from the distribution $F_{S|V}$. It is important to note that the CIPV paradigm is still within the private values

[4] Intuitively, bidder j's valuation for the good is private if learning the valuations of its competitors would not change j's valuation.

paradigm, so that bidder i's payoff from receiving the good depends on s_i alone.[5] Product quality, V, introduces affiliation (a strong form of correlation) into the bidders' private valuations, since the distribution $F_{S|V}$ is a function of V.

There are roughly three ways that the literature has treated the random variable V. The first approach assumes that V is observed to bidders and to the econometrician. I will refer to this case as observed auction heterogeneity. In this case, all that is required for estimation is for the econometrician to condition on V. For example, we can assume that product quality V depends only on observed product characteristics X_i. Since the bidders and the econometrician observe X_i, and hence V, they can both condition on V and treat the auction as though it is within the IPV paradigm. This is the approach taken in "Price Discrimination and Public Policy in the U.S. College Market." Whether or not this approach works depends on whether the observed characteristics X_i fully account for product quality.

A second assumption we can make is to assume that V is observed to bidders but not to the econometrician. This is known as "unobserved auction heterogeneity" and can best be thought of as an omitted variables problem. We would expect this to occur if bidders observe a dimension of product quality that is unobserved to the econometrician. Unobserved auction heterogeneity is an inherently econometric problem, not a modeling problem. In other words, if the econometrician had better data, then we would be back to the case of observed auction heterogeneity discussed above. Since the bidders can observe V and condition on it, from their perspective the informational environment is still one of independent private values. It is the econometrician who cannot observe V and will erroneously pool bids from auctions with different values of V in his estimation.

Finally, a third assumption we can make is that bidders do not observe V, although they know the distribution F_V from which it is drawn as well as the conditional distribution $F_{S|V}$. Since they do not observe it and cannot condition on it, V introduces affiliation into their private valuations which will affect equilibrium bidding behavior. Holding the number of bidders fixed, increasing affiliation causes bidders to bid more aggressively. The intuition is that if my valuation is high it is likely that my competitors' valuations are also high, thereby intensifying competition between bidders. However, affiliation also introduces an *affiliation effect* which causes bidders to shade their bids as though they were subject to a winner's curse.[6] Holding constant the amount of affiliation, adding another bidder may actually cause bidders to reduce their bids.

Because they are distinct issues, separate methods have been developed in the empirical auctions literature to deal with observed auction heterogeneity, unobserved auction heterogeneity, and affiliated private values. Below I discuss each issue separately and propose a strategy to address each one in turn.

[5] In other words, V only affects bidder i's valuation indirectly through its effect on s_i.

[6] To be clear, affiliation does not actually introduce a winner's curse. The winner's curse is a feature of the common values paradigm, not the private values paradigm.

F.2 Observed Auction Heterogeneity

The case of observed auction heterogeneity is the most straightforward. Whereas the estimator of Guerre et al. (2000) simply requires estimation of the cdf and pdf of the bid distribution, F_B and f_B, with observed auction heterogeneity we must estimate the cdf and pdf of the bid distribution conditional on observable characteristics, $F_{B|X}$ and $f_{B|X}$. In many applications in the literature, researchers simply divide auctions into bins based on a small number of observed characteristics X. Unfortunately, binning on covariates is empirically infeasible for even moderate numbers of covariates. The same is true of more sophisticated nonparametric methods like kernel smoothing. In section C, I provide more details about how I condition on student covariates.

F.3 Unobserved Auction Heterogeneity

Colleges do see things about students that are not observed to the econometrician. If these unobservables are idiosyncratic to each school, then a private values framework is still justified. Indeed, in the model I assume that colleges observe v_{ij}, the student's valuation for the college, which is not observed to the econometrician. However, if the unobservables (to the econometrician) are observed to colleges and common across colleges, then this would introduce unobserved auction heterogeneity. This heterogeneity would be a problem because it would cause bidder valuations (in my case, match surpluses) to be correlated. A common approach to this problem is to use multiple bids to deal with unobserved auction heterogeneity, but in my application I only have information on the winning bid.

To address unobserved auction heterogeneity, I adopt a similar approach to that of Roberts (2013). Roberts argues that, if bidders can observe the unobserved (to the econometrician) product characteristic, then it seems sensible that the seller could also observe the characteristic. He assumes that sellers incorporate the characteristic into their choice of reserve price and uses reserve prices to account for unobserved auction heterogeneity. Importantly, Roberts does not assume that sellers set optimal reserve prices or that they incorporate the characteristic optimally. He merely requires that reserve prices are a monotone function of the unobserved product characteristic. In essence, the reserve price becomes a proxy for the unobserved product characteristic. Roberts proves that, under these conditions, the IPV model in a first-price auction with unobserved auction heterogeneity is identified with data on winning bids and reserve prices. Rather than use reserve prices, which I do not have in my data, I use data on a student's *ex post* college GPA. Following Roberts, I assume that college GPA is a function of both observed student characteristics X and unobserved (to the econometrician) student quality χ. By observing the ex post realization of college GPA, I can infer unobserved student quality and then condition on it in estimation.

Of course, grades at different colleges may not be comparable. To deal with this possibility, I begin by projecting college GPA into ACT units using the fitted values from the following regression

$$ACT_{ij} = \beta_{t(j)} GPA_{ij} + \mu_j + \epsilon_{ij}$$

where ACT_{ij} denotes student i's ACT score, GPA_{ij} is student i's GPA at college j, $\beta_{t(j)}$ is a coefficient on GPA that varies with college j's type,[7] and μ_j is a college-specific fixed effect. The fixed effects in this specification allow for the possibility that some colleges may have stricter grading norms than other colleges. The specification also allows the coefficient on GPA to vary across college types.[8] Call the fitted values from this equation $\widehat{GPA}_i$.

Now that I have projected college GPA onto a common scale, I estimate χ_i as the residual from the following regression

$$\widehat{GPA}_i = X_i \delta + \chi_i$$

where X_i are the observed student characteristics. Finally, I include χ_i as a conditioning covariate when I estimate the model. Table 17 compares the results when controlling for unobserved auction heterogeneity to those in the paper. Controlling for unobserved auction heterogeneity does not affect the qualitative findings and has only a modest effect on the quantitative results.

	Baseline		Counterfactual 1		Counterfactual 2		Counterfactual 3		Counterfactual 4		Counterfactual 5	
	(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)
Panel A. Levels												
Consumer (student) surplus per student	$5,023	$5,113	$5,645	$5,687	$5,697	$5,730	$5,504	$5,555	$5,778	$5,794	$5,851	$5,889
Total surplus per student	$15,417	$15,457	$15,339	$15,379	$15,322	$15,349	$15,317	$15,376	$15,279	$15,313	$15,183	$15,211
Of those who remain at elite colleges:												
Mean student share of surplus	30.1%	30.1%	37.5%	37.6%	37.7%	37.7%	36.7%	36.8%	38.3%	38.1%	38.4%	38.5%
Mean transaction price	$13,158	$13,158	$13,066	$13,106	$13,024	$13,124	$13,305	$13,307	$13,040	$13,122	$13,088	$13,106
Panel B. Changes Relative to Baseline												
Consumer (student) surplus per student	$0		$622	$574	$673	$616	$481	$442	$755	$681	$827	$776
Total surplus per student	$0		-$78	-$78	-$95	-$108	-$100	-$81	-$138	-$144	-$234	-$246
Percent of students who inefficiently choose a non-elite college	0.0%		8.0%	8.2%	8.5%	9.3%	9.0%	8.3%	10.4%	10.7%	12.7%	13.0%
Of those who remain at elite colleges:												
Mean change in student share of surplus	0.0%		5.8%	5.7%	5.8%	5.6%	4.7%	4.8%	6.1%	5.8%	5.9%	5.8%
Mean change in transaction price	$0		-$684	-$635	-$748	-$695	-$540	-$490	-$860	-$783	-$986	-$935
Percent of students with price drop	0.0%		71.6%	70.5%	71.3%	68.5%	80.1%	79.5%	75.6%	72.7%	68.2%	66.9%
Within-college variance in price (in millions)	38.81		-15.0%	-14.7%	-16.2%	-16.5%	-9.6%	-8.4%	-16.6%	-16.0%	-22.2%	-22.7%
FAFSA Information Available												
Parent income	Yes		Poverty Dummy		No		Yes		Poverty Dummy		No	
Number of schools listed on FAFSA	Yes		Yes		Yes		No		No		No	
Whether completed FAFSA	Yes		Yes		Yes		Yes		No		No	

Subcolumn 1 contains the main estimates reported in Table 11. Subcolumn 2 repeats the analysis in subcolumn 1 after using *ex post* college GPA to control for unobserved auction heterogeneity.

SOURCE: U.S. Department of Education, National Center for Education Statistics, National Postsecondary Student Aid Study, 2008, and Beginning Postsecondary Students, 2004.

Table 17: Comparing Counterfactual Estimates When Controlling For Unobserved Auction Heterogeneity

F.4 Affiliated Private Values

What if there is a dimension of product quality that is unobserved by both the econometrician *and* the bidders? In this case, the unobserved characteristic would introduce affiliation into the

[7] College types include Very Selective Public, Very Selective Private, Moderately Selective Private, and Not Selective Private.

[8] Although I could, in principle, allow β to differ for each college, doing so would split the data rather thin.

private valuations of the bidders, which would in turn alter equilibrium bidding behavior. Athey and Haile (2002) prove that the joint distribution of bidder valuations in an affiliated private value model is nonparametrically identified only if all bids in each auction are observed.[9] Li et al. (2002) extend the approach of Guerre et al. (2000) and construct a nonparametric estimator of the APV model when all bids are observed. However, the need to observe all bids presents a severe limitation on estimating this model in practice.

The results of Athey and Haile (2002) indicate that, without being able to observe all of the bids, some parametric assumptions are unavoidable if we are going to make any progress. Hubbard et al. (2012) report promising results using a semiparametric approach that employs an Archimedean copulae. A copula is a function $\mathcal{C} : [0,1]^d \to [0,1]$ that is a proper CDF of d random variables with uniform marginal distributions. In fact, Sklar's Theorem tells us that any joint distribution $\mathcal{F}(\cdot)$ can be written as

$$\mathcal{F}(X_1, \ldots, X_d) = \mathcal{C}(F_1(X_1), \ldots, F_d(X_d))$$

for some copula function $\mathcal{C}(\cdot)$, where F_i is the marginal distribution of X_i. This means that the copula representation is fully general. Moreover, copulae allow us to separate any joint distribution into the marginal distributions of the random variables and the dependence structure between those random variables. Hubbard et al. (2012) propose estimating the marginal distributions of bidder values non-parametrically but placing parametric restrictions on the copula itself. The approach of Hubbard et al. (2012) allows the econometrician to avoid parametric assumptions about the marginal distribution of bidder private valuations while imposing parametric restrictions on the dependence (affiliation) of those valuations. In particular, they consider copulae where the degree of affiliation is governed by a single parameter.[10] Hubbard et al. (2012) find that their semiparametric estimator performs well relative to the nonparametric estimator of Li et al. (2002), even when the copula is misspecified.

F.4.1 The model with affiliation and a parametric copula

In this section, I extend the model from the paper to allow for affiliation in match surpluses with a known affiliation parameter. College j is bidding on student i. Student i evaluates her offer from college j using the utility function $u_{ij} = v_{ij} - p_{ij}$, where v_{ij} represents her valuation, in dollars, of attending the college and p_{ij} is the price college j offers her. Elite colleges compete for students on price. The college "wins" the auction—the student enrolls—if it makes the best offer, u, as judged by the student. Colleges care about maximizing both the quality of their students as well as tuition revenue. Let $\mathbf{\Pi}$ denote the space of college payoffs for enrolling a student. College j's payoff from enrolling student i is $\pi_{ij} = w_{ij} + p_{ij}$, where $w_{ij} = z_j + \omega(X_i) + \gamma I_{ij}$ represents

[9]However, it is possible to identify the joint distribution of the two highest private valuations using just the top two bids.

[10]Their approach has the added benefit of imposing affiliation on the joint distribution, which is required to guarantee the existence of equilibrium.

college j's valuation, in dollars, of enrolling the student. The vector X_i denotes characteristics of student i that are observed to both the college and the econometrician, while z_j is observed to the college only. I_{ij} is an indicator equal to one if college j is public and student i is in-state. Note that since w_{ij} is college j's valuation, $-w_{ij}$ is j's willingness to accept. That is, $-w_{ij}$ represents the lowest price that the college would be willing to offer student i because charging her less than $-w_{ij}$ would give the college a negative payoff.

College j knows z_j and learns v_{ij} and X_i, and by extension w_{ij}, during the application process, but it does not know the v's or w's of the other bidders. It also does not know the number of bidders, n_i, but it does observe a noisy signal, $\widetilde{n}_i$ which is one of the elements of the vector X_i. It also knows the probability of n bidders conditional on the characteristics of the student, $\rho(n|X)$.

College j makes a take-it-or-leave-it price offer, p_{ij}, to student i to maximize its expected surplus which is simply its payoff if it enrolls the student, π_{ij}, times the probability of enrolling her

$$\pi_{ij}\,\mathbb{P}[j \text{ wins}] = (w_{ij} + p_{ij})\,\mathbb{P}[u_{ij} \geq u_{i\ell}\ \forall \ell \neq j|X_i].$$

Up to this point, we have been thinking about the college's decision in terms of price offers. However, if we recast the college's problem in terms of utility bids, we can express the model in a way that lends itself to empirical estimation. At this point it will be convenient to focus on college $j = 1$. Define $s_{ij} \equiv u_{ij} + \pi_{ij} = v_{ij} + w_{ij}$ to be the total surplus from matching student i with college j. The joint distribution of match surpluses for exactly n bidders is given by

$$\mathcal{F}_{\mathbf{S}|X_i}(s_{i1}, \ldots, s_{in}|X_i) = \mathcal{C}^{(n)}(F_{S|X}(s_{i1}|X_i), \ldots, F_{S|X}(s_{in}|X_i))$$

where $\mathcal{C}^{(n)}$ is the n-dimensional copula and $F_{S|X}(\cdot)$ is the marginal distribution of match surpluses.[11] Note that $\mathcal{C}^{(n)}$ takes n arguments, one for each bidder. Hubbard et al. (2012) prove that, conditional on bidder 1's match surplus, the distribution of its competitors' surpluses is given by the derivative of $C^{(n)}$ with respect to the first argument

$$\mathcal{F}_{\mathbf{S}_{-1}|S_{i1},X_i}(s_{i2}, \ldots, s_{in}|s_{i1}, X_i) = \mathcal{C}_1^{(n)}(F_{S|X}(s_{i1}|X_i), \ldots, F_{S|X}(s_{in}|X_i))$$

Now we can rewrite college 1's objective function as

$$\begin{aligned}
& \{(v_{i1} + w_{i1}) - (v_{i1} - p_{i1})\}\,\mathbb{P}[u_{i1} \geq u_{i\ell}\ \forall \ell > 1|X_i] \\
= \quad & (s_{i1} - u_{i1})\,\mathbb{P}[u_{i1} \geq \beta(s_{i\ell}|X_i)\ \forall \ell > 1|X_i] \\
= \quad & (s_{i1} - u_{i1})\,\mathbb{P}[\beta^{-1}(u_{i1}|X_i) \geq s_{i\ell}\ \forall \ell > 1|X_i] \\
= \quad & (s_{i1} - u_{i1}) \left\{ \sum_{n=1}^{\overline{n}} \mathcal{F}_{\mathbf{S}_{-1}|S_{i1}}(\beta^{-1}(u_{i1}), \ldots, \beta^{-1}(u_{i1})|s_{i1})\rho(n) \right\} \\
= \quad & (s_{i1} - u_{i1}) \left\{ \sum_{n=1}^{\overline{n}} \mathcal{C}_1^{(n)}\Big(F_S(s_{i1}), F_S(\beta^{-1}(u_{i1})), \ldots, F_S(\beta^{-1}(u_{i1}))\Big)\,\rho(n) \right\}.
\end{aligned} \tag{16}$$

[11]Since bidders are symmetric, this marginal distribution does not vary across bidders.

where for notational convenience we have suppressed the fact that the joint distribution $\mathcal{F}_S$, the marginal distribution F_S, the distribution of bidders $\rho(n)$, and the equilibrium bid function $\beta(s)$ are all conditioned on observable covariates X_i. The marginal distribution of match surpluses F_S has support $\boldsymbol{S} = [\underline{s}, \bar{s}]$ ($\underline{s} \leq 0 < \bar{s}$). As is standard in the auction literature, I assume that the marginal density f_S is strictly positive over the entire support.

Taking college 1's first order condition, we get the first order condition

$$\beta'(s) = (s - \beta(s)) \frac{\sum_{n=1}^{\overline{n}} \rho(n)(n-1) f_S(s) C_{12}^{(n)}(F_S(s), \ldots, F_S(s))}{\sum_{n=1}^{\overline{n}} \rho(n) C_1^{(n)}(F_S(s), \ldots, F_S(s))} \tag{17}$$

Note that the IPV model corresponds to the case where $C^{(n)}(y_1, y_2, \ldots, y_n) = \prod_{j=1}^{n} y_j$. In this case, (17) reduces to equation (2) in the paper.

Define the college payoff function $\pi(s) \equiv s - \beta(s)$, which gives the *ex post* college payoff as a function of the total surplus s. Note that $\pi(s)$ is monotone in s with $\pi'(s) \in (0,1)$ (since $\beta'(s) \in (0,1)$). Denote the distribution of college payoffs, π by F_π. Now the derivative of the payoff function $\pi(s)$ is

$$\begin{aligned}
& \pi'(s) = 1 - \beta'(s) = 1 - (s - \beta(s)) \frac{\sum_{n=1}^{\overline{n}} \rho(n)(n-1) f_S(s) C_{12}^{(n)}(F_S(s), \ldots, F_S(s))}{\sum_{n=1}^{\overline{n}} \rho(n) C_1^{(n)}(F_S(s), \ldots, F_S(s))} \\
\Rightarrow \quad & \frac{\sum_{n=1}^{\overline{n}} \rho(n) C_1^{(n)}(F_S(s), \ldots, F_S(s))}{\sum_{n=1}^{\overline{n}} \rho(n)(n-1) f_S(s) C_{12}^{(n)}(F_S(s), \ldots, F_S(s))} \pi'(s) = \frac{\sum_{n=1}^{\overline{n}} \rho(n) C_1^{(n)}(F_S(s), \ldots, F_S(s))}{\sum_{n=1}^{\overline{n}} \rho(n)(n-1) f_S(s) C_{12}^{(n)}(F_S(s), \ldots, F_S(s))} - \pi(s) \\
\Rightarrow \quad & \pi(s) = \frac{\sum_{n=1}^{\overline{n}} \rho(n) C_1^{(n)}(F_\pi(\pi(s)), \ldots, F_\pi(\pi(s)))}{\sum_{n=1}^{\overline{n}} \rho(n)(n-1) f_\pi(\pi(s)) C_{12}^{(n)}(F_\pi(\pi(s)), \ldots, F_\pi(\pi(s)))} \left(\frac{1}{\pi'(s)} - 1 \right). \qquad (18)
\end{aligned}$$

I have now rewritten $\pi(s)$ in terms of the distribution of college payoffs rather than the surplus distribution by using the fact that $F_S(s) = F_\pi(\pi(s))$ and therefore $f_\pi(\pi(s)) = \frac{f_S(s)}{\pi'(s)}$. Solving (18) for $\pi'(s)$ gives

$$\begin{aligned}
\pi'(s) &= \left(1 + \pi(s) \frac{\sum_{n=1}^{\overline{n}} \rho(n)(n-1) f_\pi(\pi(s)) C_{12}^{(n)}(F_\pi(\pi(s)), \ldots, F_\pi(\pi(s)))}{\sum_{n=1}^{\overline{n}} \rho(n) C_1^{(n)}(F_\pi(\pi(s)), \ldots, F_\pi(\pi(s)))} \right)^{-1}, \quad 0 \leq s \qquad (19) \\
\pi(0|X_i) &= 0. \qquad (20)
\end{aligned}$$

Notice that since $\pi : \boldsymbol{S} \to \boldsymbol{\Pi}$ is monotone its inverse $\psi : \boldsymbol{\Pi} \to \boldsymbol{S}$ exists and has a derivative that is simply the reciprocal of π'. So we can write

$$\begin{aligned}
\psi'(\pi) &= 1 + \pi \frac{\sum_{n=1}^{\overline{n}} \rho(n)(n-1) f_\pi(\pi) C_{12}^{(n)}(F_\pi(\pi), \ldots, F_\pi(\pi))}{\sum_{n=1}^{\overline{n}} \rho(n) C_1^{(n)}(F_\pi(\pi), \ldots, F_\pi(\pi))} \qquad 0 \leq \pi \qquad (21) \\
\psi(0) &= 0. \qquad (22)
\end{aligned}$$

Finally, Equation (21) can be solved by simply integrating from 0 to π. $\psi(\pi)$ is the equilibrium inverse payoff function; it maps from the space of college payoffs, $\boldsymbol{\Pi}$, to the space of match surpluses, $\boldsymbol{S}$. $\psi(\pi)$ depends only on the equilibrium marginal distribution of colleges payoffs, F_π, the copula $C(\cdot)$ and the distribution of potential bidders, $\rho(n)$. Recall that we have suppressed the fact that all of these distributions and function are conditioned on student covariates X.

If I observed all the payoffs for all of the bidders in the auction (or even just a random sample of them), then I could estimate F_π directly. However, I only observe the payoff for the winning college, which is also the largest payoff among the invited bidders. Let $G_\pi(\pi)$ denote the distribution of observed (winning) college payoffs. Then the marginal distribution function F_π and its density f_π are defined implicitly by

$$G_\pi(\pi) = \sum_{n=1}^{\bar{n}} \rho(n) C^{(n)}(F_\pi(\pi), \ldots, F_\pi(\pi))$$
$$g_\pi(\pi) = \sum_{n=1}^{\bar{n}} \rho(n) n f_\pi(\pi) C_1^{(n)}(F_\pi(\pi), \ldots, F_\pi(\pi))$$

With a known copula function and observed G_π, g_π, and $\rho(n)$, we can numerically solve for F_π and f_π. Then we plug these into (21) and proceed in a similar manner as in the paper.

F.4.2 Estimating the affiliation parameter

Unfortunately, the estimator of Hubbard et al. (2012) requires multiple bids at each auction. Their idea is to use the rank correlation of bids to identify the rank correlation of bidder valuations. Since I only observe the winning bid (really college payoff) for each auction, I instead leverage a student's (*ex post*) college GPA to estimate the affiliation parameter as follows. One aspect of the Archimedean copulae employed by Hubbard et al. (2012) is that the affiliation parameter for two random variables can be expressed as a function of Kendall's-τ rank correlation between those random variables. Since rank correlations are invariant to monotonic transformations, this means that the rank correlation between college payoffs must be the same as the rank correlation between match surpluses. I infer unobserved student quality using *ex post* college GPA as described in section F.3. Thus, I observe the distribution $F_{\chi|X}$, which is the distribution of unobserved student quality, conditional on observable student characteristics.[12] Within the CIPV framework, bidder valuations (in my case, match surpluses) are independent conditional on both observed and unobserved student characteristics. Since college payoffs are a monotone function of match surpluses, we can think of college payoffs as being distributed independently, conditional on χ and X, according to some distribution $F_{\pi|\chi,X}$. Thus, for any fixed student characteristics X, we can simulate multiple college payoffs from the same auction by first drawing a value of $\chi \sim F_{\chi|X}$

[12] Since I observe the college payoff of the *winning* college (which is the largest payoff among all the bidding colleges) and not a random college, I must make a correction to account for this fact. The distribution of winning college payoffs is given by

$$G_{\pi_{ij}|\chi_i,X_i} = \sum_{n=1}^{\bar{n}} F_{\pi|\chi,X}(\pi_{ij}|\chi_i, X_i)^n \rho(n|\chi_i, X_i)$$

Given the observed distribution $G_{\pi_{ij}|\chi_i,X_i}$ and the probabilities $\rho(n|\chi_i, X_i)$, I numerically invert this expression to calculate the parent distribution $F_{\pi|\chi,X}(\pi_{ij}|\chi_i, X_i)$.

and then drawing a pair of college payoffs $\pi_1, \pi_2 \overset{iid}{\sim} F_{\pi|\chi,X}$. Conditional only on observed student characteristics X, π_1 and π_2 will be affiliated because of unobserved student quality χ. The strength of the affiliation depends on the strength of the observed relationship between college payoffs and χ. The stronger is the observed relationship between π_{ij} and χ_i, conditional on X_i, then the stronger will be the implied affiliation between π_{ij} and π_{ik}. I simply simulate 10,000 draws of (χ, π_1, π_2) and calculate Kendall's-τ rank correlation of π_1 and π_2.

I estimate a rank correlation of 0.026 between bidder match surpluses. This results from the fact that, conditional on observed covariates X, unobserved student quality (as inferred from freshmen GPA) is only weakly correlated with the college's payoff π_{ij}. Put differently, observable student characteristics X account for nearly all of the common component of student quality, and once I condition on X, nearly all of the remaining variation in match surpluses is due to idiosyncratic factors. Empirically, allowing for affiliated private values with a rank correlation of 0.026 does not produce substantially different results than assuming independent private values (see Table 18).

A skeptical reader might be concerned that college GPA does not capture all of the unobserved student quality. In order for this to be true, there must exist some other dimension of student quality that is orthogonal to multiple measures of academic performance (ACT scores, high school GPA, college GPA, AP exams). To explore this idea, I re-estimate the model by calibrating the rank correlation to 0.26 (ten times the value estimated in the data.) Even with a rank correlation of 0.26, the qualitative results of the paper are unchanged and the quantitative results change only modestly (see Table 18). In short, the data do not indicate a significant role for affiliated private values, and even at moderate levels of affiliation, the qualitative findings of the paper are unaffected.

		Counterfactual				
	Baseline	**1**	**2**	**3**	4	5
	Panel A. Levels					
Consumer (student) surplus per student	$5,023	$5,645	$5,697	$5,504	$5,778	$5,851
	$5,086	$5,706	$5,757	$5,563	$5,835	$5,903
	$5,735	$6,328	$6,378	$6,154	$6,417	$6,456
Total surplus per student	$15,417	$15,339	$15,322	$15,317	$15,279	$15,183
	$15,480	$15,402	$15,382	$15,380	$15,339	$15,248
	$16,129	$16,046	$16,024	$16,005	$15,973	$15,864
Of those who remain at elite colleges:						
Mean student share of surplus	30.1%	37.5%	37.7%	36.7%	38.3%	38.4%
	30.2%	37.7%	37.8%	36.8%	38.5%	38.5%
	32.2%	39.4%	39.5%	38.3%	39.7%	39.6%
Mean transaction price	$13,158	$13,066	$13,024	$13,305	$13,040	$13,088
	$13,158	$13,067	$13,025	$13,308	$13,046	$13,089
	$13,158	$13,115	$13,068	$13,406	$13,143	$13,210
	Panel B. Changes Relative to Baseline					
Consumer (student) surplus per student	$0	$622	$673	$481	$755	$827
	$0	$620	$670	$477	$748	$817
	$0	$593	$643	$418	$682	$720
Total surplus per student	$0	-$78	-$95	-$100	-$138	-$234
	$0	-$78	-$97	-$100	-$141	-$232
	$0	-$83	-$105	-$123	-$156	-$265
Percent of students who inefficiently choose a non-elite college	0.0%	8.0%	8.5%	9.0%	10.4%	12.7%
	0.0%	8.0%	8.5%	9.0%	10.5%	12.6%
	0.0%	8.1%	8.8%	9.7%	10.9%	13.1%
Of those who remain at elite colleges:						
Mean change in student share of surplus	0.0%	5.8%	5.8%	4.7%	6.1%	5.9%
	0.0%	5.8%	5.8%	4.7%	6.0%	5.8%
	0.0%	5.5%	5.5%	4.0%	5.2%	4.8%
Mean change in transaction price	$0	-$684	-$748	-$540	-$860	-$986
	$0	-$683	-$745	-$536	-$854	-$973
	$0	-$656	-$721	-$480	-$789	-$877
Percent of students with price drop	0.0%	71.6%	71.3%	80.1%	75.6%	68.2%
	0.0%	71.6%	71.0%	79.6%	75.2%	67.9%
	0.0%	70.6%	70.1%	73.6%	71.0%	64.7%
Within-college variance in price (in millions)	38.81	-15.0%	-16.2%	-9.6%	-16.6%	-22.2%
	38.81	-14.9%	-16.1%	-9.6%	-16.6%	-22.2%
	38.81	-14.4%	-15.1%	-9.2%	-16.2%	-22.0%
FAFSA Information Available						
Parent income	Yes	Poverty Dum	No	Yes	Poverty Dum	No
Number of schools listed on FAFSA	Yes	Yes	Yes	No	No	No
Whether completed FAFSA	Yes	Yes	Yes	Yes	No	No

Subrow 1 contains the main estimates reported in Table 11. In subrow 2, I estimate an affiliated private values model where the degree of affiliation is governed by a Frank copula with rank correlation estimated to be 0.026. In subrow 3, I calibrate the degree of affiliation to a rank correlation of 0.26, ten times the rank correlation that is estimated.

SOURCE: U.S. Department of Education, National Center for Education Statistics, National Postsecondary Student Aid Study, 2008, and Beginning Postsecondary Students, 2004.

Table 18: Comparing Counterfactual Estimates When Allowing For Affiliation

F.5 Alternative Estimates of a College's Payoff

Recall that $-w_{ij}$ is college j's willingness to receive for student i. In equilibrium, a university's willingness to receive is given by the lower support of the distribution of transaction prices, conditional on student covariates X_i. Unfortunately, although w_{ij} is nonparametrically identified, the identification proof does not immediately suggest an estimator that could be used in a finite data set. Therefore I adopt a parametric assumption about the distribution of $p|X, j$. I assume that the left tail of the cdf $F_{p|X,j}$ follows the quadratic parametric form in (9). I then estimate several quantiles of the price distribution $F_{p|X,j}$ using the quantile regression specification in equation (10) at the quantiles $q = .05, .10, \ldots, .40$. For each observation, I obtain the fitted values from these quantile regressions, giving me eight (estimated) points on the left tail of $F_{p|X,j}$. Then, separately for each observation, I fit the curve in (9) to these points via nonlinear least squares.

In this section I explore three alternative estimators for a college's willingness to receive. First, I estimate w_{ij} using a more flexible cubic specification

$$\widehat{F}_{p|X,j} = \alpha_1(X,j)(p - \underline{p}(X,j)) + \alpha_2(X,j)(p - \underline{p}(X,j))^2 + \alpha_3(X,j)(p - \underline{p}(X,j))^3$$

with $\alpha_1, \alpha_2, \alpha_3 > 0$. While the quadratic specification implies a linear PDF, the cubic specification allows for some curvature in the PDF. Nevertheless, the cubic specification gives results that are very similar to those from the quadratic specification (see Table 19). Second, I fit the quadratic specification but this time to the quantiles $q = .05, .10, \ldots, .25$. The idea is to fit a more local approximation of the CDF near the left boundary of support. Again, the results do not change dramatically from those in the paper (see Table 20). Finally, in the paper I allow college j's willingness to receive to vary with student characteristics, such as test scores, thus allowing for some students to be more "desirable" than others to the college. But if colleges did not care about student characteristics at all (i.e. they viewed all students as interchangeable), then college j's willingness to receive would be given by the lower bound of the support of prices among all students at college j. Thus, as a third alternative estimator, I estimate each college's willingness to receive by taking the lowest transaction price observed at that college in the data. The results are reported in Table 21. Using this estimator of willingness to receive, the counterfactual simulations become significantly more dramatic. For example, transaction prices fall by up to 70 percent more than in the results reported in the paper. In essence, this approach attributes *all* of the within-college price variation to price discrimination, which naturally amplifies the consequences of restricting the FAFSA information.

	Baseline		Counterfactual 1		Counterfactual 2		Counterfactual 3		Counterfactual 4		Counterfactual 5	
	(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)
Panel A. Levels												
Consumer (student) surplus per student	$5,023	$5,599	$5,645	$6,220	$5,697	$6,275	$5,504	$6,104	$5,778	$6,377	$5,851	$6,454
Total surplus per student	$15,417	$16,595	$15,339	$16,519	$15,322	$16,494	$15,317	$16,482	$15,279	$16,452	$15,183	$16,374
Of those who remain at elite colleges:												
Mean student share of surplus	30.1%	30.6%	37.5%	37.8%	37.7%	38.2%	36.7%	37.3%	38.3%	38.8%	38.4%	38.8%
Mean transaction price	$13,158	$13,158	$13,066	$13,076	$13,024	$13,047	$13,305	$13,333	$13,040	$13,068	$13,088	$13,039
Panel B. Changes Relative to Baseline												
Consumer (student) surplus per student	$0		$622	$620	$673	$676	$481	$505	$755	$778	$827	$855
Total surplus per student	$0		-$78	-$76	-$95	-$101	-$100	-$113	-$138	-$143	-$234	-$221
Percent of students who inefficiently choose a non-elite college	0.0%		8.0%	7.3%	8.5%	8.1%	9.0%	8.8%	10.4%	9.8%	12.7%	11.3%
Of those who remain at elite colleges:												
Mean change in student share of surplus	0.0%		5.8%	5.5%	5.8%	5.7%	4.7%	4.6%	6.1%	5.9%	5.9%	5.7%
Mean change in transaction price	$0		-$684	-$678	-$748	-$748	-$540	-$567	-$860	-$881	-$986	-$998
Percent of students with price drop	0.0%		71.6%	72.6%	71.3%	70.1%	80.1%	78.2%	75.6%	75.4%	68.2%	68.2%
Within-college variance in price (in millions)	38.81		-15.0%	-18.9%	-16.2%	-19.6%	-9.6%	-13.6%	-16.6%	-20.0%	-22.2%	-25.3%
FAFSA Information Available												
Parent income	Yes		Poverty Dummy		No		Yes		Poverty Dummy		No	
Number of schools listed on FAFSA	Yes		Yes		Yes		No		No		No	
Whether completed FAFSA	Yes		Yes		Yes		Yes		No		No	

Subcolumn 1 contains the main estimates reported in Table 11. Subcolumn 2 repeats the analysis in subcolumn 1 except that it estimates a college's willingness-to-receive using a cubic specification, rather than a quadratic specification.

SOURCE: U.S. Department of Education, National Center for Education Statistics, National Postsecondary Student Aid Study, 2008, and Beginning Postsecondary Students, 2004.

Table 19: Comparing Counterfactual Estimates Using a Cubic Specification to Estimate w_{ij}

	Baseline		Counterfactual 1		Counterfactual 2		Counterfactual 3		Counterfactual 4		Counterfactual 5	
	(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)
Panel A. Levels												
Consumer (student) surplus per student	$5,023	$5,557	$5,645	$6,236	$5,697	$6,301	$5,504	$6,048	$5,778	$6,367	$5,851	$6,477
Total surplus per student	$15,417	$16,882	$15,339	$16,801	$15,322	$16,762	$15,317	$16,770	$15,279	$16,730	$15,183	$16,624
Of those who remain at elite colleges:												
Mean student share of surplus	30.1%	30.1%	37.5%	37.6%	37.7%	38.1%	36.7%	36.6%	38.3%	38.1%	38.4%	38.6%
Mean transaction price	$13,158	$13,158	$13,066	$12,958	$13,024	$12,975	$13,305	$13,262	$13,040	$12,973	$13,088	$12,951
Panel B. Changes Relative to Baseline												
Consumer (student) surplus per student	$0		$622	$679	$673	$744	$481	$490	$755	$810	$827	$920
Total surplus per student	$0		-$78	-$81	-$95	-$120	-$100	-$112	-$138	-$153	-$234	-$259
Percent of students who inefficiently choose a non-elite college	0.0%		8.0%	7.4%	8.5%	8.6%	9.0%	8.6%	10.4%	10.0%	12.7%	12.4%
Of those who remain at elite colleges:												
Mean change in student share of surplus	0.0%		5.8%	5.7%	5.8%	6.0%	4.7%	4.5%	6.1%	5.8%	5.9%	5.9%
Mean change in transaction price	$0		-$684	-$741	-$748	-$830	-$540	-$550	-$860	-$921	-$986	-$1,092
Percent of students with price drop	0.0%		71.6%	70.9%	71.3%	71.1%	80.1%	77.6%	75.6%	75.0%	68.2%	70.1%
Within-college variance in price (in millions)	38.81		-15.0%	-13.2%	-16.2%	-14.4%	-9.6%	-6.7%	-16.6%	-14.1%	-22.2%	-20.1%
FAFSA Information Available												
Parent income	Yes		Poverty Dummy		No		Yes		Poverty Dummy		No	
Number of schools listed on FAFSA	Yes		Yes		Yes		No		No		No	
Whether completed FAFSA	Yes		Yes		Yes		Yes		No		No	

Subcolumn 1 contains the main estimates reported in Table 11. Subcolumn 2 repeats the analysis in subcolumn 1 except that it estimates a college's willingness-to-receive using the left 25 percent of the distribution of transaction prices, rather than the left 40 percent.

SOURCE: U.S. Department of Education, National Center for Education Statistics, National Postsecondary Student Aid Study, 2008, and Beginning Postsecondary Students, 2004.

Table 20: Comparing Counterfactual Estimates Using the Left 25 Percent of the Transaction Price Distribution to Estimate w_{ij}

	Baseline		Counterfactual 1		Counterfactual 2		Counterfactual 3		Counterfactual 4		Counterfactual 5	
	(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)
Panel A. Levels												
Consumer (student) surplus per student	$5,023	$4,999	$5,645	$5,990	$5,697	$6,077	$5,504	$5,580	$5,778	$6,124	$5,851	$6,296
Total surplus per student	$15,417	$16,158	$15,339	$16,051	$15,322	$16,005	$15,317	$16,101	$15,279	$16,025	$15,183	$15,925
Of those who remain at elite colleges:												
Mean student share of surplus	30.1%	27.2%	37.5%	36.9%	37.7%	37.7%	36.7%	34.0%	38.3%	37.6%	38.4%	38.6%
Mean transaction price	$13,158	$13,158	$13,066	$13,328	$13,024	$13,305	$13,305	$13,680	$13,040	$13,234	$13,088	$13,120
Panel B. Changes Relative to Baseline												
Consumer (student) surplus per student	$0		$622	$991	$673	$1,078	$481	$581	$755	$1,125	$827	$1,297
Total surplus per student	$0		-$78	-$107	-$95	-$153	-$100	-$58	-$138	-$133	-$234	-$233
Percent of students who inefficiently choose a non-elite college	0.0%		8.0%	15.5%	8.5%	16.8%	9.0%	13.8%	10.4%	16.5%	12.7%	18.8%
Of those who remain at elite colleges:												
Mean change in student share of surplus	0.0%		5.8%	8.0%	5.8%	8.7%	4.7%	5.3%	6.1%	8.6%	5.9%	9.4%
Mean change in transaction price	$0		-$684	-$1,193	-$748	-$1,328	-$540	-$683	-$860	-$1,372	-$986	-$1,646
Percent of students with price drop	0.0%		71.6%	78.1%	71.3%	78.1%	80.1%	88.9%	75.6%	83.1%	68.2%	78.1%
Within-college variance in price (in millions)	38.81		-15.0%	-25.7%	-16.2%	-25.7%	-9.6%	-20.4%	-16.6%	-25.1%	-22.2%	-30.0%
FAFSA Information Available												
Parent income	Yes		Poverty Dummy		No		Yes		Poverty Dummy		No	
Number of schools listed on FAFSA	Yes		Yes		Yes		No		No		No	
Whether completed FAFSA	Yes		Yes		Yes		Yes		No		No	

Subcolumn 1 contains the main estimates reported in Table 11. Subcolumn 2 repeats the analysis in subcolumn 1 except that it estimates a college's willingness-to-receive by taking the minimum transaction price observed at that college. Thus, a given college is assumed to have the same willingness-to-receive for all students.

SOURCE: U.S. Department of Education, National Center for Education Statistics, National Postsecondary Student Aid Study, 2008, and Beginning Postsecondary Students, 2004.

Table 21: Comparing Counterfactual Estimates Using the Minimum Price Observed at College j as an Estimate of w_{ij}

F.6 Treating the Number of Colleges Listed on the FAFSA as a Perfect Signal of Potential BIdders

In the paper, I assumed that the number of colleges listed on the FAFSA was an imperfect signal of the true number of potential bidders. In this section, I re-estimate the model under the alternative assumption that the number of colleges listed on the FAFSA accurately reflects the number of potential bidders. Table 22 contrasts these estimates with the main estimates in the paper. In counterfactuals 3, 4, and 5, when colleges are restricted from using the number of colleges listed on the FAFSA, I still assume that they can use other covariates to predict the number of potential bidders. When the FAFSA perfectly reveals the number of competitors, colleges are able to extract all of the surplus from students who list only one college on the FAFSA because they know that they face no other competitors for those students. On the other hand, when a student lists several colleges, bidders now know they must bid more aggressively to attract the student. Assuming that the FAFSA perfectly reveals the number of competitors primarily serves to magnify the effects I find in the paper.

	Baseline		Counterfactual 1		Counterfactual 2		Counterfactual 3		Counterfactual 4		Counterfactual 5	
	(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)
Panel A. Levels												
Consumer (student) surplus per student	$5,023	$4,845	$5,645	$5,462	$5,697	$5,486	$5,504	$5,752	$5,778	$5,952	$5,851	$5,972
Total surplus per student	$15,417	$15,239	$15,339	$15,143	$15,322	$15,064	$15,317	$15,023	$15,279	$15,005	$15,183	$14,851
Of those who remain at elite colleges:												
Mean student share of surplus	30.1%	29.4%	37.5%	36.4%	37.7%	36.7%	36.7%	38.1%	38.3%	39.4%	38.4%	39.3%
Mean transaction price	$13,158	$13,158	$13,066	$13,070	$13,024	$13,115	$13,305	$13,007	$13,040	$12,790	$13,088	$12,800
Panel B. Changes Relative to Baseline												
Consumer (student) surplus per student	$0		$622	$617	$673	$641	$481	$907	$755	$1,107	$827	$1,127
Total surplus per student	$0		-$78	-$96	-$95	-$175	-$100	-$216	-$138	-$234	-$234	-$387
Percent of students who inefficiently choose a non-elite college	0.0%		8.0%	7.8%	8.5%	9.3%	9.0%	12.6%	10.4%	12.8%	12.7%	15.6%
Of those who remain at elite colleges:												
Mean change in student share of surplus	0.0%		5.8%	6.2%	5.8%	6.0%	4.7%	7.1%	6.1%	8.1%	5.9%	7.4%
Mean change in transaction price	$0		-$684	-$673	-$748	-$711	-$540	-$1,064	-$860	-$1,288	-$986	-$1,373
Percent of students with price drop	0.0%		71.6%	66.9%	71.3%	65.5%	80.1%	77.8%	75.6%	77.0%	68.2%	71.9%
Within-college variance in price (in millions)	38.81		-15.0%	-16.4%	-16.2%	-18.4%	-9.6%	-11.7%	-16.6%	-17.5%	-22.2%	-25.9%
FAFSA Information Available												
Parent income	Yes		Poverty Dummy		No		Yes		Poverty Dummy		No	
Number of schools listed on FAFSA	Yes		Yes		Yes		No		No		No	
Whether completed FAFSA	Yes		Yes		Yes		Yes		No		No	

Subcolumn 1 contains the main estimates reported in Table 11. Subcolumn 2 repeats the analysis in subcolumn 1 except that it assumes that the number of colleges listed on the FAFSA is identical to the number of potential bidders (colleges), rather than a noisy signal of the number of potential bidders.

SOURCE: U.S. Department of Education, National Center for Education Statistics, National Postsecondary Student Aid Study, 2008, and Beginning Postsecondary Students, 2004.

Table 22: Simulating Counterfactuals Assuming That the FAFSA Perfectly Reveals the Number of Potential Bidders

F.7 Modeling College Spending by Incorporating Dynamics

When colleges price discriminate, they are able to earn more revenue than if they charged a uniform price. If the colleges invest this additional revenue into improving educational quality, then price discrimination may directly improve the quality of education for all students (although as Peña (2010) points out, the colleges may capture some of that increased consumer surplus by raising prices). In this section, I generalize the model to explicitly incorporate both tuition revenues and alumni giving. I show that adding these features to the model does not fundamentally alter the college's first order condition.

The model as presented in the paper is silent about how colleges use their tuition revenues. Modeling college spending on quality is tricky. For instance, if I assume that colleges use today's revenues on today's quality, then the problem becomes quite complicated, because a given student's demand for college j will depend on her beliefs about the classmates she will have—wealthier classmates translate into more tuition revenue and thus a better education. But if I adopt an alternative, and perhaps more realistic, timing assumption, then the model remains tractable. I assume that colleges must purchase all of their quality inputs (faculty, facilities, etc.) one period ahead. This timing assumption means that the quality of college j will not depend on today's tuition revenue. Rather, the college's quality will depend on tuition revenue from *last* period. Thus, a college's quality is fixed and common knowledge when student i is considering whether to attend.

There are a finite number I of student types indexed by i. Each type contains a mass of students.[13] Students live for two periods. In the first period, they decide which college to attend.

[13] Types i and i' need not have the same mass of students. Furthermore, the mass of all student types combined

In the second they make a donation to their alma mater. Let d_{ij} be the average donation made by type i students to college j.

Colleges are infinitely lived. College j enters the period with a stock of quality Q, an endowment E, and a set of alumni. Let a_{ij} denote the mass of type i alumni from college j. The demand for college j among type i students is given by $q_{ij}(p_{ij}; Q)$. Demand depends on both the price offered by the college as well as the college's quality. The college makes price offers to each student type and collects its tuition revenue from the students that enroll. In addition, the college receives a payoff of w_{ij} for each type i student that enrolls. w_{ij} represents the contribution that type i students make on campus minus the direct costs of enrolling the student and the opportunity costs of diverting resources away from other activities such as research. w_{ij} could be positive or negative, depending on the student and college.

The college's quality and endowment evolve according to the laws of motion

$$Q' = (1-\delta)Q + \frac{Z}{p_z} \tag{23}$$

$$E' = (1+r)\left(E - Z + \underbrace{\sum_{i=1}^{I}(p_{ij} - m_j + S_{ij})q_{ij}(p_{ij}; Q)}_{\text{Tuition Revenue}} + \underbrace{\sum_{i=1}^{I} d_{ij}a_{ij}}_{\text{Alumni Donations}} \right). \tag{24}$$

δ is the depreciation rate on the stock of college quality. If all quality inputs must be repurchased every year, then δ will be one, but if some quality inputs behave more like capital, then δ will be less than one. Z represents the college's investment in quality (in dollars), and p_z is the price of those investments. r is the interest rate earned on the endowment. m_j represents per-student instructional costs. S_{ij} represents a per-student subsidy that the college receives, and is probably most relevant for public colleges if state governments tie appropriations to enrollment levels—especially in-state enrollment.

In order to keep things simple, I will assume that Z is chosen by an external group called the Board of Trustees, and the college takes Z as given when it makes its price offers. This assumption is relatively innocuous because I will be focusing on the steady state of the model, where Z would not differ even if it were chosen by the college.

As mentioned above, the college receives a (possibly negative) payoff w_{ij} for each student it enrolls. It also receives payoff $g(Q)$ from having quality Q. The function g is increasing and concave. The Bellman equation for college j is

$$V(Q, E, a_{1j}, \ldots, a_{Ij}) = \max_{\{p_{ij}\}} \sum_{i=1}^{I} w_{ij}q_{ij}(p_{ij}; Q) + g(Q) + \beta V\left(Q', E', q_{1j}(p_{1j}; Q), \ldots, q_{Ij}(p_{Ij}; Q)\right),$$

need not be one, although it could easily be normalized to one.

subject to the laws of motion (23) and (24). In the steady state, $Q' = Q$ and $E' = E$ so that

$$Z^{ss} = \frac{r}{1+r}E^{ss} + \sum_{i=1}^{I}(p_{ij} - m_j + S_{ij})q_{ij}(p_{ij}; Q^{ss}) + \sum_{i=1}^{I} d_{ij}a_{ij}.$$

That is, in the steady state the college invests all of its tuition revenue and alumni donations, along with any interest income, into college quality. The first order condition with respect to p_{ij} for a college in the steady state is

$$0 = w_{ij}q'_{ij}(p_{ij}; Q) + \frac{1}{p_z}\beta V_Q \times \Big((p_{ij} - m_j + S_{ij})q'_{ij}(p_{ij}; Q) + q_{ij}(p_{ij}; Q)\Big) + \beta V_{a_{ij}} q'_{ij}(p_{ij}; Q),$$

which can be simplified to

$$p_{ij} = p_z \frac{-w_{ij} - \beta V_{a_{ij}}}{\beta V_Q} + m_j - S_{ij} + \frac{q_{ij}(p_{ij}; Q)}{q'_{ij}(p_{ij}; Q)}.$$

Substituting in the Euler condition $V_{a_{ij}} = \frac{1}{p_z}\beta V_Q d_{ij}$ and using the fact that we are in the steady state (so that V_Q is the same every period), we get

$$p_{ij} = \underbrace{m_j - p_z \frac{w_{ij}}{\beta V_Q} - \beta d_{ij} - S_{ij}}_{c_{ij}} + \underbrace{\frac{q_{ij}(p_{ij}; Q)}{q'_{ij}(p_{ij}; Q)}}_{m_{ij}}.$$

Note that this first order condition for the college has the same form as in the static model.[14] The only difference is in the components of the willingness to receive term c_{ij}. Willingness to receive is now given by per-student instructional costs minus i) a term capturing the payoff to the college of enrolling the student w_{ij}, ii) the net present value of a student's alumni giving, and iii) any per-student subsidy the college receives for enrolling the student (probably most relevant for in-state students at public colleges). The college has a lower willingness to receive—it is willing to charge less—for students who are expected to make larger future donations or who represent a larger per-student subsidy to the college. At colleges with a higher marginal value of quality, willingness to receive will be less sensitive to the college's payoff w_{ij}. Intuitively, if a college desperately needs money to invest in quality improvements, then it can't afford to be picky about the types of students it enrolls and will be willing to charge a low price to any student as long as the total dollar revenue $p_{ij} + \beta d_{ij} + S_{ij}$ from the student remains greater than instructional costs m_{ij}.

In this section, I have extended the model to incorporate tuition revenues as well as alumni giving. Both of these features affect the college's pricing decision by altering the components of the college's willingness to receive term c_{ij}. The net present value of future donations directly

[14] q_{ij} is analogous to $\mathbb{P}[i \text{ chooses } j]$, and the first order conditions from the static model in the paper and the dynamic model presented here have the same form, $(p - c)q' = q$.

lowers c_{ij}. In the steady state, tuition revenues are spent entirely on college quality, and the marginal value (to the college) of quality shows up in the denominator of $p_z \frac{-w_{ij}}{\beta V_Q}$. The takeaway lesson here is that alumni giving and tuition revenues both alter the *components* of c_{ij} but do not alter the way it is estimated. In any case, c_{ij} can always be interpreted as the lowest price college j is willing to receive from students of type i.