# Estimating the returns to parental time investment in children using a life-cycle dynastic model* 

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#### Abstract

This paper estimates the impact of parents' time investment in young children, their socioeconomic status and family structure on long-term outcomes of children. We developed and estimated a model of dynastic households in which altruistic individuals choose fertility, labor supply, and time investment in children sequentially, using data on two generations from the PSID. Parental behavior affects the education outcomes of children, their labor market earnings, and also their marriage market outcomes. The dynastic framework provides a natural way to aggregate the outcomes of children as it is measured by their valuation function. Time allocation patterns differ by race, gender, education and family structure (number of children and marital status). Both the returns to parental time investment and the costs vary across these groups. On average black individuals invest less time with their children. Our estimation results show that despite the fact that the valuation function is higher for whites and the fact that conditional on education, blacks earn less than whites, on the margin, there are no significant race differences in the rate of returns to paternal time investment, and blacks have a higher return to maternal time investment than whites. The main reason for the lower parental time investment by blacks is the differences in family structure. There is a significantly higher proportion of black single mothers than white single mothers and the opportunity costs of time for single mothers are higher than the opportunity costs of married mothers due to income sharing and transfers within married households. We also find that the returns to maternal time investment are significantly higher for boys. This implies that mothers act in a compensatory manner, favoring low ability children in the family. Since girls already have a higher likelihood of achieving a high level of education than boys, mothers seem to invest more time in boys than in girls as the number of children increases. Our findings suggest a significant quality-quantity trade-off. The level of investment per child is smaller the larger the number of children, thus, this decline in the per-child investment is driven by the time constraint and the opportunity costs of time and not by the properties of the production function technology of children. We, also find that quality-quantity trade-off for blacks is significantly larger than that of whites. This is mainly due to the higher fertility of single black female and the resulting greater time constraint they face.


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#### Abstract

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## 1 Introduction

Parental investment in the human capital of children plays an important role in the intergenerational persistence of wealth. One important channel for intergenerational transmission of human capital is the time parents spend with children when they are young. ${ }^{1}$ In order to understand why investment patterns differ across race, gender and education groups, we analyze the costs and returns to time investment. The opportunity cost of time depends on the parents' labor market prospect and on the family structure; married and single parents may face different tradeoffs when they allocate time between housework and labor market activities. The returns, in terms of the outcomes of children are typically measured by their educational attainment and labor market outcomes. Educational outcomes, however, also affect the marriage market outcomes. The goal of this paper is to quantify the effect of the different factors on parental inputs. We develop a dynastic model in which, altruistic adults, in each generation, sequentially choose fertility, labor supply, and time investment in children. Using data on two generations from the PSID, we estimate the model and quantify the costs and returns of time investment in children during early childhood. Only a handful of papers estimate the returns to parental time investment accounting for the opportunity costs of time and labor supply decisions of parents in a life-cycle model. To the best of our knowledge no other paper estimates the cost and returns to parental inputs, accounting for quantity-quality tradeoffs involved in fertility decisions, and assortative mating.

The theoretical framework we develop builds on the dynastic model of intergenerational transmission of human capital in Loury (1981), Becker and Tomes (1986) model of intergenerational transfers , and Becker and Barro (1988) and Barro and Becker (1989) dynastic models with endogenous fertility. Most dynastic models do not model marriage explicitly. ${ }^{2}$ In our framework individuals may be single or married, and divorce and marriage evolve according to a stochastic process. In the literature, households decisions are either framed as a single decision maker problem (this approach is pioneered by Becker $(1965,1981)$ ) or as a bargaining problem which is either modeled as a cooperative gametheoretic problem or as a non-cooperative one (e.g., Manser and Brown (1980), McElroy and Horney (1981), Chiappori (1988); see also Chiappori and Donni (2009) for a recent survey on non-unitary models of household behavior, and Lundberg and Pollak (1996) survey on non-cooperative models of allocation within households). We model household decision problem as a noncooperative game and solve for a Markov Perfect Equilibrium (for models of household allocations which are determined as a Nash Equilibrium outcomes of a non cooperative game see Lundberg and Pollak (1993), Del Boca and Flinn (1995, 2010), and Chen and Wolley (2001)). While there is no consensus in the literature regarding the process governing household decisions, there are several advantages to this approach in our framework. First, the Barro-Becker model is formalized as a single decision maker dynamic optimization problem. Since we solve for a Markov Perfect equilibrium, given any spouse strategies and characteristics, the problem reduces to a single agent optimization problem and fits naturally in

[^1]their theoretical framework as well as in the estimation framework of dynamic games which we discuss below. At the same time, in contrast to a unitary model approach, we are able to evaluate separately the value function of each individual, which is an advantage as parents utility is derived from their own children utility and not from the utility of their spouse. Second, individuals may belong to different households over their life cycle, and spouses may have children from previous marriages. In our formulations, this issue does not complicate the analysis, since parents care only about the utility of their biological children, and they simply maximize their own utilities, conditional on their spouse's strategies. To the best of our knowledge no paper has fully estimated a dynastic model with joint household decisions. ${ }^{3}$

In the model, each individual from each generation lives for $T$ periods. Over the life-cycle, each individual makes labor supply and time investment decisions in children every period; only females make birth decisions. Marriage and divorce evolve according to a stochastic process. The process depends on choices individuals make, therefore, although we do not model marriage and divorce decisions explicitly, individuals take into account the effect of their choices on future probabilities of marriage and divorce. Couples decisions are modeled as a non-cooperative game and are made simultaneously. We do not model explicitly bargaining over allocation of consumption within the households and assume that each individual receives (per-period) utility from his own income, the spouse's income and the stock of existing children in the household. This formulation is consistent with transfers of income between spouses in which the size of the transfers depends on the number of children and earnings of each individual in the household. Total time investment of both parents over the life-cycle affects the children's outcomes through several channels. Once children become adults, their education levels are realized; the education level is a stochastic function of the parental time input and the parents' education level and labor market skills. In addition, the skill level of a child and the education level of the child's spouse are a stochastic function of the child's education. Thus, parental time input and characteristics affect labor market outcomes and marriage market outcomes indirectly.

The Barro-Becker framework provides a natural way to aggregate the value of the different aspects of the outcomes of the children by measuring the returns in terms of the discounted valuation function of the child. Time investment in children involves trading off leisure and hours worked in the labor market. Earnings are the marginal productivity of the individual and depend on the skill level, education, current level of labor supply and actual labor market experience. Thus, the opportunity costs of time includes current earnings as well as future loss of earnings resulting from accumulating less experience. This formulation allows us to capture the heterogeneity in the opportunity costs of time of parents by education, skill, race and gender groups. In Barro and Becker (1988), transfer to children do not depend on the parents wealth, because wealthy individuals have more children and the transfer per-child is fixed. However, in our model because both the returns in terms of children outcomes and the opportunity costs of time depend on the parents productive characteristics the model can potentially generate decline in fertility for high earning households (see Alvarez (1999) for analysis of the assumptions that generate this result in the Barro-Becker (1989) model and Jones, Schoonbroodt and Tertilt (2008) for discussions on fertility models). ${ }^{4}$ Modelling fertility decisions in a life-cycle model allows us to capture the quantity-quality parents in different types of households make when they decide how many children to have and on the spacing of children. We model spacing

[^2]of children because parental time and timing of income are important during early childhood (see Carneiro and Heckman (2003) for evidence that, controlling for permanent family income, timing of income is important during early childhood)

We use a partial solution, multi-stage estimation procedure developed to accommodate the nonstandard features of the model. It uses the assumption of stationarity across generations and the discreteness of the state space of the dynamic programming problem to obtain an analytic representation of the valuation function. This representation is a function of the conditional choice probabilities, the transition function of the state variable, and the structural parameters of the model. The conditional choice probabilities and the transition function are estimated in a first stage and used in the generation valuation representation to form the terminal value in the life-cycle problem. The life-cycle problem is then solved by backward induction to obtain the life-cycle valuation functions. Because the game between spouses is a complete information game, a sufficient condition for the existence of equilibrium in pure strategies is super modularity. Our game is super modular if there are strategic complementarities in time investment of parents or outcome of parental time investment is independent of the spouse's investment. An additional advantage of using a multiple step estimation approach is that it allows us to estimate the children's education production function parameters separately, using a Three Stage Least Square method, and verify that the conditions for existence of equilibrium are satisfied. We then form moment conditions from the best response functions and estimate it in a third step. Finally to reduce the computational burden of the backward induction in the life-cycle problem we use the forward simulation technique developed in Hotz, Miller, Sanders, and Smith (1994), and estimate the remaining structural parameters using Generalized Methods of Moment (GMM) estimator. To the best of our knowledge this is first paper to estimate a dynamic complete information game.

Our preliminary analysis shows that parental investment in children varies significantly across gender, race, education levels, and household composition. It also shows that after controlling for gender, education levels, and household composition, the differences across race are significantly reduced. This is consistent with the literature that estimates the black-white test scores gaps (see Carniero, Heckman and Masterov (2002) and Todd and Wolpin (2007) among others). We find that both maternal and paternal time investment increase the likelihood of higher educational outcome of their children. However, the impact is complementary; fathers' time investment increases the probability of graduating from high school and getting some college education while mothers' time increases the probability of achieving a college degree. The estimates of the education production-function show that girls have a higher likelihood than boys of achieving high levels of education (consistent with most findings in the literature), and that blacks have higher variance than whites in their educational outcomes, after controlling for parental inputs. Specifically, blacks have a higher probability of not completing high school than whites, however, they also have a higher probability of graduating from college than whites.

We then quantify the returns to parental time investment using the effect of an increase in time input on the change in the valuation function of the child. We find that the overall returns to fathers' time investment is only $60 \%$ that of mothers' time investment ${ }^{5}$. Although both parents input improve the educational attainment of children, maternal time investment increases the probability of a child graduating from college, and a college degree increases the returns in both the labor and the marriage markets. Similar to Rios-Rull and Sanchez-Marcos (2002), we find that both parents education levels, all else equal, increases the outcomes of the children but the effect of fathers' education is higher than

[^3]the effect of mothers' education.
Overall, we find that the valuation function of blacks is lower than that of whites with the same education levels. One reason is that we find that there is a black labor market "tax", and another reason is that the probability of being a single parent or having a less educated spouse, both associated with lower valuation function, is higher for blacks. Despite the fact that the valuation function is higher for whites, we find that on the margin, there are no significant race differences in the rate of returns to paternal time investment, and that blacks have a higher return to maternal time investment than whites. Hence, the main reason for the lower parental time investment by blacks seems to be the family structure. There is a significantly higher proportion of black single mothers than white single mothers and the costs of time for single mothers are higher than the opportunity costs of married mothers due to income sharing and transfers in married couples households. This finding supports the explanation of the black-white test and AFQT scores gaps suggested by Neal (1996). We do not take a stand on whether this tax reflects discrimination of other factors (see discussion in Neal and Johnson (1996) and Neal (2006)). However, we can quantify the impact of early investment and socioeconomic factors as well as family structure assuming the tax is due to discrimination and interpret the estimates as a lower bound on the importance of these per-market factors.

Finally the returns to maternal time investment are significantly higher for boys than for girls. This implies that mothers act in a compensatory manner, favoring low ability children in the family. Since girls already have a higher likelihood of achieving high education outcome than boys, mothers seems to investment more time in boys than in girls as the number of children increases. These findings are consistent with the findings in Hanuschek (1992).

Our findings suggest a significant quality-quantity tradeoff. This tradeoff is measured in terms of the rate of increase in utility of parents versus the rate of the decline in the average life time utility per child resulting from having an additional child. The level of investment per child is smaller the larger the number of children, thus, this decline in the per child investment is driven by the time constraint and the opportunity costs of time and not by the properties of the production function technology of children. The negative relationship between income (education) and fertility is therefore explained by the higher opportunity cost of time of educated parents in terms of forgone earnings. We, also find that quality-quantity tradeoff for blacks is significantly larger than that of whites. This is mainly due to the higher fertility of single black female and the resulting greater time constraint they face. This explanation is in line with Chiswick (1988) evidence for quantity-quality tradeoff; he concludes that family decisions and intergenerational transfers may play a big role in the observed race gap in achievements and earnings. Neal (2006) provides evidence for the importance of these factors in the observed Black-White skill gap and its trends. Our direct estimates support this hypothesis. Greenwood, Guner and Knowles (2003), develop and calibrate an overlapping generations model with endogenous human capital investment, labor supply, fertility, marriage and divorce, in which parents derive utility from number of children and their human capital. They show that accounting for assortative mating, and family size, and quantity-quality tradeoff of children, generate a more skewed income distribution. They make the point that once quantity-quality tradeoffs are modeled, single parent families are the poorest. This is consistent with our findings that one of the main reasons for differences between outcomes of black and white families is the higher proportion of single parent households.

Interestingly, we find that females have higher valuation functions (i.e. female child value is higher than that of a male child). Despite the fact that females earn less than men with the same productive characteristics, females are more likely to obtain higher levels of education than males, given equal amount of parental inputs and education is highly compensated in the labor market. However, even given the same level of education the valuation function of females are higher than males; this is because married females receive significant transfers from their husband's income. This findings can
be explained by the fact than females are endowed with the birth decisions and males value children, but cannot make decisions to have them. This explanation is consistent with Echevarria and Merlo (1999) which finds that transfers made within households increase the returns to parental investment in girls, and that the gender gap in education outcome of children is smaller when considering endogenous investment of parents in children.

Del Boca, Flinn and Wiswall (2010) estimate the effect of parental time on children's quality. They focus on the dynamic nature of investment in young children, measuring how investment is adjusted over the life-cycle in response to the child's observed outcomes measured by test scores. In contrast, we focus on long-term outcomes of children. In addition, they estimate the model using data on households with one child, and therefore, do not capture the quantity-quality trade-off between the number of children and their quality. Kang (2010) estimates a life-cycle model with endogenous parental transfers, fertility and labor supply. The paper uses labor market hours as a proxy for parental time investment. ${ }^{6}$ In her paper parents derive utility from the quality of children measured by their education and skill which proxy for children's labor market outcomes.

In contrast to the above papers and the large literature that estimates effect of parental investment and socioeconomic characteristics on children's short-and long-term outcomes (which we discuss below), we measure the returns in terms of children's life-time utility, incorporating lifetime earnings and marriage market outcomes. Using this measure instead of test scores or education is especially useful because our goal is to compare the different returns white and black parents face when they make investment decisions, as well as the different returns for boys and girls. Test scores, however, change over time, and the racial test score gaps tend to increase with age ${ }^{7}$. Moreover, education outcomes are correlated with earnings; however, labor market returns may be different for blacks, whites, males, and females conditional on the same education level. Further, education affects stochastically the education and earnings of the child's future spouse, and even conditional on education there are differences in the matching patterns of blacks and whites, and their likelihood of being a single parent. Since spouse's education and earning may have large effect on individuals welfare, using labor market earnings may not be sufficient to proxy for the returns. We contribute to this literature by accounting for the marital outcomes of children as part of the returns to parental time investment.

Our paper is also related to literature that estimates effect of parental investment and socioeconomic characteristics and the production function of children's short-and long-term outcomes. See for example Rosenweig and Wolpin (1994) and Berman, Foster, Rosenweig and Vashishtha (1999) which estimate the impact of maternal investment on children's schooling among others; several papers estimate the dynamic process of skill formation, Todd and Wolpin (2007) measure the effect of investment on child's quality using test scores measures of outcomes; Cunha and Heckman (2008) estimate the effect of investment on cognitive and non-cognitive skills using various childhood measures of outcomes; Cunha, Heckman and Schenach (2010) measure the effect of parental investment on development of cognitive and non-cognitive skills and the impact on education and crime rate; see also Black and Devereaux (2011) for a survey on the literature. The empirical literature which uses direct measures of parental time investment in children finds a positive relationship between time investment, controlling for various socioeconomic background characteristics such as parents economic circumstances, education, number of siblings, marital status etc., and children's outcomes (see Murnane, Maynard and Ohis (1981), Guryan, Hurst and Kearney (2008), Datcher-Loury (1982,1988), Houtenville and

[^4]Smith Conway (2008), Leibowitz (1974, 1977), Hill and Stafford 1980, Kooreman and Kapteyn 1987, Haveman, Wolfe and Spaulding (1991), and Juster and Stafford (1991) for a survey on empirical evidence of time allocation). In contrast to the above literature, this paper focuses on the returns to investment measured in terms of children's life-time utility, which depends on completed education, skill, labor market and marriage market outcomes. Whereas Todd and Wolpin (2007), Cunha and Heckman (2008) and Cunha, Heckman and Schennach (2010) and Del Boca et al. (2010) estimate the dynamic process of production, our production function only measures the total time investment in children of both spouses over the life-cycle assuming children outcomes and unobserved labor market skill are determined after investment is made ${ }^{8}$.

A small number of empirical paper quantify the returns to parental investment in children using Barro-Becker type dynastic models. Rios-Rull and Sanchez-Marcos (2002) studies the returns of parental investment in children's education, their earnings and marriage market, Doepke and Tertilt (2009) allows the returns on investment in children's human capital to depend on the parents' education and Echevarria and Merlo (1999) develop a dynastic model of household bargaining which gives rise to a gender gap in parental investment in education of the children. Our paper contributes to this literature by using data on time investment in children and by incorporating life-cycle into the BarroBecker framework, thus capturing the dynamic aspects of labor supply decisions, time investment in children and fertility.

The rest of the paper is organized as follows. Section 2 describes our data and variable construction. It also presents our preliminary analysis. Section 3 presents our theoretical model. Section 4 presents our estimation technique and empirical implementation. Section 5 presents the estimation results. Section 6 presents our measures of the quality-quantity trade-off and the return to parental time investment. Section 7 summaries our findings and concludes.

## 2 Data

We used data from the Family-Individual File of the Michigan Panel Study of Income Dynamics (PSID). We selected individuals from 1968 to 1996 by setting the individual level variables "Relationship to Head" to head or wife or son or daughter. We dropped all sons or daughters if they are younger than 17 years of age. This initial selection produces a sample of 12,051 and 17,744 males and females respectively; these individuals were observed for at least one year during our sample period. Our main sample contains 423,631 individual-year observations.

We only kept white and black individuals between the ages of 17 and 55 in our sample. The earnings equation requires the knowledge of past 4 participation decisions in the labor market. This immediately eliminates individuals with less than 5 years of sequential observations. This reduces the number of individual-year observations to 139,827 . In order to keep track of parental time investment throughout a child's early life we dropped parents we only observed after their children are older than 16 years of age. We also dropped parents with missing observations during the first 16 years of their children's life. Furthermore, if there are missing observations on the spouse of a mare individual then that individual is dropped from our sample.

The PSID measures annual hours of housework for each individual, however, it does not provide data on time parents spend on child care. This variable is estimated using a variation of the approach use in the previous literature. Example of papers using this approach can be found in Hill and Stafford (1974, 1980), Leibowitz (1974), and Datcher-Loury (1988). Hours with children are computed as the deviation of housework hours in a particular year from the average housework hours of individuals

[^5]with no child by gender and education, and year. Negative values are set to zero and child care hours are also set to zero for individuals with no children. In addition, in the estimation, and the analysis we do not use levels of hours measure, but use a discrete measure with three levels of time spend with children for men and women, which may reduces the problem.

Table 1 presents the summary statistics for our sample; Column (1) summarizes the overall sample, Column (2) focuses on the parents, and Column (3) summarizes the characteristics of the their children. It shows that the first generation is on average 7 years older than the second generation in our sample. As a consequence a higher proportion are married in the first generation relative to the second generation. The male-female ratio is similar across generations (about 55 percent female), however, our sample contains a higher proportion of blacks in the second generation that in the first generation (about 29 percent in the second and 20 percent in the first generation). This higher proportion of blacks in the second generation is due to the higher fertility rate among blacks in our sample. There are no significant differences across generations in the years of completed education. As would be expected, because on average the second generation in our sample is younger that the first generation in our sample, the first generation has higher number of children, annual labor income, labor market hours, housework hours, and time spent with children. Our second generation sample does span the same age range, 17 to 55 , as our first sample.

### 2.1 Preliminary Analysis

Many studies have analyzed various dimensions of the relationship between mothers' time with children and children's outcomes (see Hill and Stafford (1974, 1980), Leibowitz (1974), Datcher-Loury (1988), among others). Few studies, however, have analyzed the effect of fathers' time with children or household labor market decisions on their children's subsequent outcomes. In this section we document some of these empirical regularities as a way of motivating and clarifying our modelling choices.

### 2.1.1 The Relationship between Time Investment in Children and Household Composition.

Figure 1 presents the kernel estimates of the density of hours spent with children by marital status, gender, and race. It shows that females provide significantly more hours than males, confirming the well documented specialization by gender in home production. The upper left hand panel shows that over the nonzero range, the distribution of hours spent with children does not differ significantly by marital status, however, there is a higher incidence of zero hours spent with children for married parents than for single parents. A closer look at the middle and bottom left hand panels shows that this higher incidence of zero hours with children for married parents versus single parents is mostly is due to the significantly higher incidence of zero hours among married versus single male parents. The middle left hand panel shows that the distribution, for time investment in children greater than 160 hours per annum, is similar across marital status for male parents. Below 160 hours per annum, married male parents are less likely to provide time with children than single male parents. Married female parents are more likely to provide high hours and are less likely to provide low hours than single female parents.

The right hand panels of Figure 1 present the distributions of child care hours by race and gender; they show that there are little to no differences in the distribution of hours spent with children of black and white parents. If anything, blacks provide more hours than whites. The pattern for the overall distribution by race is repeated for males, however, white females provide more hours than their black counterparts. This could be due to the higher incidence single mothers among blacks than whites; this is demonstrated by the similarity between the whites versus blacks' distributions and married versus
single distributions for mothers.
Figure 2 presents the kernel estimates of the density of hours invested in children by own education, spouse education, number children, and gender. The top panels show that fathers hours are increasing with fathers' education, with college educated fathers having the highest likelihood of providing time with children. However, the distributions of hours of mothers are not monotone in mothers' education; a mother with less than a high school education is most likely to provide high hours while a mother with some college education is least likely to provide high hours. The patterns observed for own education are repeated for spouse education, with the differences that a mother whose spouse has a college education is the least likely to provide high hours. This highlights the assortative mating on education in the marriage market. The bottom panels of Figure 2 present the distributions by the number children and show that hours provided by both fathers and mothers are increasing in the number of children.

### 2.1.2 The Relationship between Time Investment in Children and Labor Market Time

Time not spent taking care of children can either be spent working in the labor market or on leisure; given a fixed hours endowment day, it suffices to analyze the relationship between time investment in children and labor market time. Figure 3 presents the kernel estimate of the densities of hours spent with children by labor supply, education, and gender. The top panels of Figure 3 shows a negative relationship between hours worked and hours parents spend with their children. This may indicate some degree of substitutability between time with children hours provided by parents and market purchased child care. The second panels from the top of Figure 3 show that among parents who are not currently employed, college graduates are more likely to spend more hours with children. Parents who did not complete high school and those that have some college education but not a college degree are the least likely to spent time with children when they are not working. Surprisingly, the behavior of parents with some college is similar to those with less than high school; this may reflect some selection on unobservable which are correlated with not completing a given level of education. We seek to capture these unobserved traits by using individual specific effects that are correlated with observed individual specific variable such as the level of completed education. The third panels from the top show that this pattern is repeated for parents that are currently working part-time. The bottom panels of Figure 3 show that these patterns are very different for parents that are working full-time in the labor market. For fathers that are working full-time in the labor market there are virtually no differences by education groups; however, mothers working full-time with less than high school education are more likely to spend a high number of hours with children. On the other hand, mothers that have at least a college degree are the least likely to spend a large amount of hours with children when they are working full-time. This may reflect differences in the type of full-time jobs performed by mother with at least a college education and mothers with less education, or differences in their households dynamics. Nevertheless, these empirical findings demonstrate the interplay between time investment in children, gender, education, household composition, and the labor market hours.

## 3 Theoretical Framework

The theoretical frameworks builds on two types of dynastic models. The first type (e.g. Loury (1981) and Becker and Tomes (1986)) analyzes transfers and intergenerational transmission of human capital with exogenous fertility and the second type analyzes dynastic models with endogenous fertility developed by Becker and Barro (1988) and Barro and Becker (1989).

We formulate a partial equilibrium discrete choice model and incorporate life-cycle if individuals in each generation into the framework. Adults in each generation derive utility from their own consump-
tion, leisure, and from their children's welfare, when children become adults. Each period, they choose sequentially, fertility, labor supply and time spend with children. We assume no Borrowing or savings. As in Loury (1981), the only intergenerational transfers are transfers of human capital. Therefore, we abstract from social investment, assets, and bequests, and focus on the tradeoffs of own consumption and leisure, and children welfare parents make. Fertility decisions capture quantity-quality tradeoffs made by different types of households. Incorporating life-cycle, allows us to capture optimal spacing of children which is central in the time allocation problem, since time input is especially important when children young.

In the model, parental time and monetary inputs when children are young, parents' characteristics (such as education), and luck determine the education outcome of children. The model incorporates marriage and assortative mating by allowing for the education outcome of the child to affect who they marry. Educational outcomes of children, as well as their marriage market outcomes are determined when children become adults, after all parental investments are made. Marriage and divorce are not modeled as choice variables, however, they depend stochastically on choices. Therefore, forward looking individuals take into account the effect of their decisions on marriage and divorce probabilities; thus, these variables are endogenous in a predetermined sense.

Household structure is an important determinant of parental transfers to children. However, most dynastic models are written as a single decisions maker problem ignoring marriages. In our model couples can share costs of raising children, and income can be transferred between spouses, while a single parent consumption depends on his own income only. This allows us to capture the different costs and tradeoffs single and married parents face. For example, for a married person, an increase time with kids and a decrease in labor supply may not reduce consumption, if the spouse makes transfers and increase their labor supply in response.

As demonstrated in Bernheim and Bagwell (1988) introducing marriage into a dynastic BarroBecker framework can imply a complex dynastic links. Our model, however, is anonymous in the sense that the valuation function of each child depends on the realized outcome at the end of the parent life-cycle: that is, education, and spouse's education outcomes of children, thus, it is the child's "type" and the "type" of their spouses that affects the parent's utility, as opposed to their individual identity or the identity of the dynasty their spouse belongs to. Therefore, in the model there are different types of representative dynasties (for each combination of the spouses' characteristics).

In contrast to Becker and Barro (1989) and Barro and Becker (1988) transfers depend on parents education. As shown in Alvarez (1999), relaxing the assumptions the marginal costs of raising a child is non-increasing and the assumption that the past generation investment does not affect the marginal costs of rasing children can lead to persistence in wealth and human capital. Since we allow for nonlinearity in the disutility from home and market activities and because increase in time invested in children may reduce current and future earnings (due to decrease in labor market experience), the marginal cost of raising children may be increasing. In addition, previous generations investment in children affect their educational outcomes and therefore the opportunity costs of time. The new element is that parental time investment also affect the costs of raising children by affecting the marriage market outcomes.

To simplify the formulation of the problem of married households who have children from previous marriages, we model households decisions as a noncooperative game, in which spouses choose actions simultaneously each period. Therefore, conditional on the spouse strategies, the optimization problem is similar to that of a single agent dynamic problem. We solve the model for a stationary Markov Perfect Equilibrium (MPE) in pure strategies. We show that an equilibrium exists for some parameters of the model. ${ }^{9}$ In addition, there is a possibility of multiple equilibria, but we show that they can be

[^6]Pareto ranked, and assume the highest equilibrium is being played.
We begin by describing the choice set, preferences, the technology of children's outcomes and labor market, for the general model with single and a married households. We then characterizing the equilibrium conditions for a model with one person in the households and then we extend it to households with two agents making decisions. In section 4.3 we fully characterize the functional forms and show existence.

Choices: There are two types of individuals, female and male denoted by $\sigma=f, m$, respectively. Adults live for $T$ periods in which they make decisions, $t \in\{0,1, . ., T\}$. An adult from generation $g \in\{0, \ldots \infty\}$ makes choices of consumption $c_{\sigma t}$, and discrete labor supply decision $h_{\sigma t} \in \pi_{h}$ ( no work, part time, full time), time spent with children $d_{\sigma t} \in \pi_{h}$ ( no time, low, high) and a birth decision $b_{t} \in\{0,1\}$. We assume that only females make the birth decision, thus we omit the gender subscript. Denote the vector of choices an adult makes in period $t$ by $k_{\sigma t}$.

The gender dummy of a child born in period $t$ is denoted by $I_{\sigma t}^{\sigma^{\prime}}$, it takes the value 1 if the child of spouse $\sigma$ is of gender $\sigma^{\prime}$ and 0 otherwise. We denote the vector of past labor supply choice in period $t$ by $H_{\sigma t}=\left\{h_{\sigma 0, . .}, h_{\sigma t-1}\right\}$, to capture the labor market experience of the individual at the beginning of the period. We denote by $N_{\sigma t}$ the total number of children at the beginning of period $t$. A child's outcomes depends on inputs of both parents thus, we denote by $D_{\sigma t}=\left\{d_{\sigma 0, . .}, d_{\sigma t-1}\right\}$ the vector of time invested in each of spouse own children (including the child other biological parent) up to period $t$. An individual time invariant characteristics are denoted by $x_{\sigma}$; it includes variables such as education, race and a skill. We denote the spouse of an individual by $-\sigma$, thus $x_{-\sigma}$ is the spouse's characteristics. The vector $x_{\sigma t}$ denotes the persistent state variables at the beginning of period $t$; it includes $x_{\sigma}, N_{\sigma t}, H_{\sigma t}, D_{\sigma t}$ as well as the gender dummies of each child ( $\left.I_{\sigma 0}^{\sigma^{\prime}} . . I_{\sigma t}^{\sigma^{\prime}}\right)$ and the total time invested in each child by the other parent.

Preferences: We begin by extending the standard dynastic formulation to include life-cycle component. An adult per-period utility depends on the individual's consumption, whether there is a birth in that period, the number of children, the person characteristics (specifically we allow the utility to vary by gender). Assume that each period there are preference shocks to the utility associated with each choice (of work, time with children and birth), denoted by $\varepsilon_{\sigma t}=\left[\varepsilon_{\sigma 1 t}, . ., \varepsilon_{\sigma t K_{\sigma}}\right]$; the shocks $\varepsilon_{\sigma k t}$ are drawn independently across choices, periods, individuals and generations from a distribution function $F_{\varepsilon}$. The shocks are also conditionally independent (of all state variables). Denote by $U_{\sigma g T}$ the life-cycle component of the individual utility. It is the discounted expected lifetime utility of an individual in generation $g$ at period 0 , excluding the dynastic component:

$$
\begin{equation*}
U_{\sigma g T}=\sum_{t=0}^{T} \beta^{t}\left[u\left(c_{\sigma t}, x_{\sigma}, b_{t}, h_{\sigma t}, d_{\sigma t,} N_{\sigma t}\right)+\varepsilon_{\sigma k t}\right] \tag{1}
\end{equation*}
$$

The first element on the right hand side is the per period utility of an adult in generation $g$ of gender $\sigma$. Each individual utility depends on their consumption, and on the number of children. The utility depends on the number of children capturing the utility/disutility associated with raising a child. We further discuss the functional form assumptions below. The per-period utility also depends on whether there is a birth in the household. Because the labor supply and time spent with children choices are discrete, the current level of leisure is fully captured by the disutility associated with any combination of labor market activities and time spent with children $h_{\sigma t}, d_{\sigma t}$.

Denote by $U_{\sigma g}$ the discounted expected lifetime utility of an individual in generation $g$ at period 0 . It has two components, the first is the life-time discounted utility described above, and the second is the discounted utility of the individual's children when they become adults. The discount factor of
the valuation of the children's utility is given by $\lambda N_{\sigma}^{1-\nu}$.

$$
\begin{equation*}
U_{\sigma g}=E_{0}\left\{U_{\sigma g T}+\beta^{T} \lambda N_{\sigma}^{1-\nu} \frac{\bar{U}_{g+1}}{N_{\sigma}}\right\} \tag{2}
\end{equation*}
$$

Where $\bar{U}_{g+1}$ is the sum of the utilities of all children : $\bar{U}_{g+1} \equiv \sum_{t=0}^{T} b_{t}\left(\sum_{\sigma^{\prime}} \sigma_{\sigma t}^{\sigma^{\prime}} U_{\sigma^{\prime} g+1}\right)$. For example, if a person has one child the intergenerational discount factor is $\lambda$.Our formulation captures the ex-ante differences between men and women, therefore, the expected utility of a child depends on the child's gender. The utility is assumed to be concave in the number of children, thus $0<\nu<1$.

Children's Outcomes and Labor Market Earnings
The time invariant state variables of a child of spouse $\sigma$ is denoted by $x_{\sigma}^{\prime}$. The child's characteristics are a stochastic function which depends on both parents' total input of time up to age $5, D_{s}$, where $s$ indexes the child's year of birth, the total time invested by both parents in the child's first 5 years. Denote the stochastic outcome function of a child born in period $s$ by $m\left(x_{\sigma}^{\prime} \mid x_{f}, x_{m}, D_{s}\right)$.

The stochastic time invariant state variables of the child, such as education and the child's labor market skill $\eta_{\sigma}$ also depend on the parent's time-invariant traits such as education and their market skill level. In addition, marriage outcomes depend stochastically on the individual characteristics; thus the child's spouse characteristics depend stochastically on the child's characteristics: $G\left(x_{-\sigma}^{\prime} \mid x_{\sigma}^{\prime}\right)$. Define the intergenerational transition function of the persistent state variables of a child born in period $s$ in the parent's life cycle by

$$
M\left(x_{0}^{\prime} \mid x_{f}, x_{m}, D_{s}\right) \equiv m\left(x_{\sigma}^{\prime} \mid x_{f}, x_{m}, D_{s}\right) G\left(x_{-\sigma}^{\prime} \mid x_{\sigma}^{\prime}\right)
$$

We assume that the earnings of individuals depend on their time invariant characteristic, such as education and a given skill endowment, the human capital accumulated with experience of working full time and part time in the past, and current level of labor supply. The earnings function in periods $t$ is given by $w_{\sigma t}\left(x_{\sigma}, H_{\sigma t-1}, h_{\sigma t}\right)$. Earnings of individuals with the same productive characteristics depend on their other time invariant characteristics such as gender and race capturing possible discrimination in the labor market. The functional forms for the earnings equations and the transition function of the children outcomes are described in Section 4.3 on empirical implementation.

### 3.1 Single Decision Maker Problem

We first describe the dynastic problem assuming that each household comprised of one person, and that a child has one parent, ignoring gender and marriage issues, thus the gender subscripts are omitted. The per-period, utility of an adult is composed of the utility from current income and number of children and the utility from leisure. We assume the following functional form:

$$
\begin{equation*}
u_{1 t}=\alpha_{I} w_{t}+\alpha_{N}\left(N_{t}+b_{t}\right) \tag{3}
\end{equation*}
$$

We assume no borrowing and saving, one consumption good with price normalized to 1 , and risk neutrality. The first term represents the utility from own consumption. The second term, however, represents the net utility/cost from having young children in the household. In general, given our assumptions, we can use a budget constraint to derive the coefficients on income and number of children, and a separate non-pecuniary utility from children and monetary costs. However, since we do not have data on consumption or expenditures on children, the coefficients on the number of
children also capture non-pecuniary utility from children and cannot be identified separately from the monetary costs of raising children. ${ }^{10}$

We assume that the preferences are additive and separable in consumption and leisure. We define the per period utility/disutility from working and spending time with children as

$$
\begin{equation*}
u_{2 t}=\theta_{k_{t}} \tag{4}
\end{equation*}
$$

Where $\theta_{k_{t}}$ are the coefficients associated with each combination of time allocation choice, thus capturing the differences in the value of non-pecuniary benefits/costs associated with the different activities. The vector of decisions includes birth, thus we allow the utility associated with different time allocations to be different if there is birth.

Under the assumption of stationarity, we omit the generation index $g$. We first define the ex-ante value function $V$ as the discounted sum of expected future utilities; it is the discounted sum of future utilities before the individual-specific preference shocks are observed and actions are taken. Define by $p\left(k_{t} \mid x_{t}\right)$ the conditional ex-ante probability that a person choice profile $k_{t}$ will be chosen conditional on the state $x_{t}$. For $t<T$ the ex-ante value function can therefore be written as

$$
\begin{equation*}
V\left(x_{t}\right)=\sum_{s \in k_{t}} p\left(k_{t}=s \mid x_{t}\right)\left[u\left(k_{t}, x_{t}\right)+\beta \sum_{x_{t+1}} V\left(x_{t+1}\right) F\left(x_{t+1} \mid x_{t}, k_{t}\right)\right]+\sum_{s \in k_{t}} E_{\varepsilon}\left[\varepsilon_{t} \mid k_{t}=s\right] p\left(k_{t}=s \mid x_{t}\right) \tag{5}
\end{equation*}
$$

where $E_{\varepsilon}$ denotes the expectation operator with respect to the individual-specific preference shocks, and $F\left(x_{t+1} \mid x_{t}, k_{t}\right)$ is the stochastic transition function of state variables which depends on the current period state and actions. Note that in the case of one decision maker, the transition to $x_{t+1}$ conditional on $x_{t}, k_{t}$ is deterministic, but we keep the notation, since in the case of marriage, this function is no longer deterministic since a single person can become married in the next period.

Let $v\left(k_{t} ; x_{t}\right)$ denote individual continuation value net of the preference shocks, conditional on choosing $k_{t}$. This can be written as:

$$
\begin{equation*}
v\left(k_{t} ; x_{t}\right)=u\left(k_{t}, x_{t}\right)+\beta \sum_{x_{t+1}} V\left(x_{t+1}\right) F\left(x_{t+1} \mid x_{t}, k_{t}\right) . \tag{6}
\end{equation*}
$$

Thus, a vector of choice $k_{t}$ is optimal if $v\left(k_{t} ; x_{t}\right)+\varepsilon_{j t} \geq v\left(k_{t}^{\prime} ; x_{t}\right)+\varepsilon_{k^{\prime} t}^{\prime}$ for all $k_{t} \neq k_{t}^{\prime}$. Thus, we can characterize the probability distribution over $k_{t}$ for and write the conditional ex ante choice probabilities of the choice profile:

$$
\begin{equation*}
p\left(k_{t} \mid x_{t}\right)=\int\left[\prod_{k_{t} \neq k_{t}^{\prime}} 1\left\{v\left(k_{t} ; x_{t}\right)-v\left(k_{t}^{\prime} ; x_{t}\right) \geq \varepsilon_{k t}-\varepsilon_{k^{\prime} t}\right\}\right] d F_{\varepsilon} \tag{7}
\end{equation*}
$$

where $v\left(k_{t} ; x_{t}\right)-v\left(k_{t}^{\prime} ; x_{t}\right)$ is the differences in the ex-ante conditional valuation when individual chooses $k_{t}$ and the valuations when $k_{t}^{\prime}$ is chosen.

Time allocation decisions involve the standard tradeoffs of the non-pecuniary costs associated with the combinations of activities (representing different levels of leisure), and current consumption. Notice that reducing labor supply has dynamic effects since it reduces labor market experience. Since there is no savings in the model, the only way parents can increase consumption in the future is by accumulating

[^7]labor market experience. In addition, both income when children are young, and parental time affect the outcomes of children. These dynamic effects of time allocation on the outcomes of children makes the solution to the labor supply decisions non-trivial, despite the linearity of the per-period utility function.

Next, consider birth decisions. To focus on the quantity-quality tradeoffs consider first the birth decision at the last period of the life-cycle in which parents can make birth decisions: $T$. The conditional continuation function net of the iid taste shocks captures the stochastic outcomes of the child in terms of the child time invariant characteristics and the child's spouse characteristics, given the parent's time invariant characteristics and time investment in the child. In the final period of the life cycle $T$ it is given by

$$
\begin{equation*}
v\left(k_{T} ; x_{T}\right)=u\left(k_{T}, x_{T}\right)+\beta \lambda \frac{\left(N_{T}+b_{T}\right)^{1-v}}{\left(N_{T}+b_{T}\right)} \bar{V}_{N}\left(k_{T} ; x_{T}\right) \tag{8}
\end{equation*}
$$

Where $\bar{V}_{N}\left(x_{T}\right)$ is sum of the expected valuation over all children born up to period $T$ plus the valuation of a child born in period $T$ if there is birth (note that $k_{T}$ includes the birth decision )

$$
\begin{equation*}
\bar{V}_{N}\left(k_{j T} ; x_{T}\right) \equiv \sum_{s=0}^{T}\left[b_{s} V_{s}\left(x_{0}^{\prime}\right) M\left(x_{0}^{\prime} \mid x_{0}, D_{s}\right)\right] \tag{9}
\end{equation*}
$$

In the final period of the life cycle, the valuation function (Equation ??) depends on current utility, and the discounted expected value of the children's valuation functions. The above equation is the expected valuation of the existing children at the beginning of period $T$ (born from periods 0 to $T-1$ ), and the additional child if $b_{T}=1$. We assume that all children become adults after period $T$ and their state variables are unknown until then regardless of the time of birth. $D_{s}$ includes parental inputs of both parents for a child born in period $s$. Note that for children born after $T-5$, it includes time and income inputs in period $T$. This illustrates the quantity-quality tradeoffs and how it ties to the spacing of children decisions.

Note that increase of time with children affects $M\left(x_{0}^{\prime} \mid x_{0}, D_{s}\right)$. The utility from an additional child, holding the average valuation function per child, $\frac{\bar{V}_{N}\left(k_{T} ; x_{T}\right)}{N_{T}}$ constant, is positive and decreasing with the number of children $\beta \lambda(1-v) N_{T}^{-v}$. Quantity-Quality tradeoffs occur in our model endogenously if the average valuation function per child is lower as a results of reduced inputs (time and income).

As clear from equations 9 and 6 , the timing of birth and spacing of children involve several tradeoffs. As in standard models of life-cycle labor supply, the opportunity cost of time rises with age (if wages increase), but there are additional factors in our model. Large age differences of children implies that the limited time resources of parents when children are young are divided between less children. Having children early in the life-cycle has the advantage of allowing for large age differences of children. At the same time if wages increase with age, for a given level of labor supply, income is higher.

### 3.2 Married Couples Households

We now extend the framework to account for decisions of decisions married couples. The per-period utility of a single person is the same as described in Equations 3 and 4 , except that all the coefficients on consumption and the time allocated to home and market activities are gender-specific. The difference between the utility of a single and married person is that a married person's utility depends on the spouse's income. We assume the following functional forms for the utility from income for a married (or for cohabitation) individual in period $t$

$$
\begin{equation*}
u_{1 \sigma t}=\alpha_{I \sigma} w_{\sigma t}+\alpha_{I \sigma}^{\prime} w_{-\sigma t}+\alpha_{\sigma N}\left(N_{\sigma t}+b_{t}\right) \tag{10}
\end{equation*}
$$

This formulation is consistent with each spouse consuming a share of their income net of their share of costs of children and a transfer from the spouse. As before, assuming no borrowing and saving, one can restrict the coefficients on the income, spouse's income and number of children so that the total value of consumption equals the total household income net of costs of children and the per-period budget constraint is satisfied. However, since we do not have data on consumption or costs of children, the coefficients on the number of children also captures non-pecuniary utility from children and cannot be identified separately from the monetary costs of raising children. We allow for the coefficient on the number of children and own incomes to differ across gender and household structure. ${ }^{11}$

### 3.2.1 Timing and Information

Let $x_{t}=\left(x_{f t}, x_{m t}\right)$ denote the persistent state variables of the spouses in the household and $\varepsilon_{t}=$ $\left(\varepsilon_{f t}, \varepsilon_{m t}\right)$ the vectors of preference shocks of both spouses. Denote the specific choice, $j$, made in each period by person $\sigma$ by $k_{\sigma j t}$; the spouse's choice, $i$, is denoted by $k_{-\sigma i t}$. The vector of choices made by both spouses in the household in period $t$ is denoted by $k_{j i t}=\left(k_{\sigma j t}, k_{-\sigma i t}\right)$ with $j$ denoting the choices of individual $\sigma$ and $i$ denoting the choices of their spouse $(-\sigma)$. As before denote by $F\left(x_{t+1} \mid x_{t}, k_{j i t}\right)$ the stochastic transition function of the state variables, conditional on last period household state variables and choices. We assume that all the transition functions are known to all individuals in all periods and generations. At the beginning of the period, all the household state variables are common knowledge, including the individual taste shocks. The state variables include divorce and marriage.

At the beginning of each period, each spouse makes labor supply and time investment in children decisions. Females also make birth decisions. All the decisions are made simultaneously. After the decisions are made, consumption is allocated according to the known sharing rule, at the end of the period.

### 3.2.2 Strategies

A Markov strategy profile for spouse $\sigma$ in the game is a vector $k_{\sigma}=\left[k_{\sigma 0}\left(x_{t}, \varepsilon_{t}\right), ., k_{\sigma T}\left(x_{T}, \varepsilon_{T}\right)\right]$, which describes the action for all possible household states variables $x_{t}, \varepsilon_{t}$ in every period, where $k_{f t}\left(x_{t}, \varepsilon_{t}\right)=\left(d_{f t}\left(x_{t}, \varepsilon_{t}\right), h_{f t}\left(x_{t}, \varepsilon_{t}\right), b_{t}\left(x_{t}, \varepsilon_{t}\right)\right)$ and $k_{m t}\left(x_{t}, \varepsilon_{t}\right)=\left(d_{m t}\left(x_{t}, \varepsilon_{t}\right), h_{m t}\left(x_{t}, \varepsilon_{t}\right)\right)$ are the period $t$ decisions in every state. Note that $k_{\sigma t}\left(x_{t}, \varepsilon_{t}\right)$ is a mapping from all possible states to $K_{\sigma}$ possible combination of choices every period: $k_{0}, . ., k_{K_{\sigma}}$. Let $k_{t}=\left(k_{\sigma t}\left(x_{t}, \varepsilon_{t}\right), k_{-\sigma t}\left(x_{t}, \varepsilon_{t}\right)\right)$ denote an element $t$ in a specific strategy profile of both spouses. Note that the strategy profile maps the state variables into choices of both spouses, whereas $k_{j i t}$ is a specific set of choices.

### 3.2.3 Valuation and Best Response Functions

As before, we first define the ex-ante value function $V_{\sigma}$ as the discounted sum of future utilities; it is the discounted sum of future utilities for household member $\sigma$ before the individual-specific preference shocks are observed and actions are taken. Define by $p\left(k_{t} \mid x_{t}\right)$ the conditional ex-ante (again before $\varepsilon_{t}$ is observed) probability that from a strategy profile for each spouse, the action profile $k_{t}$ a will be

[^8]chosen conditional on the state $x_{t}$. For $t<T$ the ex-ante value function can be written as
\[

$$
\begin{align*}
V_{\sigma}\left(x_{t}\right)=\sum_{s \in k_{t}} p\left(k_{t}=s \mid x_{t}\right)\left[u\left(k_{t}, x_{\sigma t}\right)\right. & \left.+\beta \sum_{x_{t+1}} V_{\sigma}\left(x_{t+1}\right) F\left(x_{t+1} \mid x_{t}, k_{j i t}\right)\right]  \tag{11}\\
& +\sum_{s \in k_{t}} E_{\varepsilon}\left[\varepsilon_{\sigma t} \mid k_{t}=s\right] p\left(k_{t}=s \mid x_{t}\right)
\end{align*}
$$
\]

where $E_{\varepsilon}$ denotes the expectation operator with respect to the individual-specific preference shocks, conditional on action choice profile $s$.

Note that the difference between the single and married individual problem is that the ex-ante valuation function depends on both spouses state variables, and that the states include both spouses state variables, the transitions functions now depend on both spouses actions, and state variables. In addition, the per-period utility depends on the spouses' actions as well. Let $v_{\sigma}\left(k_{j i t} ; x_{t}\right)$ denote individual $\sigma$ 's best response continuation value net of the preference shocks playing strategy $k_{\sigma j t}$ conditional on the spouse playing strategy $k_{-\sigma i t}$. This can be written as:

$$
\begin{equation*}
v_{\sigma}\left(k_{j i t} ; x_{t}\right)=u\left(k_{j i t}, x_{\sigma t}\right)+\beta \sum_{x_{t+1}} V_{\sigma}\left(x_{t+1}\right) F\left(x_{t+1} \mid x_{t}, k_{j i t}\right) . \tag{12}
\end{equation*}
$$

Recall that a vector of choices for a household is given by $k_{j i t}=\left(k_{\sigma j t}, k_{-\sigma i t}\right)$. Again the difference between the best response continuation function and the individual one is that it is conditional on the spouse's choice and state variables. In addition, notice that spouse choices affect the next period state variables, which include divorce, so when making choices, spouses take into account the effect these have on the probability of divorce. Thus, given a spouse strategy $k_{-\sigma i t}$ a vector of choice $k_{\sigma j t}$ is optimal if $v_{\sigma}\left(k_{\sigma j t}, k_{-\sigma i t} ; x_{t}\right)+\varepsilon_{\sigma j t} \geq v\left(k_{\sigma j^{\prime} t}, k_{-\sigma i t} ; x_{t}\right)+\varepsilon_{\sigma j^{\prime} t}$ for $k_{\sigma j^{\prime} t}$. Thus, we can characterize the probability distribution over $k_{\sigma j t}$ for all $j$ and write the conditional ex ante choice probabilities of the choice profile given a spouse's strategy profile:

$$
\begin{equation*}
p_{\sigma j t}\left(k_{\sigma j t} \mid k_{-\sigma i t}, x_{t}\right)=\int\left[\prod_{k_{\sigma j t} \neq k_{j^{\prime} i t}} 1\left\{v_{\sigma}\left(k_{j i t} ; x_{t}\right)-v_{\sigma}\left(k_{j^{\prime} i t} ; x_{t}\right) \geq \varepsilon_{\sigma j t}-\varepsilon_{\sigma j^{\prime} t}\right\}\right] d F_{\varepsilon} \tag{13}
\end{equation*}
$$

where $v_{\sigma}\left(k_{j i t} ; x_{t}\right)-v_{\sigma}\left(k_{0 i t} ; x_{t}\right)$ is the differences in the ex-ante conditional valuation when individual $\sigma$ chooses $k_{\sigma j t}$ and the valuations when $k_{\sigma j^{\prime} t}$ is chosen given that the spouse chooses $k_{-\sigma i t}$. Notice that the choices $k_{\sigma j t}$ and $k_{\sigma j^{\prime} t}$ are chosen according to the strategy $k_{\sigma}$ which maps for every period state variables $\left(x_{t}, \varepsilon_{t}\right)$ into choices, and given a spouse choices, we describe the probability distribution over the choices of an individual when the strategy is optimal. Because the conditional independence of the shocks, the household strategies probabilities are given by

$$
\begin{equation*}
p\left(k_{t} \mid x_{t+1}\right)=p_{\sigma j t}\left(k_{\sigma j t} \mid k_{-\sigma i t}, x_{t}\right) \times p_{-\sigma i t}\left(k_{-\sigma i t} \mid x_{t}\right) . \tag{14}
\end{equation*}
$$

The ex-ante conditional best response function net of the preference shock in the final period of the life cycle $T$ is given by

$$
\begin{equation*}
v_{\sigma}\left(k_{j i T} ; x_{T}\right)=u\left(k_{j i T}, x_{\sigma T}\right)+\beta \lambda \frac{\left(N_{\sigma T}+b_{T}\right)^{1-v}}{\left.N_{\sigma T}+b_{T}\right)} \bar{V}_{N \sigma}\left(k_{j i T} ; x_{T}\right) \tag{15}
\end{equation*}
$$

Where $\bar{V}_{N}\left(x_{T}\right)$ is sum of the expected valuation over all children born up to period $T$ plus the valuation of a child born in period $T$ if there is birth

$$
\begin{equation*}
\bar{V}_{N}\left(k_{j i T} ; x_{T}\right) \equiv \sum_{s=0}^{T-1}\left[b_{s} \sum_{\sigma^{\prime}} I_{\sigma s}^{\sigma^{\prime}} \sum_{x_{0}^{\prime}} V_{\sigma s}\left(x_{0}^{\prime}\right) M\left(x_{0}^{\prime} \mid x_{f}, x_{m}, D_{s}\right)\right]+b_{T} \sum_{\sigma} p_{\sigma} \sum_{x_{0}^{\prime}} V_{\sigma T}\left(x_{0}^{\prime}\right) M\left(x_{0}^{\prime} \mid x_{f}, x_{m}, D_{T}\right) \tag{16}
\end{equation*}
$$

Note that $D_{T}$ and $D_{s}$ for $s<T$ are both functions of $k_{j i T}$. In the final period of the life cycle, the valuation function (Equation 15) depends on current utility, and the discounted expected value of the children's valuation functions. The first element of Equation 16 is the expected valuation of the existing children at the beginning of period $T$, which state variables depend on past parental time input and the current period inputs. The second element is the expected value of a child born in period $T$ for which the gender is unknown at the beginning of the period. Thus, this element depends on the birth decision and parental time input. We assume that all children become adults after period $T$ and their state variables are unknown until then regardless of the time of birth.

As clear from equation 10, married individuals are affected by the action of a spouse from a different dynasty. The income externalities with a household imply that the utility of individual in generation $g$ will depend on future spouses of own children and their children's spouses from different dynasties. As showed by Bernheim and Bagwell (1988), it is possible that in a few generations there will be links between most or all dynasties, in which case, the representation of the problem may be complicated .Notice that we circumvent this problem because our formulation of dynasties is anonymous in the sense that it is only the state variables of future generations that affect individual utilities and not their identity. Similarly, future offsprings' spouses affect individual's utility through their state variables, and not the identity of the dynasty they come from. By stationarity, the valuation function of a person with state variable $x_{0}$ (which includes spouse's characteristics) is the same across generation. Ex-ante, individuals with different characteristics, have different probability distribution over different "types" of offsprings $\left(x_{0}^{\prime}\right)$. This create different "types" of dynasties, each has different life-time expected utility, different expected number of offspring, and a different distribution probabilities over their children's "types."

### 3.2.4 Equilibrium

We solve for a Markov Perfect Equilibrium of the game; restricting attention to pure strategies equilibria.

Definition 1 (Markov perfect equilbrium) A strategy profile $k^{\circ}$ is said to be a Markov perfect equilibrium if for any $t \leq T, \sigma \in\{m, f\}$, and $\left(x_{t}, \varepsilon_{t}\right) \in\left(X, R^{K_{f}+K_{m}}\right)$ : (1) $v_{\sigma}\left(k_{j i t}^{\circ} ; x_{t}\right)+\varepsilon_{\sigma j t} \geq$ $v_{\sigma}\left(k_{j^{\prime} i T}^{0} ; x_{t}\right)+\varepsilon_{\sigma j^{\prime} t} ;$ (2) all players play Markovian Strategies.

The tradeoffs individual make when they are married and single are different. First, marriage allows for some degree of specialization (not necessarily full) within the household. For example, it is possible that in equilibrium one spouse increases the time spent with children and decrease labor supply, but own consumption may not decline if the partner increases labor supply, since transfers are proportional to the income. In the single agent problem, decreasing labor supply implies lower consumption. A second point is that we assume that women make fertility decisions; in the household framework, this does not mean that men cannot affect fertility decisions. It is possible that females best response to males working longer hours when there are kids in the home is to increase fertility.

In our model, the tradeoffs that time investment in children and fertility decisions are different for individuals with different education. Thus, in equilibrium parents with higher education levels may have less children and will invest more time in each child than lower education parents, generating intergenerational persistence in earnings. A similar result is in Loury (1981) model of intergenerational investment in human capital with exogenous fertility. However, Barro and Becker (1999) show that once fertility is endogenous, transfers to children may not depend on parent's wealth, because wealthy parents have more children an may not make higher transfer per child then less wealthy parent. The Barro-Becker result, however, depends on assumptions which are not satisfied in our framework. As
demonstrated in Alvarez (1999) a model of intergenerational human capital transmission (as in Loury 1981) and endogenous fertility Barro and Becker (1989) if the assumption that the cost of rasing a child is not constant per child or there are nonlinearity in the budget sets, the independence of transfers and parental wealth breaks down. In our model, the opportunity cost of raising a child is higher for more educated parents, and since we have non-pecuniary values of each time allocation choice (in the case where there is birth and when there is no birth separately), the cost in general is not constant (depending on the parameters of the utility function). In addition, increasing time spent with children, may reduce current and future returns to earning due to reduced stock of market experience. Second, costs of raising children for the future generation, depends on parental current transfers, because high time investment increases educational attainment of children, thus affecting their opportunity costs of time.

These tradeoffs are more involved in our model, because in contrast to all the above mentioned models, parents choose how to space children. However, since the time available to have children is limited and the opportunity costs of time varies over the life-cycle, our model does not in general, predicts that time with children is independent of the parent's education. The models discussed above are models of a single decision maker, thus, there are additional elements in our model, related to the marriage market and the interactions between spouses within households. The cost of raising a child in our model depends on the equilibrium outcome, thus investment of time by each parent depends on the education of both spouses and the resulting allocation or resources, and degree of specialization in time with children and labor market activities and how they vary by education level of spouses.

Interestingly, the investment of parents affect the costs of raising children and the feasible set of the children through the effect on the marriage market. The educational outcomes of a child may change the probability of the child being a single parent, changing the costs of investment directly (recall that the coefficients in the utility function on children depends on marital status). It also affects the education of the spouse of the child, taking into account assortative mating.

In general a pure strategy Markovian perfect equilibrium for complete information stochastic games may not exist, however, we imposed sufficient conditions on the primitives of our game and show that there exist at least one pure strategies Markov perfect equilibrium. To show this results, we use some of the properties and definitions of super modular games on lattice theory (see Milgrom and Roberts(1990), Milgrom and Shannon (1994), and Tokis(1998) for examples these properties). A binary relation $\geq$ on a non-empty set is a partial order if it is reflexive, transitive, and anti-symmetric. A partially ordered set is said to be a lattice if for any two elements the supremum and infimum are elements of the set. A 2 person game is said to be super modular if the set of actions for each player $\sigma$ is a compact lattice and the payoff function is super modular in $k_{\sigma}$ for fixed $k_{-\sigma}$ and satisfies increasing differences in $\left(k_{\sigma}, k_{-\sigma}\right)$. Following Watanabe and Yamashita (2010), if the continuation values in every period and state satisfy the conditions below, the game is super modular and there exists a pure strategies Markov perfect equilibrium. Following the convention, we use $\vee$ to denote the supremum of two elements and $\wedge$ to denote the infimum of two elements.

Condition 1 (S) $v_{\sigma}\left(k_{\sigma t}, k_{-\sigma t}, x_{t}\right)$ is super modular in $k_{\sigma t}$ for any $x_{\sigma t}$ and $k_{-\sigma t}$ if

$$
\begin{equation*}
v_{\sigma}\left(k_{\sigma t}^{\prime} \vee k_{\sigma t}, k_{-\sigma t}, x_{\sigma t}\right)+v_{\sigma}\left(k_{\sigma t}^{\prime} \wedge k_{\sigma t}, k_{-\sigma t}, x_{\sigma t}\right) \geq v_{\sigma}\left(k_{\sigma t}^{\prime}, k_{-\sigma t}, x_{\sigma t}\right)+v_{\sigma}\left(k_{\sigma t}, k_{-\sigma t}, x_{\sigma t}\right) \tag{17}
\end{equation*}
$$

for all $\left(k_{\sigma t}^{\prime}, k_{\sigma t}\right)$.
Condition 2 (ID) $v_{\sigma}\left(k_{\sigma t}, k_{-\sigma t}, x_{\sigma t}\right)$ has increasing differences in $\left(k_{\sigma}, k_{-\sigma}\right)$ for any $x_{\sigma t}$ if

$$
\begin{equation*}
v_{\sigma}\left(k_{\sigma t}^{\prime}, k_{-\sigma t}^{\prime}, x_{\sigma t}\right)-v_{\sigma}\left(k_{\sigma t}, k_{-\sigma t}^{\prime}, x_{\sigma t}\right) \geq v_{\sigma}\left(k_{\sigma t}^{\prime}, k_{-\sigma t}, x_{\sigma t}\right)-v_{\sigma}\left(k_{\sigma t}, k_{-\sigma t}, x_{\sigma t}\right) \tag{18}
\end{equation*}
$$

for all $k_{\sigma t}^{\prime} \geq k_{\sigma t}$ and $k_{-\sigma t}^{\prime} \geq k_{-\sigma t}$.

Watanabe and Yamashita (2010) provide sufficient conditions on the stochastic transitions functions and the per period utility for existence of a pure strategy Markov perfect equilibrium. These conditions impose restrictions on the functional forms of the per period utility sharing rules, wage functions, value of kids, and the return investment in children in our model. In the implementation section we discuss these restrictions further once the functional of these primitives are specified and provide a proof.

## 4 Functional Forms and Empirical Implementation

We describe the choice set specifications, functional forms of model which we estimate. Since existence of equilibrium depends on the functional forms, we include discussion on existence in this section.

### 4.1 Choice sets

We set the number of periods in each generation $T=39$ and measure the individual's age where $t=0$ is age 17. Below we summarize the decision process of males and females for possible choice combinations. Define an indicator variable $\mathbb{I}_{k_{\sigma t}}$ where $\mathbb{I}_{k_{\sigma t}}=1$ if the action $k_{\sigma t}$ is chosen and $\mathbb{I}_{k t \sigma}=0$ otherwise. Females have 16 mutually exclusive choices each includes a level of labor market time, time spent with children and a birth decision. There are 3 levels of labor supply corresponding to no work, part time work, and full time work (i.e. $h_{f t} \in\{0,1,2\}$ ). These levels are defined using the 40 hours week; an individual working less than three hours per week is classified as not working, individuals working between 3 and 20 hours per week are classified as working part time, while individuals working more than 20 hours per week are classified as working full time. There are 3 levels of parental time spent with children corresponding to no time, low time, and high time. To control for the fact females spend significantly more time with children than males, we used a gender-specific categorization. We used the 50th percentile of the distribution of parental time with spent children as the threshold for low versus high parental time with children, and the third category is 0 time with children (i.e. $\left.d_{\sigma t} \in\{0,1,2\}\right)$. This classification is done separately for males and females. Finally, birth is a binary variable equal one if the mother gives birth in that year and zero otherwise (i.e. $b_{t} \in\{0,1\}$ ). Table 2 presents the summary of these 16 mutually exclusive choices.

Males have 9 mutually exclusive choices since they do not have a birth decision (three labor supply categories and three categories for time spent with children). The second panel in Table 2 presents the summary of the males choice set. Denote by $\mathcal{H}_{P \sigma}$ and $\mathcal{H}_{F \sigma}$ index the sets of choices that involve working part time and full time, respectively, and denote by $\mathcal{H}_{\sigma}$ be the choice set for each gender $\sigma$.

### 4.2 Labor Market Earnings

Individual's earnings depend on his/her characteristics, $x_{\sigma t}$. Let $z_{\sigma t}$, be a subset of $x_{\sigma t}$, which includes age, age squared and $E d_{\sigma}$, an education dummy variables indicating whether the individual has high school, some college or college (or more) education interacted with age respectively ${ }^{12}$. Let $\eta_{\sigma}$ be the individual specific ability which is assumed to be correlated with the individual specific time invariant observed characteristics.. Earnings are assumed to be the marginal productivity of workers, and are assumed to be exogenous, linear additive and separable across individuals in the economy. The

[^9]earnings equations are given by:
\[

$$
\begin{equation*}
w_{\sigma t}=\exp \left(\delta_{0 \sigma} z_{\sigma t}+\sum_{s=0}^{\rho} \delta_{\sigma, s}^{p t} \sum_{k_{t-s} \in \mathcal{H}_{P \sigma}} \mathbb{I}_{k_{t-s} \sigma}+\sum_{s=1}^{\rho} \delta_{\sigma, s}^{f t} \sum_{k_{t-s} \in \mathcal{H}_{F m}} \mathbb{I}_{k_{t-s} \sigma}+\eta_{\sigma}\right) \tag{19}
\end{equation*}
$$

\]

where the earnings equation depends on experience accumulated while working part time and full time, and the current level of labor supply. We assume $\rho=4$. Thus, $\delta_{\sigma, s}^{p t}$ and $\delta_{\sigma, s}^{f t}$ capture the depreciation of the value of human capital accumulated while working part-time and full time, respectively.

### 4.3 Production Function of Children

Parental time investment in children affect the future educational outcome of the child which is denoted by $E d_{\sigma}^{\prime}$. and innate ability $\eta_{\sigma}^{\prime}$, both affecting the child's earnings (see Equation 19).

The state vector for the child in the first period of her life cycle $x_{0 \sigma}^{\prime}$ determined by the intergenerational state transition function $M\left(x_{0}^{\prime} \mid x_{f}, x_{m}, D_{s}\right)$; specifically we assume that,

$$
\begin{equation*}
M\left(x_{0}^{\prime} \mid x_{f}, x_{m}, D_{s}\right)=\left[\operatorname{Pr}\left(\eta_{\sigma}^{\prime} \mid E d_{\sigma}^{\prime}\right), \operatorname{Pr}\left(E d_{-\sigma 0}^{\prime} \mid E d_{\sigma}^{\prime}\right), \operatorname{Pr}\left(\eta_{-\sigma}^{\prime} \mid E d_{-\sigma}^{\prime}\right), 1\right] \operatorname{Pr}\left(E d_{\sigma}^{\prime} \mid x_{f}, x_{m}, D_{s}\right) \tag{20}
\end{equation*}
$$

Thus, we assume that the parental inputs and characteristics (parents education and fixed effects) determines educational outcomes according to probability distribution $\operatorname{Pr}\left(E d_{\sigma}^{\prime} \mid x_{f}, x_{m}, D_{s}\right)$. The state vector of inputs contains the cumulative investment variables (low time and high time) of each parent up to period $T$. In the data, we only observe total time devoted to children each period, thus we assign each child under the age of 5 in the household the average time investment assuming all young children in the household receive the same time input.

Parents' characteristics include the education of the father and mother, their individual-specific effects and race. Once the education level is determined, it is assumed that the ability $\eta_{\sigma}^{\prime}$ is determined according to the probability distribution $\operatorname{Pr}\left(\eta_{\sigma}^{\prime} \mid E d_{\sigma}^{\prime}\right)$. The spouse's education is also determined after the realization of the child's education according to the distribution $\operatorname{Pr}\left(E d_{-\sigma 0}^{\prime} \mid E d_{\sigma}^{\prime}\right)$, potentially capturing assortative mating and the spouse's fixed effect conditional on their education, $\operatorname{Pr}\left(\eta_{-\sigma}^{\prime} \mid\right.$ $E d_{-\sigma}^{\prime}$ ) . The above form of the transition allows us to estimate the equations separately for the production function of children given as the first two probabilities, and the marriage market matching given as the last term.

### 4.4 Existence of MPE in Pure Strategies

We need one final assumption to guarantee that there exist a MPE in pure strategies.
Assumption 1: For an increasing levels of $\widehat{E d_{\sigma}}$

$$
\operatorname{Pr}\left(\widehat{E d_{\sigma}} \mid k_{\sigma t}^{\prime}, k_{-\sigma t}^{\prime}, x_{\sigma t}\right)-\operatorname{Pr}\left(\widehat{E d_{\sigma}} \mid k_{\sigma t}, k_{-\sigma t}^{\prime}, x_{\sigma t}\right) \geq \operatorname{Pr}\left(\widehat{E d_{\sigma}} \mid k_{\sigma t}^{\prime}, k_{-\sigma t}, x_{\sigma t}\right)-\operatorname{Pr}\left(\widehat{E d_{\sigma}} \mid k_{\sigma t}, k_{-\sigma t}, x_{\sigma t}\right)
$$

for all $k_{\sigma t}^{\prime} \geq k_{\sigma t}$ and $k_{-\sigma t}^{\prime} \geq k_{-\sigma t}$.
The property implies that the differences in outcomes of children in terms of higher $x_{0}^{\prime}$ are weakly higher the larger the existing stock of investment. Thus, if there are complementarities in time investment of parents or if the increase in outcomes is independent of the spouse's investment, the condition is satisfied. Table 3 shows that this condition is satisfied.

It is important that we estimate the education production function (and the earnings equations and the conditional best response probabilities) outside the main estimation (of the utility parameters), because it allows us to verify that the conditions for existence of a MPE in pure strategies that are imposed on the stochastic transition functions and all the parameters, except for the utility function parameters, are satisfied. This guarantees that our estimator is well define over the parameters space.

Proposition 1 Under Assumption 1 and given the specification in equation (10),(4),(19) and (20); there exist a MPE in Pure Strategy.

## 5 A Generic Estimator of the Dynamic Complete Information Game with Altruistic Preferences

The estimation of dynamic incomplete information games has become routine over the last five years (see Aguirregabiria and Mira (2007), Bajari, Benkard, and Levin (2007), and Pesendorfer and SchmidtDengler (2008) among others for examples) in the empirical economics literature. However, the estimation of dynamic games of complete information has received little or no attention in the literature. This is mainly due to the fact that, in addition to the multiple equilibria problem in incomplete information games, complete and asymmetric information games do not always have an equilibrium ${ }^{13}$. If one could overcome the non-existence of equilibrium then, depending on the data available, one can also overcome the multiple equilibria problem using the same techniques used in the literature on the estimation of dynamic games of incomplete information. The solution to the non-existence of equilibrium problem used in the estimation of static complete information games is to imposed sufficient conditions for the existence of equilibrium on the parameters of the model (See for example Assumptions 2 of Theorem 1 in Tamer (2003)). These conditions are normally difficult to derive in the dynamic equivalent of complete information games on a case by case basis. The sufficient conditions used in the estimation of the static complete information games are particular examples of a boarder set of sufficient conditions for existence of a pure strategy equilibrium in complete information games. There are dynamic analogues to these sufficient conditions in the game-theoretical literature; specifically, these conditions are strategic complementarities or substitutabilities conditions (see Vives (2005) for a survey of the theoretical literature). Since we have data on the actions of all (potential) players in our game, we can nonparametrically identify the ex-ante best-reply probabilities under the following equilibrium selection assumption. The equilibrium selection criterion assume that the data is generated by the same equilibrium within and across generations conditional on observed (to the econometrician) data partition ${ }^{14}$.

We use a partial solution, multi-stage estimation procedure to accommodate the non-standard features of the model. It uses the assumption of stationarity across generations and the discreteness of the state space of the dynamic programming problem to obtain an analytic representation of the valuation function. This representation is a function of the conditional choice probabilities, the transition function of the state variable, and the structural parameters of the model. The conditional choice probabilities and the transition function are estimated in a first stage and used in the generation valuation representation to form the terminal value in the life-cycle problem. The life-cycle problem is then solved by backward induction to obtain the life-cycle valuation functions. We imposed conditions on the payoff and transition functions to ensure that our game is super modular; super modularity is a sufficient condition for the existence of equilibrium in pure strategies. Our game is super modular if there are strategic complementarities in time investment of parents or outcome of parental time investment is independent of the spouse's investment. An additional advantage of using a multiple step estimation approach is that it allows us to estimate the children's education production function parameters separately, using a Three Stage Least Square method, and verify that the conditions for existence of equilibrium are satisfied. We then form moment conditions from the best response functions

[^10]and estimate it in a third step. Finally to reduce the computational burden of the backward induction in the life-cycle problem we use the forward simulation technique developed in Hotz, Miller, Sanders and Smith (1994), and estimate the remaining structural parameters using Generalized Methods of Moment (GMM) estimator.

### 5.1 An Alternative Representation of the Problem

The alternative representation of the continuation value of the intergenerational problem we develop below enables us to adopt the Hotz and Miller estimation technique for standard single agent problems to the dynastic problem. We begin with the following representation of the problem,

$$
\begin{align*}
v_{\sigma}\left(k_{j i t} ; x_{t}\right)= & u_{\sigma}\left(k_{j i t}, x_{t}\right) \\
& +\sum_{s=t+1}^{T} \beta^{s-t} \sum_{x_{s}}\left\{\left(\sum_{k_{s}}\left[u_{\sigma}\left(k_{s}, x_{s}\right)+E_{\varepsilon}\left(\varepsilon_{\sigma s} \mid k_{s}=s\right)\right] p\left(k_{s}=s \mid x_{s}\right)\right) F\left(x_{s} \mid x_{t}, k_{j i t}\right)\right\} \\
& +\lambda \beta^{T-t} \sum_{x_{0}} V\left(x_{0}^{\prime}\right) H\left(x_{0}^{\prime} \mid x_{t}, k_{j i t}\right) \tag{21}
\end{align*}
$$

where $F\left(x_{s} \mid x_{t}, k_{j i t}\right)$ is the $s-t$ transitions, $H\left(x_{0}^{\prime} \mid x_{t}, k_{j i t}\right)$ is weighted generation transitions, and $V\left(x_{0}\right)$ $\left(=\left[V_{f}\left(x_{0}\right), V_{m}\left(x_{0}\right)\right]^{\prime}\right)$ is a vector of the ex-ante. The transition function $H\left(x_{0}^{\prime} \mid x_{t}, k_{j i t}\right)$ can write as recursive function of $F\left(x_{t+1} \mid x_{t}, k_{j i t}\right), M\left(x_{0}^{\prime} \mid x_{f}, x_{m}, D_{s}\right), N_{\sigma T}, b_{s}, p_{\sigma}$ and $1-\nu$. Define the ex-ante conditional lifetime utility as period $t$, excluding the dynastic component, as:

$$
\begin{aligned}
U_{\sigma}\left(k_{j i t}, x_{t}\right)= & u_{\sigma}\left(k_{j i t}, x_{t}\right) \\
& +\sum_{s=t+1}^{T} \beta^{s-t} \sum_{x_{s}}\left\{\left(\sum_{k_{s}}\left[u_{\sigma}\left(k_{s}, x_{s}\right)+E_{\varepsilon}\left(\varepsilon_{\sigma s} \mid k_{s}=s\right)\right] p\left(k_{s}=s \mid x_{s}\right)\right) F\left(x_{s} \mid x_{t}, k_{j i t}\right)\right\}
\end{aligned}
$$

Therefore we can write an alternative representation for the ex-ante value function as time $t$ :

$$
\begin{align*}
V_{\sigma}\left(x_{t}\right)= & \sum_{k_{-\sigma i t}}\left\{p\left(k_{-\sigma i t} \mid x_{t}\right) \sum_{k_{\sigma j t}}\left[U_{\sigma}\left(k_{j i t}, x_{t}\right)+E_{\varepsilon}\left(\varepsilon_{\sigma j t} \mid k_{j i t}, x_{t}\right)\right] p_{t}\left(k_{\sigma j i t} \mid x_{t}\right)\right\}  \tag{22}\\
& +\sum_{k_{-\sigma i t}}\left\{p\left(k_{-\sigma i t} \mid x_{t}\right) \sum_{k_{\sigma j t}}\left[\lambda \beta^{T-t} \sum_{x_{0}} V\left(x_{0}\right) H\left(x_{0} \mid x_{t}, k_{j i t}\right)\right] p_{t}\left(k_{\sigma j i t} \mid x_{t}\right)\right\}
\end{align*}
$$

Equation (22) is satisfied at every state vector $x_{t}$, and since the problem is stationarity over generation at period 0 we express it as a matrix equation:

$$
\begin{align*}
V\left(X_{0}\right) & =P\left(X_{0}\right) U\left(X_{0}\right)+e\left(X_{0}, P\left(X_{0}\right)\right)+\lambda \beta^{T} P\left(X_{0}\right) H\left(X_{0}\right) V\left(X_{0}\right) \\
& =\left[I_{2 S(X)}-\lambda \beta^{T} P\left(X_{0}\right) H\left(X_{0}\right)\right]^{-1}\left[P\left(X_{0}\right) U\left(X_{0}\right)+e\left(X_{0}, P\left(X_{0}\right)\right)\right] \tag{23}
\end{align*}
$$

The terms on the right hand side of Equation 23 are the intergenerational and the per period discount factors, the household choice probability matrix, the intergenerational state transition matrix, the ex-ante conditional lifetime utility, and the expected purveyances shocks. In matrix notation $V\left(X_{0}\right)=\left[V\left(x_{0}\right)\right]_{x_{0} \in X_{0}}$ is $2 S\left(X_{0}\right) \times 1$ vector of expected discounted sum of future utility; $P\left(X_{0}\right)$ is $2 S\left(X_{0}\right) \times\left(S(K) \cdot 2 S\left(X_{0}\right)\right)$ dimensional matrix consisting if the household choice probability $p\left(k \mid x_{0}\right)$ in rows $x_{0}$ and $S(X)+x_{0}$ and columns $\left(k, x_{0}\right)$ and $\left(k, S(X)+x_{0}\right)$, zeros in rows $x_{0}$ and $S(X)+x_{0}$ and
columns $\left(k, x_{0}^{\prime}\right)$ and $\left(k, S(X)+x_{0}^{\prime}\right)$ with $x_{0}^{\prime} \neq x_{0} ; e\left(X_{0}, P\left(X_{0}\right)\right)$ is the $2 S\left(X_{0}\right) \times 1$ vector of expected preference shocks with element $\left[\sum_{k_{-f i}} E_{\varepsilon}\left(\varepsilon_{f j} \mid k_{j i}, x\right) p\left(k_{f j i} \mid x\right) p\left(k_{-f i} \mid x\right)\right.$,
$\left.\sum_{k_{-m i}} E_{\varepsilon}\left(\varepsilon_{m j} \mid k_{j i}, x\right) p\left(k_{m j i} \mid x_{t}\right) p\left(k_{-m i} \mid x_{t}\right)\right]_{x \in X_{0}}^{\prime} ;$ and $I_{2 S(X)}$ denotes the $2 S\left(X_{0}\right)$-dimensional identity matrix. The second line in Equation (23) is a direct implication of the dominant diagonal property, which implies that the matrix $\left[I_{2 S(X)}-\lambda \beta^{T} P\left(X_{0}\right) H\left(X_{0}\right)\right]$ is invertible.

Under the assumption that $\varepsilon_{\sigma s}$ is distributed i.i.d. type I extreme value then Hotz and Miller inversion implies that
$\log \left(\frac{p_{\sigma j t}\left(k_{\sigma j t} \mid k_{-\sigma i t}, x_{t}\right)}{p_{\sigma j t}\left(k_{\sigma 0 t} \mid k_{-\sigma i t}, x_{t}\right)}\right)=U_{\sigma}\left(k_{j i t}, x_{t}\right)-U_{\sigma}\left(k_{0 i t}, x_{t}\right)+\lambda \beta^{T} \sum_{x_{0}} V\left(x_{0}\right)\left[H\left(x_{0} \mid x_{t}, k_{j i t}\right)-H\left(x_{0} \mid x_{t}, k_{0 i t}\right)\right]$
for $\sigma \in\{f, m\}, k_{j i t} \neq k_{0 i t}$. A similar set of conditions can be derived for single individuals except that the best reply ex-ante probabilities are replace with the single agent conditional choice probabilities.

### 5.2 Moment Conditions

Using equation (24) we then use a simulated method of moment estimation techniques developed in Hotz, Miller, Sanders and Smith (1994). In the first step we estimate the transition functions and conditional best response probabilities from the data. Starting at age seventeen we use the estimate in the first step to simulate lifetime paths for each value of the state space. Using the formulate in equation (23), we compute and estimate of $V\left(X_{0}\right)$ from the simulated data. Similarly we simulated paths for each value of the state space at age greater seventeen to obtain estimates of $U_{\sigma}\left(k_{j i t}, x_{t}\right)$. Using the estimates of the conditional best response probabilities, transition functions, $V\left(X_{0}\right)$, and $U_{\sigma}\left(k_{j i t}, x_{t}\right)$, we form an empirical counterpart to equation (24) and estimate the parameters of our model using a 2 -step GMM estimator.

The moment conditions in our framework can be obtained from the difference in the conditional valuation functions calculated for choice $j$ versus base choice 1 . Therefore, the following moment conditions are produced for an individual who is single at age $t \in\{17, \ldots ., 55\}$ :

$$
\begin{equation*}
v_{\sigma}\left(j, x_{n t}\right)-v_{\sigma}\left(1, x_{n t}\right)-\ln \left(\frac{p\left(j, x_{n t}\right)}{p\left(1, x_{n t}\right)}\right)=0 \tag{25}
\end{equation*}
$$

Therefore single males have 8 ( 9 choices) orthogonality conditions at age $t$ while single females have 15 ( 16 choices) orthogonality conditions. Let $\theta \in \Theta$ denote all the parameters of the models which have not been estimated in previous stages of the estimation procedure. Let $\xi_{n t}^{m, s}(\theta)$ and $\xi_{n t}^{f, s}(\theta)$ be the vector of moment conditions for single males and females respectively at $t$, these vectors are defined as follows:

$$
\xi_{n t}^{m, s}(\theta)=\left(\begin{array}{c}
v_{m}\left(2, x_{n t}\right)-v_{m}\left(1, x_{n t}\right)-\ln \left(\frac{p\left(2, x_{n t}\right)}{p\left(1, x_{n t}\right)}\right)  \tag{26}\\
v_{m}\left(3, x_{n t}\right)-v_{m}\left(1, x_{n t}\right)-\ln \left(\frac{p\left(3, x_{n t}\right)}{p\left(1, x_{n t}\right)}\right) \\
\vdots \\
v_{m}\left(9, x_{n t}\right)-v_{m}\left(1, x_{n t}\right)-\ln \left(\frac{p\left(2, x_{n t}\right)}{p\left(1, x_{n t}\right)}\right)
\end{array}\right)
$$

and

$$
\xi_{n t}^{f, s}(\theta)=\left(\begin{array}{c}
v_{f}\left(2, x_{n t}\right)-v_{f}\left(1, x_{n t}\right)-\ln \left(\frac{p\left(2, x_{n t}\right)}{p\left(1, x_{n t}\right)}\right)  \tag{27}\\
v_{f}\left(3, x_{n t}\right)-v_{f}\left(1, x_{n t}\right)-\ln \left(\frac{p\left(3, x_{n} t\right.}{p\left(1, x_{n t}\right)}\right) \\
\vdots \\
v_{f}\left(16, x_{n t}\right)-v_{f}\left(1, x_{n t}\right)-\ln \left(\frac{p\left(16, x_{n t}\right)}{p\left(1, x_{n t}\right)}\right)
\end{array}\right)
$$

Let $I_{n t}^{(s)}=1$ if individual $n$ is single at age $t$ and 0 otherwise. Therefore $E\left[\xi_{n t}^{\sigma, s}\left(\theta_{0}\right) \mid I_{n t}^{(s)}=1, x_{n t}\right]=0$ for $\sigma \in\{m, f\}, t \in\{17, \ldots, 55\}$ and where $\theta_{0}$ is the true parameter of the model. Because of conditional independence implies covariance independence then $E\left[\xi_{n t}^{\sigma, s}\left(\theta_{0}\right) I_{n t}^{(s)} \mid x_{n t}\right]=0$. Married couples are playing a complete information game, the orthogonality conditions come off the conditional best response function instead of the conditional valuation function of single agent optimization. Therefore, the following moment conditions are produced for individuals who are married at age $t \in\{17, \ldots ., 55\}$ :

$$
\begin{equation*}
v_{\sigma}\left(i, x_{n t} \mid k_{n^{\prime} t}=j\right)-v_{\sigma}\left(1, x_{n t} \mid k_{n^{\prime} t}=j\right)-\ln \left(\frac{p\left(i, x_{n t} \mid k_{n^{\prime} t}=j\right)}{p\left(1, x_{n t} \mid k_{n^{\prime} t}=j\right)}\right) \tag{28}
\end{equation*}
$$

A married man has $128(=8 \times 16)$ orthogonality conditions at age $t$, this because the differences of best response are conditional on the actions (female has 16 possible actions) of his spouse. Similarly a married woman has female $135(=15 \times 9)$ orthogonality conditions at age $t$ and let $\xi_{n t}^{m, c}(\theta)$ and $\xi_{n t}^{f, c}(\theta)$ be the vector of moment conditions for males and females members of a married couple respectively at $t$. These vectors are defined as follows:
and

$$
\xi_{n t}^{f, c}(\theta)=\left(\begin{array}{c}
v_{f}\left(2, x_{n t} \mid k_{n^{\prime} t}=1\right)-v_{f}\left(1, x_{n t} \mid k_{n^{\prime} t}=1\right)-\ln \left(\frac{p\left(2, x_{n} \mid k_{n^{\prime} t}=1\right)}{p\left(1, x_{n t} \mid k_{n^{\prime} t}=1\right)}\right)  \tag{30}\\
\vdots \\
v_{f}\left(16, x_{n t} \mid k_{n^{\prime} t}=1\right)-v_{f}\left(1, x_{n t} \mid k_{n^{\prime} t}=1\right)-\ln \left(\frac{p\left(16, x_{n t} \mid k_{n^{\prime} t}=1\right)}{p\left(1, x_{n t} \mid k_{n^{\prime} t}=1\right)}\right) \\
\vdots \\
\vdots \\
v_{f}\left(2, x_{n t} \mid k_{n^{\prime} t}=9\right)-v_{f}\left(1, x_{n t} \mid k_{n^{\prime} t}=9\right)-\ln \left(\frac{p\left(2, x_{n t} \mid k_{n^{\prime} t}=9\right)}{p\left(1, x_{n+\mid} \mid k_{t}=9\right)}\right) \\
v_{f}\left(16, x_{n t} \mid k_{n^{\prime} t}=9\right)-v_{f}\left(1, x_{n t} \mid k_{n^{\prime} t}=9\right)-\ln \left(\frac{p\left(16, x_{n t} \mid k_{n} t\right.}{p\left(1, x_{n t} \mid k_{n^{\prime} t}=9\right)}\right)
\end{array}\right)
$$

Let $I_{n t}^{(c)}=1-I_{n t}^{(s)}$ be an indicator equal one if individual $n$ is married at age $t$ and 0 otherwise. Similar to the orthogonality conditions for single individuals we have that $E\left[\xi_{n t}^{\sigma, c}\left(\theta_{0}\right) \mid I_{n t}^{(c)}=1, x_{n t}\right]=0$ for $\sigma \in\{m, f\}, t \in\{17, \ldots, 55\}$ and where $\theta_{0}$ is the true parameter of the model and the covariance implies that $E\left[\xi_{n t}^{\sigma, c}\left(\theta_{0}\right) I_{n t}^{(c)} \mid x_{n t}\right]=0$.

Let $\xi_{n t}(\theta) \equiv\left(\xi_{n t}^{m, s}(\theta)^{\prime} I_{n t}^{(s)}, \xi_{n t}^{f, s}(\theta)^{\prime} I_{n t}^{(s)}, \xi_{n t}^{m, c}(\theta)^{\prime} I_{n t}^{(c)}, \xi_{n t}^{f, c}(\theta)^{\prime} I_{n t}^{(c)}\right)^{\prime}$ be the $286(=8+15+128+135) \times 1$ vector of the complete orthogonality conditions and let $T_{3}$ denote the set of periods for which the necessary conditions for equilibrium are valid ${ }^{15}$. Define $\xi_{n}(\theta) \equiv\left(\xi_{n 1}(\theta)^{\prime}, \ldots, \xi_{n T_{3}}(\theta)^{\prime}\right)^{\prime}$ as the the vector of moment restrictions for a given individual over time. Similarly, define $\Phi(\theta) \equiv E_{t}\left[\xi_{n}(\theta) \xi_{n}(\theta)^{\prime}\right]$. Notice

[^11]that the matrix $\Phi(\theta)$ is block diagonal with diagonal elements defined as $\Phi_{t} \equiv E_{t}\left[\xi_{n t}(\theta) \xi_{n t}(\theta)^{\prime}\right]$, and offdiagonal elements that are zero because $E_{t}\left[\xi_{n t}(\theta) \xi_{n t}(\theta)^{\prime}\right]=0$ for $s \neq t, s<t$. The $286 \times 286$ conditional heteroskedasticity matrix $\Phi_{t}$ associated with the individual-specific errors $\xi_{n t}(\theta)$ is evaluated using a nonparametric estimator based on the estimated residuals, $\xi_{n t}(\theta)$, using an initial consistent estimator of $\theta$.This estimator is similar to Robinson(1987) estimator except we use a kernel based nonparametric regressions instead of a Nearest neighbor regression approach. To ensure none zero variance the data should be trimmed. The optimal GMM estimator for, $\theta$ satisfies
\[

$$
\begin{equation*}
\widehat{\theta}=\underset{\theta}{\arg \min }\left[1 / N \sum_{n=1}^{N} \xi_{n}(\theta)\right]^{\prime} \widehat{\Phi}\left[1 / N \sum_{n=1}^{N} \xi_{n}(\theta)\right] . \tag{31}
\end{equation*}
$$

\]

The above description of the moment conditions can be partially linearized conditional on some parameters of the model. The conditional valuation function of the individual at age $t$ can be written as a finite sum of future expected utilities. This will simplify the forward simulation use constructs the conditional valuation functions.

## 6 Estimation Results

As noted in the estimation section we used a multi-stage estimation technique. As such we present the results in three stages. The first stage (section 5.1) presents the estimates of the earnings equation, and the unobserved skills function, the intergenerational education production function, the marital status transition functions, and the marriage assignment functions. All these functions are fundamental parameters of our model which are estimated outside the main estimation of the preference, discounts factors, household sharing rules (coefficient on own and spouse earnings in the utility function), and the net costs of raising children parameters. The first stage estimates also include equilibrium objects such as the conditional choice probabilities and the best response functions. Note that the unobserved skill is controlled for in all the first stage functions that are estimated. ${ }^{16}$ The second stage (section 5.2) presents estimates of the intergenerational and intertemporal discount factors, the preference parameters, the household sharing rules, and child care cost parameters. The third and final stage (section 6) uses counterfactual analysis and presents the estimates of the return to parental time investment and the value of children.

### 6.1 First Stage Estimates

### 6.1.1 Earnings Equation and Unobserved Skills

Table 4 presents the estimates of the earnings equation and the function of unobserved (to the econometrician) individual skill. The top panel of the first column shows that the age-earnings profile is significantly steeper for higher levels of completed education; the slope of the age-log-earnings profile for a college graduate is about 3 times that of an individual with less than a high school education. However, the largest gap is due to being a college graduate; the of the age-log-earnings profile for a college graduate is about twice that of an individual with only some college. These results confirm that there are significant returns to parental time investment in kids in terms of labor market because parental investment significantly increases the likelihood of higher education outcomes which significantly increases life time labor market earnings.

The bottom panel of the first column and the second of column of Table 4 show that full time workers earn 2.6 times more than part time workers for males, and 2.3 times more than part time workers for females. It also shows that there are significant returns to past full time employment for

[^12]both genders; however, females have higher returns to full time labor market experience than males. The same is not true for part time labor market experience; males' earnings are lower if they work part time in the past while the there are positive returns to the most recent female part time experience. However, part time experiences 2 and 3 years in the past are associated with lower earnings for females, these rates of reduction in earnings are however lower than that of males. These results are similar to those find in Gayle and Golan (2012) and perhaps reflect statistical discrimination in the labor market in which past labor market history affects beliefs of employers on workers' labor market attachment in the presence of hiring costs. ${ }^{17}$ These results imply that there are significant costs in the labor market in terms of loss of human capital from spending time with kids, if spending more time with kids comes at the expense of working more in the labor market. This cost may be smaller for female than males because part time work reduces compensation less for females than males. If a female works part time for 3 years, for example, she loses significantly less human capital than a male working part time for 3 years instead of full time. This may give rise to females specializing in child care; this specialization comes from the labor market and production function of child's outcome as is the current wisdom.

The unobserved skill (to the econometrician) is assumed to be a parametric function of the strictly exogenous time-invariant components of the individual variables. This assumption is used in other papers such as MaCurdy (1981) Chamberlain (1986), Nijman and Verbeek (1992), Zabel (1992), Newey (1994), Altug and Miller (1988), and Gayle and Viauroux (2007). It allows us to introduce unobserved heterogeneity to the model but at the same time maintain the assumption on the discreteness of the state space of the dynamic programming problem needed for the estimation of the structural parameters from the dynastic model. The Hausman statistic shows that we cannot reject this correlated fixed effect specification. Column 3 of Table 4 presents the estimate of the skill as function of unobserved characteristics; it shows that blacks and females have lower unobserved skill than whites and males. This could capture labor market discrimination. Education increases the level of the skill but it increases at a decreasing rate in the level of completed education. The rate of increase for blacks and females with some college and a college degree are higher than their white and male counterparts. This pattern is reversed for blacks and females with a high school diploma. Notice that the skill is another transmission mechanism through which parental time investment affects labor market earnings in addition to education.

### 6.1.2 Intergenerational Education Production Function

A well known problem with the estimation of production functions is the simultaneity of the inputs (time spent with children and income). As is clear from the structural model the intergenerational education production function suffers from a similar problem. However, because the output of the intergenerational education production (i.e. completed education level) is determined across generations while the inputs, such as parental time investment, are determined over the life cycle of each generation, we can treat these inputs as predetermined and use instruments from within the system to estimate the production function.

Table 3 presents results of a Three Stage Least Square estimation of the system of individual educational outcomes. The system includes the education outcomes equation as well as labor supply, income and time with children equations. The estimation uses mother's and father's labor market hours over the first 5 years of the child's life as well as linear and quadratic terms of mother's and father's age on the 5th birthday of the child as instruments. The estimation results show that a child who's mother has a college education has a significantly higher probability of graduating from college

[^13]and a lower probability of only being a high school graduate, while if a child's father has some college or college education the child has a higher probability of graduating from college.

We measure parental time investment as the sum of the parental time investment over the first 5 years of the child's life. Total time investment is a variable that ranges between 0 and 10 since low parental investment is coded as 1 and high parental investment is code as 2 . The results in Table 3 shows that while mothers time investment significantly increases the probability of a child graduating from college, fathers time investment significantly increases the probability of the child graduating from high and going to college. These estimates suggest that while mothers' time investment increases the probability of a high educational outcome, fathers' time investment truncates low educational outcome. However, both parents' time investment is productive in terms of children education outcomes. It is important to note that mothers' and fathers' hours spent with children are at different margins, with mothers providing significantly more hours than fathers. Thus the magnitudes of the discrete levels of time investment of mothers and fathers are not directly comparable since what constitutes low and high investment differs across genders.

The results in Table 3 also show that females are more likely to enter and graduate college than males. Interestingly, controlling for parental characteristics and time investment, black children have a higher probability of graduating from college as well as a higher probability of not graduating from high school than white children.

Table 3B presents the predicted probabilities of a child's education outcomes by parents education and time investment for a white male child. This exercise illustrates the quantitative magnitude of the effect of parental time investment on education outcomes. It shows that if both parents have less than a high school education and invest no parental time over the child's first five years of life, the child has a $14 \%$ chance of not completing high school and $86 \%$ chance of graduating college. However, if both parents invest the average time observed in our sample then while the chance of not completing high school does not change, the probability of some college increases to $24 \%$ and the chance of graduating college increases to $3 \%$. If both parents invest the maximum amount of time then the probabilities of not graduating from high school or only graduating from high school are zero, the probability of some college is $23 \%$ and the probability of graduating from college is $77 \%$. This pattern is repeated for other education groups; if both parents are college graduates but do not invest then the child has no chance of going to or graduating from college. These results suggest that there are significant returns to parental time investment and in the rest of the paper we quantify these returns.

### 6.1.3 Marriage Transitions and Assignment

Table 5 presents the logit coefficient estimates of the one period transition from single to marriage. It shows that blacks of both genders are less likely to be married next period if they are currently single. The level of education does not have any effect on the male's transition from single to married. However a single female with a high school education is more likely to transition to marriage next period than any other level of education, while a single female with a college degree is less likely to transition to marriage next period than any other education group. This result may mean that while college education for females is valuable in the labor market it may not be as valuable in the marriage market, however, another option is that college education implies a better outside options and a higher value of being single.

Table 5 also shows that the single to married transition probabilities are concave in age for both genders. The number of children, while not affecting the female transition, increases the probability of a single male transition to marriage next period. Working part time in the past does not have any significant effect on males' transition from single to marriage. However, working part time or full time last period reduces the probability that a single female will transition to marriage next period, while
working full time 2 year in the past reduces the probability that a single male transition to marriage next period. The age distribution of current children or the time spent with them do not have a significant effect on the transition probability of a single female, however, the older the second child of a single male the more likely he is to get marry next period.

The right hand panel of Table 5 shows that all the current choices of a single female increase the probability she will transition to married next period relative to choosing "no work-no birth-no time with children". For males all choices except those that involve a choice of not working while spending time with children (i.e. choices 4 and 7) increase the likelihood he will transition to marriage next period relative to not working while providing no parental time. In fact we find that if a single male chooses to work part time and supply low parental time he will transition to marriage next period with probability one.

Table 6 presents the logit coefficient estimates of the one period divorce rates. It shows that black females have a higher divorce rate than their white counterpart while there are no differences between the black and white males one period divorce rates. There is also no effect of a person's education on the one period divorce rate. For females the one period divorce rate is convex in age while age does not have any significant effect on the one period divorce rate of males. Similar patterns hold for the number of children. Table 6 also shows that if a female worked full time last period she is more likely to get divorce next period than a female who did not work or worked part time last period. Past work behavior does not have any significant effect on males' one period divorce rate. The age distribution of current children does not have any effect on female's one period divorce rate, however, the older a male's 4th child, the less likely he will get divorced next period. The time spent with current kids in the past or the number of female kids does not have any effect on the one period divorce rates of females. However, the more time a male spends with his 3rd child the higher the one period divorce rate while the more time he spends with his 4th child reduces the divorce rate. Overall it seems that if a male has four kids he is less likely to get divorced next period.

Table 6 also shows that males' whose spouse has some college or a college degree are more likely to get divorced while the opposite is true for females. The older a female's spouse, the less likely she is to get divorce next period. A male whose spouse worked part or full time last period is less likely to get divorce next period relative to one with a spouse who did not work; the same is true for a female whose spouse worked part or full time 4 years in the past. This pattern is reversed for males whose spouse worked full or part time 2 or 4 years in the past. Males whose spouse provide high parental time investment in the 1st and 4th child are more likely to get divorce next period.

Females who work part time, give birth, and do not provide any child care hours in the current period (i.e. Choice 4) are more likely to divorce next period. The same is true for females who work full time, do not give birth, and provide low child care hours in the current period (i.e. Choice 7) . The opposite is true for a female who does not work or give birth, but provides high child care hours (i.e. Choice 11). On the other hand, a male who works full or part time and provides no child care hours (i.e. Choices 2 and 3 ) has a lower probability of divorce next period relative to a male who does not work or provide any parental time investment. The same is true for a male who worked full time and provide high parental time investment (i.e. Choice 4). Again, we find that males that worked part time and provide low parental time never get divorce in our sample.

When it comes to the choices of females' spouse the patterns are not so clear. We find that a female whose spouse works full or part time and does not provide any child care (i.e. Choices 2 and 3) has a higher probability of remaining married next period relative to a female whose spouse does not work or provides parental time investment. The same is true for a female whose spouse works full time and provides some parental time investment (i.e. Choices 6 and 9 ) or does not work but provides high parental time investment (i.e. Choice 7). For males all spouse choices lead to a lower divorce rate relative to choosing no work, no birth, and no parental time.

### 6.1.4 Conditional Choice Probabilities of Single Females

Table 7 presents the logit coefficient estimates of the conditional probability for single females. The excluded category is choice 1 , which in not participating in the labor market, not giving birth, and not providing parental time investment. It shows that black females are less likely to choose choices $2,3,7$, and 13 ; the first two involve working full or part time while not giving birth or investing time in children and the last two involve working full time while not giving birth and providing high or low parental time investment. On the other hand, black females are more likely to choose choices 4 , 8 , and 9 ; the predominant feature of these choices is giving birth. Therefore single black females are more likely to give birth than single white females.

It also shows that single female college graduates are less likely to choose choices $5,8,11$, and 14 which involve not working. At the same time they are more likely to choose choices 3 and 7 which involve working full time, not giving birth, and providing no or low levels of parental time investment. While not as strong, a similar pattern holds for females with high school or some college education. The number of children increases the likelihood of any choice other than 1 , at a decreasing rate. The same is true for all form of labor market experience.

Table 7 also shows that the older the 1st child of a single female, the more likely she chooses choice 1 relative to all the other choices, while the age of the 2nd child only has a significant positive effect on choice 3 (i.e. full time work and no birth or parental time investment) relative to the choice 1 . The age of the 3 rd child has a significant positive effect on choice 2 (i.e. part time work, no birth, and no parental time invest) and choice 4 (i.e. full time work, birth, and no parental time investment); however, the effect on choice 4 is much greater than on choice 2 . The age of the 4 th child has a significant positive effect on choices 2 and 4 , which is similar to the effect of the age of the 3 th child. Unlike the effect of the age of the 3rd child, the effect of the age of the 4th child on the likelihood of choices $8,9,10,14,15$, and 16 is negative. The predominant features of all these choice are giving birth and providing positive parental investment. Past time investment in the 1st child has a positive effect on the likelihood of choice 5 through 16 relative to choice 1 ; these are all choices that involve providing positive amount of parental time investment. The only negative effect of past parental time investment in the 1st child is on choice 4 , which is full time work, giving birth, and providing no parental time investment. Past parental investment in the 2nd child has a significant negative effect on the likelihood of choices 3,5 , and 6 relative to choice 1 , all involving not giving birth. The effects of parental time investment in the 3th child are similar to those of parental time investment in the 1st child except they are not as significant. There are no clear patterns to the effect of parental time investment in the 4th child (there are both negative and positive effects on different aspects of the choices). Finally, the number of female children reduces the likelihood of choice 9 and 12, which all involve working part time with positive parental time investment.

### 6.1.5 Conditional Choice Probabilities of Single Males

Table 8 presents the logit coefficient estimates of the conditional choice probability for single males. It shows that black males are less likely than white males to choose choices $3,4,5$, and 9 relative to the choice 1 (i.e. not working and providing parental time investment). It seems black males are less likely to specialized in parental time investment than white males and they are less likely to work full time.

Table 8 also shows that a college educated and high school graduate single males are more likely overall to work full time than single men with only some college. College graduates are more likely to make choices 3 and 5; these choices involve either full time work with no parental time investment or part time work with low parental time investment. A similar pattern holds for high school graduates
or some college. On the other hand college graduate is less likely than single male with less than a high school education to choose choices 4, 7, and 8; these choices involve specialization in parental time investment to some extent. Similar patterns hold for high school graduate and some college. Similar to single females, the number of children increases the likelihood of single males making choices 4 through 9 relative to choice 1. All these choices involve providing some parental time investment. Therefore even single males with child are more likely to invest time in their children. The only negative effects of any type of labor market experience are on choices 4,5 , and 7 ; these are all choices that involve not working or working part time with low parental time investment. Therefore as with single female's labor market experience increases the likelihood of continue labor market participation. The only positive effect of the age distribution of kids on the choices of single males is the positive effect of the age of the 1 st child on the probability of full time work while providing low parental time investment. Finally the number of female children increases the likelihood that a single male would choose choices $3,5,6$, and 9 ; that is either working full time while not providing any parental time investment or working and working with some parental time investment.

### 6.1.6 Best Response Functions

Unlike single individuals, married couples are engaged in a non-corporative game of complete information, therefore we have to estimate the best response function of each spouse. These best response functions do not only depend on the individual's state space but also on the state space and choices of their spouses.

Females' Ex-ante Best Response Probabilities Table 9 presents the logit coefficient estimates of ex-ante conditional best response probabilities of a married female. It shows that the behavior of single black females and married black females differs significantly. Specifically, married black females are less likely to choose $3,5,6,7,11,12,13$ and 14 relative to their white counterparts. The first choice is working full time while doing nothing else; the next three choices (i.e. choices 5, 6, and 7 ) involve not giving birth while providing low parental time investment; and the last four (i.e. choices $11,12,13$, and 14) involve high time investment while either giving birth, working, or doing nothing. So while they behave differently from white married females it is hard to make any generalization as the choices include different combinations of work, birth, and parental time investment, however, overall black married females are less likely to make choices involving high parental time investment relative to white married women. Similar to single female, college educated married females are more likely to choose almost all other choices relatives to choice 1. This pattern is similar for high school graduates and some college education. The same is true for the effect of the number of children. Again all types of labor market experiences make it more likely to work in the current period.

Table 9 also shows that the age of the 1st child has a significant negative effect on the likelihood of choices $8,11,14,15$, and 16 ; most of these choices involve giving birth in the current period. The effects of the age distribution of older children are not as striking as those of the age of the 1st child. Parental time investment in the 1st child has a significant positive effect on the likelihood of choices 5 through 16; therefore past parental time investment in the 1st child leads to higher likelihood of current parental time investment. The pattern is reversed for parental time investment in the 2nd child, in fact the likelihood of the choices relative to doing nothing, except choice 2 which is statistically insignificant, increases in the time invested in the second child. This may be because most families have only 2 children. This pattern is repeated for parental time investment in the 3rd and 4th child.

The second panel of Table 9 presents the effect of spouse's characteristics on the ex-ante conditional best response of married female. If a female's spouse is a college graduate, the female has a higher likelihood of choosing $3,5,6,8,11$, and 14 . As usual similar patterns hold for high school or some
college education. Therefore education of the spouse increases the likelihood of specialization either in the labor market or at home. Spouse's labor market experience has the opposite effect on the likelihood choices relative to the female's own labor market. All else equal, the more labor market experience a female's spouse has, the more likely that the female will choose not to work. The more parental investment a female's spouse made in their 1st child, the lower the likelihood of the female choosing 11 through 16. These are all choices involving high parental time investment. This shows that fathers' parental investment seems to be a substitute for mothers' parental investment. A similar pattern holds for the spouse's parental time investment in the 3rd child, except that there is also a reduced likelihood of the female choosing choices 5,6 , and 7 . The additional choices involve low parental time investment of the female. The effect of the 4th child is similar to those above except that higher spouse parental time investment in the 4th child increases the likelihood of female choosing not to work while giving birth and providing high parental time investment.

The final panel of Table 9 presents the reaction function of spouse's choices on the female ex-ante probability of choices. It shows that if the spouse choose to work part time (i.e. spouse choices 2,5 , and 8) the female is more likely to work. If the spouse works full time (i.e. spouse choices 3,6 , and 9 ) the female is still more likely to work but is also more likely to give birth or provide positive parental time investment. If the spouse chooses not to work and provide low parental time investment, the female is less likely to choose 2,4 , and 11 . These choices involve either not providing parental time investment and work full time (whether the female chooses to give birth or not) or provide high time investment in children and not work.

Males' Ex-ante Best Response Probabilities Table 10 presents the logit coefficient estimates of the ex-ante best response probabilities of a married male. Most of the effects of male's own variables on these probabilities are similar to that of single males. Table 10, however, shows that a male with a spouse who is college educated is less likely to choose not to work and provide high parental time investment. The same is true if his spouse is a high school graduate or attended some college. Apart from the effect of parental time investment in their 4th child, which reduces the likelihood of a male choosing not to work and provide low parental time investment, none of the other spouse characteristics has any effect on his choices.

The final panel in table 10 represents the reaction function of the male's choice probabilities to his spouse's choices. It shows that if the spouse chooses to work part time and not provide parental time investment or give birth (i.e. female's choice 2) then the male is less likely to choose choice 4 , 5 , and 9 ; that is he is less likely to work part time and provide high or low child care and less likely not to work and provide high time investment in children, and is more likely to choose to work full time and do nothing else. If the spouse chooses to work full time and give birth while not provide parental time investment the husband is least likely to choose not to work and provide low parental time investment. However, he is more likely to choose 5 or 7 , which involve providing low parental investment while working full time or not working while providing high parental time investment. This is a case where the female is the main bread winner and gives birth, and the husband responds by providing the parental investment.

If the female choose to work part time while not giving birth, but provides low parental time investment, then the husband is more likely to choose choices 6 through 9 ; the first (i.e. male's choice 6 ) involves working full time while providing low parental time investment while the last three involve high parental time investment. A similar pattern holds for choice 7 (i.e. female choosing full time work, no birth, and low parental investment) except that there is a higher likelihood of choosing choices 3 and 4 . If the female chooses choices 8 (i.e. not working, birth, and low parental time investment) then the male is least likely to choose 7 (i.e. not working and high parental time investment) and most
likely to choose 4 (i.e. not working and low parental time investment). This highlights the fact that if the female does not work then the male has a higher probability of working. If the female chooses to work part time, give birth, and provides low parental time investment, then the husband has a higher likelihood of working in all possible combinations of parental time investment. On the other hand if the female chooses to work full time, give birth, and provide low parental time investment (i.e. choice 10) then the husband is more likely to provide the parental time investment (i.e. choices 4 through 9). This type of substitution pattern is highlighted through the other male's reactions to the female choices. Overall the reaction functions of both males and females display a certain degree of cooperation in their behavior. However, in cases in which females either do not give birth or provide no parental time investment, both spouses seem to focus on the maximizing labor income and leisure.

### 6.2 Preference Parameter Estimates

Table 11 presents the GMM estimates of the parameters characterizing the utility of functions along with the various discount factors of the model. First, the top left hand panel of Table 11 shows that there are per-period utility costs of giving birth for females. This is demonstrated by the universal significant and negative coefficients associated with all choices in the per-period utility function that involve giving birth in the current period. This finding rationalizes the low frequency of these choices in the data and conforms to the finding of previous literature on fertility behavior (see Wolpin (1984) and Hotz and Miller (1988) for example).

While the utility for female is monotonically declining in the level of labor market work for no birth and low level of parental time (i.e. choices 5 through 7), this is not always the case for other choice permutations. This seems to be caused by the interaction of labor market choice with parental time investment; some levels of parental time investment seem to be preferred to no parental time if these choices do not involve low levels of leisure. This implies that there may be some level of consumption value to maternal time investment. For example, conditional on working part time in the labor market and not giving birth in the current period, the utility of mothers are increasing in the level of parental time investment. This monotonic relationship is not present conditional on working full time in the labor market and not giving birth in the current period. This may be due to the nonlinear nature of time requirements of jobs or occupations chosen by females. That is, the full time and part time classification does not fully capture the degree of effort or flexibility of hours associated with female job choices.

The top right hand panel of Table 11 presents the estimates for males. It shows that the disutility from working in nonlinear in the level of labor market work activities. Conditional on providing zero paternal time investment, males prefer working part time to either not working or working full time. Males, however, prefer not working in the labor market to working full time in the labor market. A similar pattern holds conditional on providing low paternal time investment. This pattern, however, is reversed conditional on providing high paternal time investment. This seemingly counter intuitive finding, that males prefer some work to not working, is the way the model rationalizes the low proportion of males that not work in our data. Similar to females, there seems to be some level of consumption associated with paternal time investment in children.

The second panel of Table 11 presents the discount factors. It shows that the intergenerational discount factor (i.e. 0.90 ) is larger than the intertemporal discount factor (i.e. 0.85). This implies that in the second to last period of their life, a parent value their child $90 \%$ of their own utility next period. The discount factor on the number child shows that the marginal increase in the value of the second child is 0.87 and of the third child is 0.82 . Although the estimated discount factor of children is larger than estimates in the literature, it cannot be compared directly to these estimates because other models do not include the life cycle dimension. For example, in our model, a parent with horizon
of 10 years, discounts the consumption of an only child, for example, by an additional time discount $\beta^{10}$ which is less that 0.2 . Thus, without taking into account the time dimension involved in trade-offs parents make when they are young, these investments may seem to be consistent with a much lower discount factor on the children's utility.

The bottom panel of Table 11 presents the estimates of the utility from earnings and the per-period net cost of existing children. It shows that, as expected, utility is increasing with own earnings for both genders, irrespective of marital status. The coefficient on spouse earning for male is, however, negative and large in magnitude; this means that males utility declines in the earnings of their spouse. Since our model specification implies transferable utility between spouses in the game, these estimates imply that there is a transfer of utility to the spouse the higher the earnings of the spouse. This may also implies higher outside option for higher earning spouses. There is a similar effect for female however of a much lower magnitude. Finally, the bottom panel of Table 11 shows that for both married male and female there is a per-period net cost of existing children. However, there is a per-period net benefit from a single father; this may be because the fact that most children stay with their mother hence the fathers utility is higher when they are not living in the same household.

## 7 Measuring the Quality-Quantity Trade-offs and The Return to Parental Investment

The dynastic model provides a natural measure of the quality-quantity trade-offs and the returns to parental time investment. Consider a parent entering the final period of his/her life and assume that he/she has completed fertility decisions. ${ }^{18}$ Taking the expectation over the choices of the last term in equation (16), we can write the expected value of children at age T as

$$
\begin{equation*}
\bar{V}_{N \sigma}\left(x_{T}\right)=\sum_{i}\left(p_{-\sigma i T}\left(k_{-\sigma i T} \mid x_{T}\right)\left[\sum_{j} p_{\sigma j T}\left(k_{\sigma j T} \mid k_{-\sigma i T}, x_{t}\right) \bar{V}_{N \sigma}\left(k_{j i T} ; x_{T}\right)\right]\right) \tag{32}
\end{equation*}
$$

The discounted valuation of children is $\frac{N_{T}^{1-v} \bar{V}_{N \sigma}\left(x_{T}\right)}{N_{T}}$ where the average quality of a child is given by $\frac{\bar{V}_{N \sigma}\left(x_{T}\right)}{N_{T}}$; we can therefore measure the quality-quantity trade-off as

$$
\begin{align*}
\Lambda_{N \sigma}\left(x_{t}\right) & \equiv \frac{\partial \log \left(\frac{N_{T}^{1-v} \bar{V}_{N \sigma}\left(x_{T}\right)}{N_{T}}\right)}{\partial N_{T}}  \tag{33}\\
& =\left[1-v+\frac{\partial\left(\frac{\bar{V}_{N \sigma}\left(x_{T}\right)}{N_{T}}\right)}{\partial N_{T}} \frac{N_{T}}{\left(\frac{\bar{V}_{N \sigma}\left(x_{T}\right)}{N_{T}}\right)}\right] \frac{1}{N_{T}}
\end{align*}
$$

This measure of quantity-quality trade-off has two components: the first element in Equation 33, $1-v$, reflects the (decreasing) rate of increase in utility associated with an additional child, and the elasticity component reflects the rate of decline in the average quality per child. The model then exhibits a quality-quantity trade-off if the elasticity of the average quality is negative, and the rate of the increase in parental utility is lower then the one implied by the the discount rate. ${ }^{19}$ Next, we

[^14]measure the return to parental time investment as
\[

$$
\begin{align*}
\Lambda_{D \sigma}\left(x_{t}\right) & \equiv \frac{\partial \log \left(\frac{N_{T}^{11^{1-v}} \bar{V}_{N \sigma}\left(x_{T}\right)}{N_{T}}\right)}{\partial D_{T}} \\
& =\left[\frac{\partial \bar{V}_{N \sigma}\left(x_{T}\right)}{\partial D_{T}}\right] \frac{1}{\overline{V_{N \sigma}\left(x_{T}\right)}} . \tag{34}
\end{align*}
$$
\]

This measures the aggregated returns to parental time investment which includes the impact of parental time input on educational attainment of children, their skills and therefore life time earnings, as well as their marriage market outcomes and life time choices. If a parent provides an additional unit of time, each child under the age of 5 in the household receives an equal share of the time. Thus, the above measure depends on the number of children under the age of 5 in the household. ${ }^{20}$

The valuation function of the next generation (from the entire stock of children), $\bar{V}_{N \sigma}\left(x_{T}\right)$, is calculated by using the estimated structural parameters to simulate the model for each individual in our data and calculate their terminal valuation as age 55. Table 12 presents the estimates of these aggregate return to parental time investment and columns 1 and 2 in Table 13 present the estimates of the quantity-quality tradeoff. The standard errors are model errors which account for the variation in the outcome of the model's predictions as well as estimation errors. We discuss the results below.

The Return to Parental Time Investment The coefficients on the parental time investment in Table 12 summarize our estimates of the return to parental time investment. They show that maternal time investment has a significantly higher return than paternal time investment; the estimated elasticity of father's time investment is about $60 \%$ of that of mother's time investment. This is despite the fact that we found no clear patterns suggesting that mothers' time is more valuable than fathers' time in terms of the education production function. In principle, this can be a result of increasing returns to scale in the production function of children, combined with labor market "tax" on females. However, the specification of our model does not allow for increasing return to scale in the education production or skill function. Therefore, this result is driven by the differential impact of maternal and paternal time on the education outcomes of children. The estimates of the education production function Table 3 show that paternal time increases the probability of graduating from high school and getting some college education while mothers' time increases the probability of having a college degree. Thus paternal time truncates bad outcomes (i.e., not graduating from high school) while maternal time investment increases the probability of being a high achiever. Our estimates reveal that maternal time has a higher impact overall than paternal time because of the higher returns of graduating from college in both the labor and the marriage markets. This result illustrates the advantage of aggregating the different outcomes of children when measuring the returns to parental time investment.

Turning to race, we find that the return to maternal time investment is significantly higher for blacks than for whites, however, there are no significant differences in the return to paternal time investment across race. The difference across race stems from the differences in the education production function. Blacks have a higher variance in their educational outcome than whites; while blacks have a higher probability of not completing high school, they also have a higher probability of graduating from college. This result, combined with the finding that maternal time investment increases the probability of graduating college, explains the higher returns to time investment of black mothers relative to whites. So if the return to blacks' maternal time investment is significantly higher than whites, why

[^15]does black provide lower maternal time investment? Again, the lower levels of time investment are driven by the family structure differences between black and whites. As we discussed above, there is a significantly higher number of black single mothers than white single mothers, and single mothers invest less in their children because it is more costly for them to specialize in parental investment.

Table 12 also shows that while there are no differences in the return to paternal time investment between boys and girls, the returns to maternal time investment are significantly higher for boys. This suggests that mothers act in a compensatory manner, favoring low ability children in the family. Since girls have a higher likelihood of high education outcome than boys, mothers seem to investment more in boys than in girls as the number of children increases. These findings confirm the finding in Hanuschek (1992) that parents seem to act in compensatory or neutral manner. Like Hanuscek (1992), we do not find any evidence that parents are "achievement maximizers". Our results hold for both blacks and whites while the results in Hanuscek (1992) were restricted to blacks.

Quantity-Quality Trade-offs To measure the quantity-quality tradeoff we regress the log of the discounted valuation from children on the number of children and the number of children squared, controlling for education, column 1 in Table 13 presents the results for white individuals and column 2 in Table 13 presents the results for black individuals. The coefficients on the number of children measure the quality-quantity trade-of. Specifically, the data is obtained from a simulation where choices are an equilibrium outcome, thus the coefficients on the number of children reflect the rate of increase in the discounted valuation from all children allowing for the optimal amount of hours to adjust. Since the four education classes are controlled for, the coefficients on number of children is for the omitted education class, less than high school. The coefficients on the quadratic term shows that this effect is nonlinear in nature and that for black individuals the decline is faster.

The coefficients on the number of children and the squared term correspond to the measure in Equation 33, and demonstrate that there is a trade-off (defined as a negative effect of increase in number of children on average quality). To illustrate more clearly the effect of number of children on the average quality of children, and also look at the effect of gender on the average quality, we regress directly the measure of average quality of a child, $\left.\left(\bar{V}_{N}\left(x_{T}\right)\right) /\left(N_{T}\right)\right)$ on the number of children, the number of girls and the quadratic terms,respectively. The results are presented in Table 13 Columns 3 (whites) and 4 (blacks).The coefficients on the number of children is negative and smaller for black individuals. The coefficients on number of girls is positive and larger for blacks, however the coefficient on the quadratic terms for girls is negative and larger in absolute value for blacks, showing that once the average quality declines with the number of female children, it declines faster. Using the regression results in Table 13 columns 3 and 4, Figure 4 shows the log average value of a child as a function of the number of children for black and white, when all children are the same sex (and both parents have less than high school education). It shows that for girls, there is only quantity quality tradeoff after the third female child. For boys, the average quality declines with each additional male child. The figure also show that the decline for both boys and girls is sharper in black families. Figure 5 shows the $\log$ valuation per child as a function of the number of girls in families with four children, for black and white individuals (both parents have less than high school education). It demonstrates a substantially larger decline in the valuation per child as the number of boys increases in black families. The higher quantity-quality rate for boys is partly a result of the fact that for a given time investment educational outcomes of girls are better. However, it also reflects the fact that conditional on the same education females valuations are higher then the valuations of boys, due to the marriage market outcomes and income sharing within the household.

By comparing the estimates across race, we see that the quality-quantity trade-offs for black are significantly larger than for whites and an increase in number of children implies a larger reduction
in the average valuation function of each child. This is mainly due to the fact that the fertility rates among single black mothers are higher than the one for white females and because the cost of time for single mothers is higher than the cost of time in households of married couples. It can also explain the sharper decline in average quality of a child as the number of boys increase, as boys require higher time investment to achieve a high education outcome. These results confirms the explanation in Neal (2006) which demonstrates how family structure differences contribute to the black-white skill gap.

## 8 Conclusion

In this paper we developed and estimated a model of dynastic households in which altruistic individuals choose fertility, labor supply, and time investment in children sequentially, using data on two generations from the PSID. We then use the estimates to quantify the quality-quantity trade-offs and the return to parental time investment in children. Our preliminary analysis shows that parental investment in children varies significantly across gender, race, education levels, and the household composition. It also shows that after controlling for gender, education levels, and household composition, the differences across race are significantly reduced.

The structural estimates show that there are significant transfers between spouses within households and that females with higher earnings potential receive larger transfers. The production function estimates show that both maternal and paternal time investment increase the likelihood of higher educational outcome of their children. However, the impact is complementary; fathers' time investment increases the probability of graduating from high school and getting some college education while mothers' time increases the probability of achieving a college degree. The estimates of the education production-function show that girls have a higher likelihood than boys of achieving higher education levels, and that blacks have higher variance than whites in their educational outcomes, after controlling for parental inputs. Specifically, blacks have a higher probability of not completing high school as well graduating from college.

We find that the intergenerational discount factor (i.e. 0.90) is larger than the intertemporal discount factor (i.e. 0.85). This implies that in the second to last period of their life, parents value their child $90 \%$ of their own utility next period. The discount factor on the number children shows that the marginal increase in the value from the second child is 0.87 and from the third child is 0.82 . Although the estimated discount factor of children is significantly larger than previous estimates in the literature, it cannot be compared directly to these estimates because other models do not include life cycle. For example, in our model, a parent with horizon of 10 years, discounts the consumption of an only child by an additional time discount $\beta^{10}$ which is less that 0.2 . Thus, without taking into account the time dimension involved in trade-offs parents make when they are young, these investments may seem to be consistent with a much lower discount factor on the children's utility.

We find significant evidence for quality-quantity trade-off. This trade-off is measured in terms of the rate of the increase in the utility of parents and the rate of the decline in the average life-time utility per child resulting from having an additional child. The level of investment per child is smaller the larger the number of children, thus, this decline in the per child investment is driven by the time constraint and the opportunity costs of time and not by the properties of the production function technology of children. The negative relationship between income (education) and fertility is therefore explained by the higher opportunity cost of time of educated parents in terms of forgone earnings. We, also find that quality-quantity trade-off for blacks are about twice as large as that of whites. This is mainly due to the higher fertility rates of single black female and the resulting greater time constraint they face.

Interestingly, we find that females have higher valuation functions (i.e. female child value is higher
than that of a male child). Despite the fact that females earn less than men with the same productive characteristics, females are more likely to obtain a higher level of education than males, given equal amount of parental inputs, and education is highly compensated in the labor market. However, even given the same levels of education the valuation function of females are higher than males because they receive significant transfers from their husband's income. These findings can be rationalized by the fact that females are endowed with birth decisions and males value children.

We find that the overall returns to fathers' time investment is only $60 \%$ that of mothers' time investment. Maternal time investment increases the probability of a child graduating from college, and a college degree increases the returns in both the labor and the marriage markets. While there are no significant race differences in the returns to paternal time investment, blacks have a higher return to maternal time investment than whites. Our results suggest that the observed gaps between black and white are driven to a large extent by the fact that there are more single mothers among blacks and that the opportunity costs of time for single mothers are higher than the costs of married mothers.

Finally, the returns to maternal time investment are significantly higher for boys. This implies that mothers act in a compensatory manner, favoring low ability children in the family. Since girls have a higher likelihood of achieving a high level of education than boys, mothers seems to invest more time in boys than in girls as the number of children increases.

## 9 Appendix A: Existence of Pure Strategy Equilibrium

Proof of Proposition 1. To show that the continuation values are super modular it suffices to show that the per-period utility is super modular and that the transition functions are super-modular. First we show that the per-period is super modular, i.e. $u\left(k_{\sigma t}, k_{-\sigma t}, x_{\sigma t}\right)$ is super-modular in $k_{\sigma t}$ for any $x_{\sigma t}$ and $k_{-\sigma t}$ if;

$$
\begin{equation*}
u\left(k_{\sigma t}^{\prime} \vee k_{\sigma t}, k_{-\sigma t}, x_{\sigma t}\right)+u\left(k_{\sigma t}^{\prime} \wedge k_{\sigma t}, k_{-\sigma t}, x_{\sigma t}\right) \geq u\left(k_{\sigma t}^{\prime}, k_{-\sigma t}, x_{\sigma t}\right)+u\left(k_{\sigma t}, k_{-\sigma t}, x_{\sigma t}\right) \text { for all }\left(k_{\sigma t}^{\prime}, k_{\sigma t}\right) . \tag{35}
\end{equation*}
$$

Without loss of generality let $k_{\sigma t}^{\prime} \succeq k_{\sigma t}$, given that the choice set satisfies partial order

$$
u\left(k_{\sigma t}^{\prime} \vee k_{\sigma t}, k_{-\sigma t}, x_{\sigma t}\right)=u_{1 \sigma t}\left(k_{\sigma t}^{\prime}, k_{-\sigma t}, x_{\sigma t}\right)+u_{2 \sigma t}\left(k_{\sigma t}^{\prime}, k_{-\sigma t}, x_{\sigma t}\right)+\varepsilon_{k_{\sigma t}^{\prime}}=u\left(k_{\sigma t}^{\prime}, k_{-\sigma t}, x_{\sigma t}\right)
$$

and similarly

$$
u\left(k_{\sigma t}^{\prime} \wedge k_{\sigma t}, k_{-\sigma t}, x_{\sigma t}\right)=u_{1 \sigma t}\left(k_{\sigma t}, k_{-\sigma t}, x_{\sigma t}\right)+u_{2 \sigma t}\left(k_{\sigma t}, k_{-\sigma t}, x_{\sigma t}\right)+\varepsilon_{k_{\sigma t}}=u\left(k_{\sigma t}, k_{-\sigma t}, x_{\sigma t}\right)
$$

Thus the condition holds.
Next we show that the transition functions are super-modular. Let $P_{F t}(\widehat{X} \mid x, k)$ and $P_{M T}(\widehat{X} \mid x, k)$ be the probabilities of the set $\widehat{X} \subseteq X$ occurring with respect to $F\left(x_{t+1} \mid x_{t}, k_{t}\right)$ and $M\left(x_{0}^{\prime} \mid x, D\right)$, i.e.

$$
\begin{aligned}
P_{F t}(\widehat{X} \mid x, k) & =\sum_{x^{\prime} \in \widehat{X}} F_{t}\left(x^{\prime} \mid x, k\right) \\
P_{M s}(\widehat{X} \mid x, k) & =\sum_{x_{0}^{\prime} \in \widehat{X}} M\left(x_{0}^{\prime} \mid x, D\right)
\end{aligned}
$$

We say that $\widehat{X} \subseteq X$ is an increasing set if $x^{\prime} \in \widehat{X}$ and $x^{\prime \prime} \geq x^{\prime}$ imply $x^{\prime \prime} \in \widehat{X}$. Therefore $F_{t}\left(x^{\prime} \mid x, k\right)$ and $M\left(x_{0}^{\prime} \mid x, D\right)$ are stochastically super-modular in $k_{\sigma t}$ for any $x_{\sigma t}$ and $k_{-\sigma t}$ if:

$$
\begin{align*}
& P_{F t}\left(\widehat{X} \mid k_{\sigma t}^{\prime} \vee k_{\sigma t}, k_{-\sigma t}, x_{\sigma t}\right)+P_{F t}\left(\widehat{X} \mid k_{\sigma t}^{\prime} \wedge k_{\sigma t}, k_{-\sigma t}, x_{\sigma t}\right) \\
\geq & P_{F t}\left(\widehat{X} \mid k_{\sigma t}^{\prime}, k_{-\sigma t}, x_{\sigma t}\right)+P_{F t}\left(\widehat{X} \mid k_{\sigma t}, k_{-\sigma t}, x_{\sigma t}\right) \text { for all }\left(k_{\sigma t}^{\prime}, k_{\sigma t}\right), \tag{36}
\end{align*}
$$

and

$$
\begin{array}{ll} 
& P_{M t}\left(\widehat{X} \mid k_{\sigma t}^{\prime} \vee k_{\sigma t}, k_{-\sigma t}, x_{\sigma t}\right)+P_{M t}\left(\widehat{X} \mid k_{\sigma t}^{\prime} \wedge k_{\sigma t}, k_{-\sigma t}, x_{\sigma t}\right) \\
\geq & P_{M t}\left(\widehat{X} \mid k_{\sigma t}^{\prime}, k_{-\sigma t}, x_{\sigma t}\right)+P_{M t}\left(\widehat{X} \mid k_{\sigma t}, k_{-\sigma t}, x_{\sigma t}\right) \text { for all }\left(k_{\sigma t}^{\prime}, k_{\sigma t}\right) \tag{37}
\end{array}
$$

for any increasing set $\widehat{X} \subseteq X$. Without loss of generality assume that for $k_{\sigma t}^{\prime} \geq k_{\sigma t}, F_{t}\left(x^{\prime} \mid k_{\sigma t}^{\prime} \vee\right.$ $\left.k_{\sigma t}, k_{-\sigma t}, x_{t}\right)=F_{t}\left(x^{\prime} \mid k_{\sigma t}^{\prime}, k_{-\sigma t}, x_{t}\right)$ and $F_{t}\left(x^{\prime} \mid k_{\sigma t}^{\prime} \wedge k_{\sigma t}, k_{-\sigma t}, x_{t}\right)=F_{t}\left(x^{\prime} \mid k_{\sigma t}, k_{-\sigma t}, x_{t}\right)$, therefore

$$
P_{F t}\left(\widehat{X} \mid k_{\sigma t}^{\prime} \vee k_{\sigma t}, k_{-\sigma t}, x_{\sigma t}\right)=\sum_{x^{\prime} \subseteq \widehat{X}} F_{t}\left(x^{\prime} \mid k_{\sigma t}^{\prime} \vee k_{\sigma t}, k_{-\sigma t}, x_{t}\right)=\sum_{x^{\prime} \subseteq \widehat{X}} F_{t}\left(x^{\prime} \mid k_{\sigma t}^{\prime}, k_{-\sigma t}, x_{t}\right)
$$

and

$$
P_{F t}\left(\widehat{X} \mid k_{\sigma t}^{\prime} \wedge k_{\sigma t}, k_{-\sigma t}, x_{\sigma t}\right)=\sum_{x^{\prime} \subseteq \widehat{X}} F_{t}\left(x^{\prime} \mid k_{\sigma t}^{\prime} \wedge k_{\sigma t}, k_{-\sigma t}, x_{t}\right)=\sum_{x^{\prime} \subseteq \widehat{X}} F_{t}\left(x^{\prime} \mid k_{\sigma t}, k_{-\sigma t}, x_{t}\right)
$$

and the condition is satisfied for. $M\left(x_{0}^{\prime} \mid x, D\right)$ is defined in Equation 20. Recall that

$$
\operatorname{Pr}\left(e_{\sigma}^{\prime} \mid x_{f}, x_{m}, D_{s}\left(k_{\sigma t}^{\prime} \vee k_{\sigma t}, x_{t}, k_{-\sigma t}\right)=\operatorname{Pr}\left(e_{\sigma}^{\prime} \mid x_{f}, x_{m}, D_{s}\left(k_{\sigma t}^{\prime}, x_{t}, k_{-\sigma t}\right)\right)\right.
$$

and

$$
\operatorname{Pr}\left(e_{\sigma}^{\prime} \mid x_{f}, x_{m}, D_{s}\left(k_{\sigma t}^{\prime} \wedge k_{\sigma t}, x_{t}, k_{-\sigma t}\right)=\operatorname{Pr}\left(e_{\sigma}^{\prime} \mid x_{f}, x_{m}, D_{s}\left(k_{\sigma t}, x_{t}, k_{-\sigma t}\right)\right)\right.
$$

Thus, $\operatorname{Pr}\left(e_{\sigma}^{\prime} \mid x_{f}, x_{m}, D_{\sigma}\right)$ is stochastically super-modular in $k_{\sigma t}$ for any $x_{\sigma t}$ and $k_{-\sigma t}$. These conditions are trivially satisfied for $\operatorname{Pr}\left(\eta_{\sigma}^{\prime} \mid e_{\sigma}^{\prime}\right), \operatorname{Pr}\left(e_{-\sigma 0}^{\prime} \mid e_{\sigma}^{\prime}\right)$ from the conditional independence assumption. Therefore,

$$
\begin{aligned}
& M\left(x_{0}^{\prime} \mid x, D_{\sigma}\left(k_{\sigma t}^{\prime} \vee k_{\sigma t}, x_{t}, k_{-\sigma t}\right)\right)=\operatorname{Pr}\left(e_{\sigma}^{\prime} \mid x_{f}, x_{m}, D_{s}\left(k_{\sigma t}^{\prime} \vee k_{\sigma t}, x_{t}, k_{-\sigma t}\right) \operatorname{Pr}\left(\eta_{\sigma}^{\prime} \mid e_{\sigma}^{\prime}\right) \operatorname{Pr}\left(e_{-\sigma 0}^{\prime} \mid e_{\sigma}^{\prime}\right)\right. \\
= & \operatorname{Pr}\left(e_{\sigma}^{\prime} \mid x_{f}, x_{m}, D_{s}^{\prime}\left(k_{\sigma t}^{\prime}, x_{t}, k_{-\sigma t}\right)\right) \operatorname{Pr}\left(\eta_{\sigma}^{\prime} \mid e_{\sigma}^{\prime}\right), \operatorname{Pr}\left(e_{-\sigma 0}^{\prime} \mid e_{\sigma}^{\prime}\right)=M\left(x_{0}^{\prime} \mid x, D_{s}\left(k_{\sigma t}^{\prime}, x_{t}, k_{-\sigma t}\right)\right)
\end{aligned}
$$

And similarly $M\left(x_{0}^{\prime} \mid x, D_{\sigma}\left(k_{\sigma t}^{\prime} \wedge k_{\sigma t}, x_{t}, k_{-\sigma t}\right)\right)=M\left(x_{0}^{\prime} \mid x, D_{s}\left(k_{\sigma t}, x_{t}, k_{-\sigma t}\right)\right)$.Thus,

$$
P_{F t}\left(\widehat{X}_{0} \mid k_{\sigma t}^{\prime} \vee k_{\sigma t}, k_{-\sigma t}, x_{\sigma t}\right)=\sum_{x^{\prime} \subseteq \widehat{X}_{0}} M\left(x_{0}^{\prime} \mid x, D_{\sigma}\left(k_{\sigma t}^{\prime} \vee k_{\sigma t}, x_{t}, k_{-\sigma t}\right)\right)=\sum_{x^{\prime} \subseteq \widehat{X}_{0}} M\left(x_{0}^{\prime} \mid x, D_{s}^{\prime}\right)
$$

and similarly $P_{F t}\left(\widehat{X_{0}} \mid k_{\sigma t}^{\prime} \wedge k_{\sigma t}, k_{-\sigma t}, x_{\sigma t}\right)=P_{F t}\left(\widehat{X}_{0} \mid k_{\sigma t}^{\prime} \wedge k_{\sigma t}, k_{-\sigma t}, x_{\sigma t}\right)$ for any set $\widehat{X}_{0} \subseteq X$.
Next we need to show that condition Condition (ID) holds. For females, for any $k_{f t}^{\prime} \succeq k_{f t}$, and given any $k_{m t}^{\prime} \succeq k_{m t}, x_{f t}$ the continuation value $v\left(k_{\sigma t}^{\prime}, k_{-\sigma t}, x_{\sigma t}\right)$ has increasing differences for every state $x_{t}$, and age $t \leq T$. First note that that the the per period utility $u\left(k_{\sigma t}, k_{-\sigma t}, x_{\sigma t}\right)$ has increasing differences,

$$
\begin{aligned}
& u\left(k_{\sigma t}^{\prime}, k_{-\sigma t}, x_{\sigma t}\right)-u\left(k_{\sigma t}, k_{-\sigma t}, x_{\sigma t}\right)=\alpha_{\sigma}\left(w_{f t}\left(k_{\sigma t}^{\prime}\right)-w_{f t}\left(k_{\sigma t}\right)\right)+\alpha_{f N}\left(b_{t}\left(k_{\sigma t}^{\prime}\right)-b_{t}\left(k_{\sigma t}\right)\right)+ \\
& \theta_{f k_{t}^{\prime}}-\theta_{f k_{t}}+\varepsilon_{k_{\sigma t}^{\prime}}-\varepsilon_{k_{\sigma t}}=u\left(k_{\sigma t}^{\prime}, k_{-\sigma t}^{\prime}, x_{\sigma t}\right)-u\left(k_{\sigma t}, k_{-\sigma t}^{\prime}, x_{\sigma t}\right)
\end{aligned}
$$

Similarly for males for any $k_{m t}^{\prime} \succeq k_{m t}$, and given any $k_{f t}^{\prime} \succeq k_{f t}, x_{m t}$

$$
\begin{aligned}
& u\left(k_{\sigma t}^{\prime}, k_{-\sigma t}, x_{\sigma t}\right)-u\left(k_{\sigma t}, k_{-\sigma t}, x_{\sigma t}\right)=\alpha_{\sigma}\left(w_{f t}\left(k_{\sigma t}^{\prime}\right)-w_{f t}\left(k_{\sigma t}\right)\right)+ \\
& \theta_{f k_{t}^{\prime}}-\theta_{f k_{t}}+\varepsilon_{k_{\sigma t}^{\prime}}-\varepsilon_{k_{\sigma t}}=u\left(k_{\sigma t}^{\prime}, k_{-\sigma t}^{\prime}, x_{\sigma t}\right)-u\left(k_{\sigma t}, k_{-\sigma t}^{\prime}, x_{\sigma t}\right)
\end{aligned}
$$

We begin by deriving that for period $T$, the conditions for increasing differences in $\left(k_{\sigma t}, k_{-\sigma t}\right)$ of the continuation value. Note that it is also the per period utility, but unlike all other periods, it includes the expected valuations of the children.

$$
\begin{aligned}
& v_{\sigma}\left(k_{\sigma T}^{\prime}, k_{-\sigma T}^{\prime} ; x_{T}\right)-v_{\sigma}\left(k_{\sigma T}, k_{-\sigma T}^{\prime} ; x_{T}\right)=\left(u\left(k_{\sigma T}^{\prime}, k_{-\sigma T}^{\prime}, x_{\sigma T}\right)-u\left(k_{\sigma T}, k_{-\sigma T}^{\prime}, x_{\sigma T}\right)\right)+ \\
& \beta \lambda\left(\frac{\left(N_{\sigma T}+b_{T}^{\prime}\right)^{1-v}}{\left(N_{\sigma T}+b_{T}^{\prime}\right)} \bar{V}_{N \sigma}\left(k_{\sigma t}^{\prime}, k_{-\sigma t}^{\prime} ; x_{T}\right)-\frac{\left(N_{\sigma T}+b_{T}\right)^{1-v}}{\left(N_{\sigma T}+b_{T}\right)} \bar{V}_{N \sigma}\left(k_{\sigma t}, k_{-\sigma t}^{\prime} ; x_{T}\right)\right)
\end{aligned}
$$

We showed above that $u\left(k_{\sigma t}^{\prime}, k_{-\sigma t}^{\prime}, x_{\sigma T}\right)-u\left(k_{\sigma t}, k_{-\sigma t}^{\prime}, x_{\sigma T}\right)$ exhibits increasing differences thus it is suffices to establishes conditions for the second element to exhibit increasing difference, that is that

$$
\begin{aligned}
& \frac{\left(N_{\sigma T}+b_{T}^{\prime}\right)^{1-v}}{\left(N_{\sigma T}+b_{T}^{\prime}\right)} \bar{V}_{N \sigma}\left(k_{\sigma T}^{\prime}, k_{-\sigma T}^{\prime} ; x_{T}\right)-\frac{\left(N_{\sigma T}+b_{T}\right)^{1-v}}{\left(N_{\sigma T}+b_{T}\right)} \bar{V}_{N \sigma}\left(k_{\sigma T}, k_{-\sigma T}^{\prime} ; x_{T}\right) \geq \\
& \frac{\left(N_{\sigma T}+b_{T}^{\prime}\right)^{1-v}}{\left(N_{\sigma T}+b_{T}^{\prime}\right)} \bar{V}_{N \sigma}\left(k_{\sigma T}^{\prime}, k_{-\sigma T} ; x_{T}\right)-\frac{\left(N_{\sigma T}+b_{T}\right)^{1-v}}{\left(N_{\sigma T}+b_{T}\right)} \bar{V}_{N \sigma}\left(k_{\sigma T}, k_{-\sigma T} ; x_{T}\right)
\end{aligned}
$$

First note that labor supply decisions only enter $u\left(k_{\sigma t}^{\prime}, k_{-\sigma t}^{\prime}, x_{\sigma T}\right)-u\left(k_{\sigma t}, k_{-\sigma t}^{\prime}, x_{\sigma T}\right)$, thus, we only need to verify the property for choices $\left(k_{\sigma t}^{\prime} \geq k_{\sigma t}\right)$ and ( $k_{-\sigma t}^{\prime} \geq k_{-\sigma t}$ ) which have higher birth and time spent with children decisions. We begin with $\left(k_{\sigma t}^{\prime} \geq k_{\sigma t}\right)$ and $\left(k_{-\sigma t}^{\prime} \geq k_{-\sigma t}\right)$ for which $k_{\sigma t}^{\prime}, k_{-\sigma t}^{\prime}$ have higher time spent with children (suppose birth decisions are similar). We need to show that

$$
\left[\bar{V}_{N \sigma}\left(k_{\sigma t}^{\prime}, k_{-\sigma t}^{\prime} ; x_{T}\right)-\bar{V}_{N \sigma}\left(k_{\sigma t}^{\prime}, k_{-\sigma t} ; x_{T}\right)\right] \geq\left[\bar{V}_{N \sigma}\left(k_{\sigma t}, k_{-\sigma t}^{\prime} ; x_{T}\right)-\bar{V}_{N \sigma}\left(k_{\sigma t}, k_{-\sigma t} ; x_{T}\right)\right]
$$

Note that $D_{s}\left(k_{\sigma t}, k_{-\sigma t}\right)$, is increasing in $k_{\sigma t}, k_{-\sigma t}$. The above condition can be written as:

$$
\begin{aligned}
& \sum_{s=0}^{T-1}\left[b_{s} \sum_{\sigma} I_{\sigma s} \sum_{x_{0}^{\prime}} V_{\sigma s}\left(x_{0}^{\prime}\right)\left(M\left(x_{0}^{\prime} \mid x_{T}, D_{s}\left(k_{\sigma T}^{\prime}, k_{-\sigma T}^{\prime}\right)\right)-M\left(x_{0}^{\prime} \mid x_{T}, D_{s}\left(k_{\sigma T}, k_{-\sigma T}^{\prime}\right)\right)\right)\right]+ \\
& b_{T} \sum_{\sigma} p_{\sigma} \sum_{x_{0}^{\prime}} V_{\sigma T}\left(x_{0}^{\prime}\right)\left(M\left(x_{0}^{\prime} \mid x_{T}, D_{T}\left(k_{\sigma T}^{\prime}, k_{-\sigma T}^{\prime}\right)\right)-M\left(x_{0}^{\prime} \mid x_{T}, D_{T}\left(k_{\sigma T}, k_{-\sigma T}^{\prime}\right)\right)\right) \geq \\
& \sum_{s=0}^{T-1}\left[b_{s} \sum_{\sigma} I_{\sigma s} \sum_{x_{0}^{\prime}} V_{\sigma s}\left(x_{0}^{\prime}\right)\left(M\left(x_{0}^{\prime} \mid x_{T}, D_{s}\left(k_{\sigma T}^{\prime}, k_{-\sigma T}\right)\right)-M\left(x_{0}^{\prime} \mid x_{T}, D_{s}\left(k_{\sigma T}, k_{-\sigma T}\right)\right)\right)\right]+ \\
& b_{T} \sum_{\sigma} p_{\sigma} \sum_{x_{0}^{\prime}} V_{\sigma T}\left(x_{0}^{\prime}\right)\left(M\left(x_{0}^{\prime} \mid x_{T}, D_{T}\left(k_{\sigma T}^{\prime}, k_{-\sigma T}\right)\right)-M\left(x_{0}^{\prime} \mid x_{T}, D_{T}\left(k_{\sigma T}, k_{-\sigma T}\right)\right)\right)
\end{aligned}
$$

Thus as long as $M\left(x_{0}^{\prime} \mid x_{f}, x_{m}, D_{s}\left(k_{\sigma t}, k_{-\sigma t}^{\prime}\right)\right.$ exhibits increasing differences in $D$, the condition is satisfied. Thus, as long as $V_{\sigma s}\left(x_{0}^{\prime}\right)$ is weakly increasing in $\eta_{\sigma}^{\prime}, E d_{-\sigma 0}^{\prime}, E d_{\sigma}^{\prime}$ and $\operatorname{Pr}\left(\eta_{\sigma}^{\prime} \mid E d_{\sigma}^{\prime}\right) \operatorname{Pr}\left(E d_{-\sigma 0}^{\prime} \mid E d_{\sigma}^{\prime}\right)$ weakly increase in $E d_{\sigma}^{\prime}$ the condition is that $\operatorname{Pr}\left(E d_{\sigma}^{\prime} \mid x_{f}, x_{m}, D_{s}\right)$ satisfied increasing differences which is satisfied by Assumption 1.. Therefore the valuation function is weakly increasing in $x_{0}^{\prime}$.

Next consider $k_{\sigma t}^{\prime} \geq k_{\sigma t}$ and $k_{-\sigma t}^{\prime} \geq k_{-\sigma t}$ for which let $b_{T}^{\prime}=1$ and $b_{T}=0$. We need to show that given the highest difference in time spent with kids in one period, the decline in the mean quality of any existing child is small enough. We already know that we have increasing differences for all other dimensions of the state space except for birth. Denote by $\underline{d}$ and $\bar{d}$ the lowest and highest investment level possible in one period by one spouse. Suppose spouse $\sigma$ strategies $k_{\sigma t}^{\prime} \geq k_{\sigma t}$ involve same $d$ and only differ by birth decisions. Suppose $k_{-\sigma t}^{\prime}$ involve $\bar{d}$ and that $k_{-\sigma t}$ involve $\underline{d}$, the condition needed for increasing differences is therefore

$$
\frac{\left(N_{\sigma T}+1\right)^{1-v}}{\left(N_{\sigma T}+1\right)}\left[\bar{V}_{N \sigma}\left(k_{\sigma t}^{\prime}, k_{-\sigma t}^{\prime} ; x_{T}\right)-\bar{V}_{N \sigma}\left(k_{\sigma t}^{\prime}, k_{-\sigma t} ; x_{T}\right)\right] \geq \frac{\left(N_{\sigma T}\right)^{1-v}}{N_{\sigma T}}\left[\bar{V}_{N \sigma}\left(k_{\sigma t}, k_{-\sigma t}^{\prime} ; x_{T}\right)-\bar{V}_{N \sigma}\left(k_{\sigma t}, k_{-\sigma t} ; x_{T}\right)\right]
$$

Define the average quality of the stock of children:

$$
\begin{aligned}
\widehat{V}_{N_{T}}\left(k_{\sigma t}, k_{-\sigma t}^{\prime} ; x_{T}\right) \equiv & \frac{1}{N_{\sigma T}+1} \sum_{s=0}^{T-1}\left[b_{s} \sum_{\sigma} I_{\sigma s} \sum_{x_{0}^{\prime}} V_{\sigma s}\left(x_{0}^{\prime}\right) M\left(x_{0}^{\prime} \mid x_{f}, x_{m}, D_{s}\left(\frac{d}{N_{\sigma T}}, \frac{\bar{d}}{N_{\sigma T}}\right)\right)\right] \\
& +\frac{1}{N_{\sigma T}+1} \sum_{\sigma} p_{\sigma} \sum_{x_{0}^{\prime}} V_{\sigma T}\left(x_{0}^{\prime}\right) M\left(x_{0}^{\prime} \mid x_{f}, x_{m}, D_{T}\left(\frac{d}{N_{\sigma T}+1}, \frac{\bar{d}}{N_{\sigma T}+1}\right)\right)
\end{aligned}
$$

Then sufficient conditions for increasing differences are:

$$
\begin{array}{r}
\frac{\left(N_{\sigma T}+1\right)^{1-v}}{\left(N_{\sigma T}+1\right)}\left[\left(N_{\sigma T}+1\right)\left(\widehat{V}_{N_{T}}\left(\frac{d}{N_{\sigma T}+1}, \frac{\bar{d}}{N_{\sigma T}+1} ; x_{T}\right)-\widehat{V}_{N_{T}}\left(\frac{d}{N_{\sigma T}+1}, \frac{\underline{d}}{N_{\sigma T}+1} ; x_{T}\right)\right)\right] \geq \\
\frac{\left(N_{\sigma T}\right)^{1-v}}{N_{\sigma T}}\left[N_{\sigma T}\left(\widehat{V}_{N_{T}}\left(\frac{d}{N_{\sigma T}}, \frac{\bar{d}}{N_{\sigma T}} ; x_{T}\right)-\widehat{V}_{N_{T}}\left(\frac{d}{N_{\sigma T}}, \frac{\underline{d}}{N_{\sigma T}} ; x_{T}\right)\right)\right]
\end{array}
$$

Rearranging the condition for all $0 \leq N_{\sigma T} \leq T$ :

$$
\left(\frac{N_{\sigma T}+1}{N_{\sigma T}}\right)^{1-v} \geq \frac{\left(\widehat{V}_{N_{T}}\left(\frac{d}{N_{\sigma T}}, \frac{\bar{d}}{N_{\sigma T}} ; x_{T}\right)-\widehat{V}_{N_{T}}\left(\frac{d}{N_{\sigma T}}, \frac{d}{N_{\sigma T}} ; x_{T}\right)\right)}{\left(\widehat{V}_{N_{T}}\left(\frac{d}{N_{\sigma T}+1}, \frac{\bar{d}}{N_{\sigma T}+1} ; x_{T}\right)-\widehat{V}_{N_{T}}\left(\frac{d}{N_{\sigma T}+1}, \frac{d}{N_{\sigma T}+1} ; x_{T}\right)\right)}
$$

That is, the highest ratio of the right hand side is obtained for the largest difference in time investment of a spouse, for a one period investment, and a strategy of an individual in which the higher one has birth. The conditions says that the increase difference in average quality of a child cause be investment difference of $\frac{\bar{d}-\underline{d}}{N_{\sigma T}}$ versus $\frac{\bar{d}-\underline{d}}{N_{\sigma T}+1}$ is bounded by the left hand side (which takes the lowest value at $N_{\sigma T}=T$ by concavity assumption). Note that this assumption can be translated to an assumption on the transition function $M\left(x_{0}^{\prime} \mid x_{f}, x_{m}, D_{s}\left(k_{\sigma t}^{\prime}, k_{-\sigma t}^{\prime}\right)-M\left(x_{0}^{\prime} \mid x_{f}, x_{m}, D_{s}\left(k_{\sigma t}, k_{-\sigma t}^{\prime}\right)\right.\right.$. We already assumed that the marginal increase in investment in a child is weakly increasing in the existing stock of investment (and the spouse's investment ), thus the left hand side of the above inequality is weakly larger than 1 . The additional condition therefore bounds the increase in probability of outcomes as a function of a one period investment. In addition valuations functions of the child are weakly increasing in parental investment. Since consumption rises in wages and since education increase expected wage as well as spouses' education (assortative matching) and expected wage, this is satisfied.

Finally solving backwards, we established conditions for increasing differences of $v_{\sigma}\left(k_{\sigma T}^{\prime}, k_{-\sigma T}^{\prime} ; x_{T}\right)$. Assuming that $F\left(x_{t+1}^{\prime} \mid x_{t}, k_{t}\right)$ satisfies stochastic increasing differences, we show that for period $T-1$, the continuation value $v_{\sigma}\left(k_{\sigma T-1}, k_{-\sigma T-1} ; x_{T-1}\right)$ satisfies increasing differences in ( $k_{\sigma T-1}, k_{-\sigma T-1}$ ). Thus, since $u\left(k_{\sigma T-1}^{\prime}, k_{-\sigma T-1}^{\prime}, x_{T-1}\right)$ satisfies increasing differences, $F\left(x_{T} \mid x_{T-1}, k_{T-1}^{\prime}\right)$ satisfies stochastic increasing differences and $v_{\sigma}\left(k_{\sigma T}^{\prime}, k_{-\sigma T}^{\prime} ; x_{T}\right)$ also satisfies stochastic increasing differences, it is left to show that $p\left(k_{T} \mid x_{T}\right)$ in equation 14 satisfies stochastic increasing differences. Because $\varepsilon^{\prime} s$ are conditionally independent across spouses, time and choices, it suffices to show that the individual choice probabilities satisfy increasing differences:

$$
p\left(k_{\sigma T}^{\prime} \mid k_{-\sigma T}^{\prime}, x_{T}\right)=\int\left[\prod_{k_{\sigma T}^{\prime} \neq k_{\sigma T}} 1\left\{v_{\sigma}\left(k_{\sigma T}^{\prime}, k_{-\sigma T}^{\prime} ; x_{T}\right)-v_{\sigma}\left(k_{\sigma T}, k_{-\sigma T}^{\prime} ; x_{T}\right) \geq \varepsilon_{\sigma k^{\prime} t}-\varepsilon_{\sigma k^{\prime} t}\right\}\right] d F_{\varepsilon}
$$

That is

$$
\sum_{k_{\sigma T}^{\prime}} p\left(k_{\sigma T}^{\prime} \mid k_{-\sigma T}^{\prime}, x_{T}\right)-\sum_{k_{\sigma T}} p\left(k_{\sigma T} \mid k_{-\sigma T}^{\prime}, x_{T}\right) \geq \sum_{k_{\sigma T}^{\prime}} p\left(k_{\sigma T}^{\prime} \mid k_{-\sigma T}, x_{T}\right)-\sum_{k_{\sigma T}} p\left(k_{\sigma T} \mid k_{-\sigma T}, x_{T}\right)
$$

Define

$$
v_{\sigma}\left(k_{\sigma T}^{\prime}, k_{-\sigma T}^{\prime} ; x_{T}\right)-v_{\sigma}\left(k_{\sigma T}, k_{-\sigma T}^{\prime} ; x_{T}\right) \equiv \Delta v_{\sigma}\left(k_{\sigma T}^{\prime}, k_{\sigma T} ; k_{-\sigma T}^{\prime}, x_{T}\right)
$$

Thus, we need to show that

$$
\begin{aligned}
& \int_{\varepsilon}\left[\prod_{k_{\sigma T}^{\prime} \neq k_{\sigma T}} 1\left\{\Delta v_{\sigma}\left(k_{\sigma T}^{\prime}, k_{\sigma T} ; k_{-\sigma T}^{\prime}, x_{T}\right) \geq \varepsilon_{\sigma k^{\prime} t}-\varepsilon_{\sigma k t}\right\}\right] d F_{\varepsilon}- \\
& \int_{\varepsilon}\left[\prod_{k_{\sigma T}^{\prime} \neq k_{\sigma T}} 1\left\{\Delta v_{\sigma}\left(k_{\sigma T}^{\prime}, k_{\sigma T} ; k_{-\sigma T}, x_{T}\right) \geq \varepsilon_{\sigma k^{\prime} t}-\varepsilon_{\sigma k t}\right\}\right] d F_{\varepsilon}= \\
& \int_{\varepsilon}\left[\prod_{k_{\sigma T}^{\prime} \neq k_{\sigma T}} 1\left\{\Delta v_{\sigma}\left(k_{\sigma T}^{\prime}, k_{\sigma T} ; k_{-\sigma T}^{\prime}, x_{T}\right)-\Delta v_{\sigma}\left(k_{\sigma T}^{\prime}, k_{\sigma T} ; k_{-\sigma T}, x_{T}\right) \geq 0\right\}\right] d F_{\varepsilon}
\end{aligned}
$$

Since for all $\left(k_{\sigma T}^{\prime}, k_{-\sigma T}^{\prime},\right) \geq\left(k_{\sigma T}, k_{-\sigma T},\right)$,

$$
\Delta v_{\sigma}\left(k_{\sigma T}^{\prime}, k_{\sigma T} ; k_{-\sigma T}^{\prime}, x_{T}\right)-\Delta v_{\sigma}\left(k_{\sigma T}^{\prime}, k_{\sigma T} ; k_{-\sigma T}, x_{T}\right) \geq 0
$$

And from conditional independence of $\varepsilon^{\prime} s, p\left(k_{\sigma T}^{\prime} \mid k_{-\sigma T}^{\prime}, x_{T}\right)$ has increasing differences. By backwards induction, the same proof applies for all $t<T-1$ thus the continuation value $v_{\sigma}\left(k_{\sigma T}, k_{-\sigma T}^{\prime} ; x_{T}\right)$ satisfies increasing differences for all $0 \leq t \leq T$.

By backwards induction, the same proof applies for all $t<T-1$ thus the continuation value $v_{\sigma}\left(k_{\sigma T}, k_{-\sigma T}^{\prime} ; x_{T}\right)$ satisfies increasing differences for all $0 \leq t \leq T$.

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Table 1 : Summary Statistics
(Standard Deviation are in parentheses)

|  | $(1)$ |  | $(2)$ |  |  | $(3)$ |  |
| :--- | :---: | ---: | :---: | ---: | ---: | ---: | :---: |
| Variable | N | Mean | N | Mean | N | Mean |  |
|  |  |  |  |  |  |  |  |
| Female | 115,280 | 0.545 | 86,302 | 0.552 | 28,978 | 0.522 |  |
| Black | 115,280 | 0.223 | 86,302 | 0.202 | 28,978 | 0.286 |  |
| Married | 115,280 | 0.381 | 86,302 | 0.465 | 28,978 | 0.131 |  |
| Age | 115,280 | 26.155 | 86,302 | 27.968 | 28,978 | 20.756 |  |
|  |  | $(7.699)$ |  | $(7.872)$ |  | $(3.511)$ |  |
| Education | 115,280 | 13.438 | 86,302 | 13.516 | 28,978 | 13.209 |  |
|  |  | $(2.103)$ |  | $(2.138)$ |  | $(1.981)$ |  |
| Number of children | 115,280 | 0.616 | 86,302 | $(0.766)$ | 28,978 | 0.167 |  |
|  |  | $(0.961)$ |  | $(1.028)$ |  | $(0.507)$ |  |
| Annual labor income | 114,871 | 16,115 | 86,137 | 19,552 | 28,734 | 5,811 |  |
|  |  | $(24,622)$ |  | $(26,273)$ |  | $(14,591)$ |  |
| Annual labor market hours | 114,899 | 915 | 86,185 | 1078 | 28,714 | 424 |  |
|  |  | $(1041)$ |  | $(1051)$ |  | $(841)$ |  |
| Annual housework hours | 66,573 | 714 | 58,564 | $(724)$ | 8,009 | 641 |  |
|  |  | $(578)$ |  | 585 |  | $(524)$ |  |
| Annual time spent on children | 115,249 | 191 | 86,275 | 234 | 28,974 | 63.584 |  |

Source: Data from the Family-Individual File of the Michigan Panel Study of Income Dynamics (PSID), and include individuals surveyed between 1968 and 1997. Column (1) contains the summary statistics for the full sample; column (2) contains the summary statistics for the parents generation; column (3) contains the summary statistics of the off spring of the parents in column (2). Annual labor income is measured in 2005 dollars. Education measures year of completed education. There are less observations for annual housework hours than time spent on children because single individuals with no child are coded as missing for housework hours but by definition are set to zero for time spent on children

Table 2: Discrete Choice Set of Structural Model

| Decisions |  |  |  |
| :---: | :---: | :---: | :---: |
| Choice | Labor Market Work | Child Birth | Child Care Hours |
| Female |  |  |  |
| 1 | None | None | None |
| 2 | Part time | None | None |
| 3 | Full Time | None | None |
| 4 | Full Time | Yes | None |
| 5 | None | None | Low |
| 6 | Part Time | None | Low |
| 7 | Full Time | None | Low |
| 8 | None | Yes | Low |
| 9 | Part Time | Yes | Low |
| 10 | Full Time | Yes | Low |
| 11 | None | None | High |
| 12 | Part Time | None | High |
| 13 | Full Time | None | High |
| 14 | None | Yes | High |
| 15 | Part Time | Yes | High |
| 16 | Full Time | Yes | High |
| Male |  |  |  |
| 1 | None | NA | None |
| 2 | Part Time | NA | None |
| 3 | Full Time | NA | None |
| 4 | None | NA | Low |
| 5 | Part Time | NA | Low |
| 6 | Full Time | NA | Low |
| 7 | None | NA | High |
| 8 | Part Time | NA | High |
| 9 | Full Time | NA | High |

Table 3: 3SLS System Estimation the Education Production Function (Standard Errors in parenthesis; Exclude class is Less than High School)

| Variable | High <br> School | Some <br> College | College |
| :--- | ---: | ---: | ---: |
| High School Father | 0.008 | 0.023 | 0.155 |
| Some College Father | $(0.068)$ | $(0.104)$ | $(0.128)$ |
| College Father | -0.012 | 0.057 | 0.162 |
|  | $(0.047)$ | $(0.074)$ | $(0.086)$ |
| High School Mother | -0.014 | 0.021 | 0.229 |
|  | $0.071)$ | $(0.110)$ | $(0.135)$ |
| Some College Mother | $(0.057)$ | 0.093 | 0.083 |
|  | -0.016 | $0.089)$ | $(0.107)$ |
| College Mother | $(0.054)$ | $(0.036$ | -0.089 |
| Mother's Time | -0.122 | $0.098)$ | 0.03 |
|  | $(0.076)$ | $(0.116)$ | $(0.140)$ |
| Father's Time | -0.091 | -0.048 | 0.299 |
|  | $(0.075)$ | $(0.114)$ | $(0.130$ |
| Mother's Labor Income | 0.153 | 0.273 | -0.108 |
|  | $(0.069)$ | $(0.103)$ | $(0.131)$ |
| Father's Labor Income | 0.021 | -0.014 | -0.004 |
| Female | $0.025)$ | $(0.039)$ | $(0.048)$ |
| Black | 0.015 | 0.018 | -0.023 |
|  | $(0.010)$ | $(0.016)$ | $(0.020)$ |
| Constant | 0.034 | 0.158 | 0.110 |
|  | $(0.030)$ | $(0.045)$ | $(0.056)$ |
| Observations | -0.227 | -0.236 | 0.324 |

Source: Data from the Family-Individual File of the Michigan Panel Study of Income Dynamics (PSID), and include individuals surveyed between 1968 and 1997. Instruments: Mother's and father's labor market hours over the child's first 8 years of life, linear and quadratic terms of mother's and fathers age when the child was 5 years old.

Table 3B: The probability of white male child's education outcome

|  |  | CHILD'S EDUCATION |  |  |  |  |
| :--- | :--- | :--- | :---: | :---: | :---: | :---: |
| Mother Education | Father's Education | Investment | Less than <br> high school | High <br> School | Some <br> College | College <br> Graduate |
| Less than high school | Less than high school | NO | 0.14 | 0.86 | 0.00 | 0.00 |
| High School | High School | NO | 0.13 | 0.87 | 0.00 | 0.00 |
| Some College | Some College | NO | 0.16 | 0.84 | 0.00 | 0.00 |
| College Graduate | College Graduate | NO | 0.29 | 0.71 | 0.00 | 0.00 |
|  |  |  |  |  |  | 0.0 .24 |
| Less than high school | Less than high school | AVG | 0.14 | 0.59 | 0.03 |  |
| High School | High School | AVG | 0.13 | 0.48 | 0.12 | 0.27 |
| Some College | Some College | AVG | 0.15 | 0.36 | 0.14 | 0.34 |
| College Graduate | College Graduate | AVG | 0.00 | 0.00 | 0.21 | 0.79 |
|  |  |  |  |  |  |  |
| Less than high school | Less than high school | MAX | 0.00 | 0.00 | 0.23 | 0.77 |
| High School | High School | MAX | 0.00 | 0.00 | 0.00 | 1.00 |
| Some College | Some College | MAX | 0.00 | 0.00 | 0.00 | 1.00 |
| College Graduate | College Graduate | MAX | 0.00 | 0.00 | 0.00 | 1.00 |

Table 4: Estimates of Earnings Equation: Dependent Variable: Log of Yearly Earnings
(Standard Errors in Parenthesis)

| Variable | Estimate | Variable | Estimate | Variable | Estimate |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Demographic Var | bes |  |  | Fixed Effect |  |
| Age Squared | $\begin{gathered} -4.0 \mathrm{e}-4 \\ (1.0 \mathrm{e}-5) \end{gathered}$ | Female x Full time work | $\begin{aligned} & -0.125 \\ & (0.010) \end{aligned}$ | Black | $\begin{aligned} & -0.154 \\ & (0.009) \end{aligned}$ |
| Age x LHS | $\begin{gathered} 0.037 \\ (0.002) \end{gathered}$ | Female x Full time work (t-1) | $\begin{gathered} 0.110 \\ (0.010) \end{gathered}$ | Female | $\begin{aligned} & -0.484 \\ & (0.007) \end{aligned}$ |
| Age x HS | $\begin{gathered} 0.041 \\ (0.001) \end{gathered}$ | Female x Full time work (t-2) | $\begin{gathered} 0.025 \\ (0.010) \end{gathered}$ | HS | $\begin{gathered} 0.136 \\ (0.005) \end{gathered}$ |
| Age x SC | $\begin{gathered} 0.050 \\ (0.001) \end{gathered}$ | Female x Full time work (t-3) | $\begin{gathered} 0.010 \\ (0.010) \end{gathered}$ | SC | $\begin{gathered} 0.122 \\ (0.006) \end{gathered}$ |
| Age x COL | $\begin{gathered} 0.096 \\ (0.001) \\ \hline \end{gathered}$ | Female x Full time work (t-4) | $\begin{gathered} 0.013 \\ (0.010) \end{gathered}$ | COL | $\begin{gathered} 0.044 \\ (0.006) \end{gathered}$ |
| Current and Lags of P | icipation | Female x Part time work (t-1) | 0.150 | Black x HS | -0.029 |
| Full time work | $\begin{gathered} \hline 0.938 \\ (0.010) \end{gathered}$ | Female x Part time work (t-2) | $\begin{gathered} (0.010) \\ 0.060 \end{gathered}$ | Black x SC | $\begin{gathered} (0.010) \\ 0.033 \end{gathered}$ |
| Full time work (t-1) | $\begin{gathered} 0.160 \\ (0.009) \end{gathered}$ | Female x Part time work (t-3) | $\begin{gathered} (0.010) \\ 0.040 \end{gathered}$ | Black x COL | $\begin{gathered} (0.008) \\ 0.001 \end{gathered}$ |
| Full time work (t-2) | $\begin{gathered} 0.044 \\ (0.010) \end{gathered}$ | Female x Part time work (t-4) | $\begin{aligned} & (0.010) \\ & -0.002 \end{aligned}$ | Female x HS | $\begin{gathered} (0.011) \\ -0.054 \end{gathered}$ |
| Full time work (t-3) | $\begin{gathered} 0.025 \\ (0.010) \end{gathered}$ | Individual Specific Effects | $\begin{gathered} (0.010) \\ \text { Yes } \end{gathered}$ | Female x SC | $\begin{gathered} (0.008) \\ 0.049 \end{gathered}$ |
| Full time work (t-4) | $\begin{gathered} 0.040 \\ (0.010) \end{gathered}$ |  |  | Female x COL | $\begin{gathered} (0.006) \\ 0.038 \end{gathered}$ |
| Part time work (t-1) | $\begin{aligned} & -0.087 \\ & (0.010) \end{aligned}$ |  |  | Constant | $\begin{gathered} (0.007) \\ 0.167 \end{gathered}$ |
| Part time work (t-2) | $\begin{aligned} & -0.077 \\ & (0.010) \end{aligned}$ |  |  |  | (0.005) |
| Part time work (t-3) | $\begin{aligned} & -0.070 \\ & (0.010) \end{aligned}$ |  |  |  |  |
| Part time work (t-4) | $\begin{array}{r} -0.010 \\ (0.010) \\ \hline \end{array}$ | Hausman Statistics Hausman P-Value | $\begin{aligned} & \hline 2296 \\ & 0.000 \\ & \hline \end{aligned}$ |  |  |
| N |  |  | 134,007 |  |  |
| Number of Individuals |  |  | 14,018 |  |  |
| R-squared |  |  | 0.44 |  | 0.278 |

Source: Data from the Family-Individual File of the Michigan Panel Study of Income Dynamics (PSID), and include individuals surveyed between 1968 and 1997. Yearly earnings is measured in 2005 dollars. LHS is a dummy variable indicating that the individual has completed education of less than high school; HS is a dummy variable indicating that the individual has completed education of high school but college; SC is a dummy variable indicating that the individual has completed education of greater than high school but is not a college graduate; COL is a dummy variable indicating that the individual has completed education of at least a college graduate.

Table 5: Logit Coefficient Estimates Transition from Single to Married
Dependent Variable: Dummy equal one if married and zero otherwise
(Standard Error in parenthesis)

| State Variables |  |  | Choice Variables |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Variables | Female | Male | Choice | Female | Male |
| Black | -1.339 | -1.952 | 2 | 1.365 | 0.951 |
|  | (0.066) | (0.168) |  | (0.132) | (0.289) |
| High School | 0.300 | 0.172 | 3 | 1.005 | 1.774 |
|  | (0.101) | (0.153) |  | (0.092) | (0.134) |
| Some College | 0.108 | 0.029 | 4 | 1.552 | 0.320 |
|  | (0.104) | (0.158) |  | (0.333) | (1.072) |
| College Graduate | -0.297 | 0.167 | 5 | 0.820 |  |
|  | (0.109) | (0.157) |  | (0.205) |  |
| Age | 0.324 | 0.408 | 6 | 1.251 | 1.646 |
|  | (0.040) | (0.064) |  | (0.237) | (0.299) |
| Age Sq | -0.006 | -0.007 | 7 | 1.249 | 0.622 |
|  | (0.001) | (0.001) |  | (0.162) | (1.063) |
| No. of Children | -0.338 | 1.849 | 8 | 1.303 | 1.410 |
|  | (0.205) | (0.412) |  | (0.240) | (1.115) |
| No. of Children Sq | 0.078 | -0.216 | 9 | 1.555 | 2.406 |
|  | (0.069) | (0.144) |  | (0.331) | (0.301) |
| Part time work (t-1) | -0.268 | -0.128 | 10 | 1.183 |  |
|  | (0.135) | (0.270) |  | (0.411) |  |
| Part time work (t-2) | 0.060 | -0.399 | 11 | 1.210 |  |
|  | (0.130) | (0.289) |  | (0.223) |  |
| Part time work (t-3) | 0.143 | -0.201 | 12 | 1.754 |  |
|  | (0.132) | (0.361) |  | (0.301) |  |
| Part time work (t-4) | -0.105 | -0.144 | 13 | 1.450 |  |
|  | (0.136) | (0.358) |  | (0.209) |  |
| Full time work (t-1) | -0.264 | 0.025 | 14 | 1.400 |  |
|  | (0.102) | (0.159) |  | (0.243) |  |
| Full time work (t-2) | 0.166 | -0.530 | 15 | 1.763 |  |
|  | (0.106) | (0.178) |  | (0.431) |  |
| Full time work (t-3) | -0.129 | 0.100 | 16 | 1.781 |  |
|  | (0.113) | (0.207) |  | (0.309) |  |
| Full time work (t-4) | -0.146 | 0.014 |  |  |  |
|  | (0.101) | (0.189) |  |  |  |
| Age of 1st Child | $\begin{gathered} 0.026 \\ (0.018) \end{gathered}$ | $\begin{array}{r} 0.008 \\ (0.032) \end{array}$ |  |  |  |
| Age of 2nd Child | 0.007 | -0.082 |  |  |  |
|  | (0.029) | (0.050) |  |  |  |
| Age of 3rd Child | 0.030 |  |  |  |  |
|  | (0.050) |  |  |  |  |
| Age of 4th Child | $\begin{gathered} 0.170 \\ (0.128) \end{gathered}$ |  |  |  |  |
| Time with 1st Child | -0.010 | -0.013 |  |  |  |
|  | (0.032) | (0.058) |  |  |  |
| Time with 2nd Child | -0.020 | -0.356 |  |  |  |
|  | (0.044) | (0.116) |  |  |  |
| Time with 3nd Child | -0.046 |  |  |  |  |
|  | (0.070) |  |  |  |  |
| Time with 4th Child | -0.316 |  |  |  |  |
|  | (0.184) |  | Constant | -6.527 | -9.457 |
| No. of Female Children | -0.053 | -0.111 |  | (0.498) | (0.810) |
|  | (0.073) | (0.179) | N | 30,875 | 30,492 |

Source: Data from the Family-Individual File of the Michigan Panel Study of Income Dynamics (PSID) between 1968 and 1997. Choice 5 for male is deterministic and is excluded; meaning if single male chose to work part time and supply low child care hours he will get married next period with probability one.

Table 6: Logit Coefficient Estimates Transition from Married to Married Dependent Variable: Dummy equal one if married and zero otherwise
(Standard Error in parenthesis)

| Variables | State VariablesIndividual |  | Spouse |  | Choice | Choice Variables |  |  | Spouse |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Female | Male | Female | Male |  | Female | Male | Female | Male |
| Black | -0.825 | -0.397 |  |  | 2 | -0.483 | 1.042 | 0.488 | 2.619 |
|  | (0.098) | (0.289) |  |  |  | (0.197) | (0.553) | (0.159) | (0.527) |
| High School | 0.037 | 0.038 | 0.019 | -0.407 | 3 | -0.665 | 1.112 | 1.860 | 3.525 |
|  | (0.130) | (0.224) | (0.111) | (0.271) |  | (0.158) | (0.408) | (0.122) | (0.330) |
| Some College | -0.118 | 0.223 | 0.129 | -0.610 | 4 | -0.213 | 0.518 | 0.136 |  |
|  | (0.137) | (0.240) | (0.121) | (0.284) |  | (0.514) | (1.085) | (0.248) |  |
| College Graduate | 0.161 | 0.431 | 0.576 | -0.552 | 5 | -0.034 |  | 0.012 | 3.508 |
|  | (0.164) | (0.258) | (0.146) | (0.313) |  | (0.224) |  | (0.253) | (0.345) |
| Age | -0.155 | -0.047 | 0.190 | -0.136 | 6 | -0.041 | 0.673 | 2.114 | 3.875 |
|  | (0.067) | (0.140) | (0.053) | (0.169) |  | (0.238) | (0.434) | (0.163) | (0.456) |
| Age Square | 0.003 | 0.000 | -0.003 | 0.002 | 7 | -0.461 | -0.536 | 0.814 | 3.745 |
|  | (0.001) | (0.002) | (0.001) | (0.003) |  | (0.193) | (0.616) | (0.296) | (0.279) |
| No. of Children | -0.349 | -0.637 |  |  | 8 | -0.125 | 0.553 | 0.378 | 2.759 |
|  | (0.179) | (0.425) |  |  |  | (0.257) | (0.820) | (0.272) | (0.528) |
| No. of Children Sq | 0.039 | 0.146 |  |  | 9 | -0.269 | 0.894 | $1.654$ | 3.020 |
|  | (0.053) | (0.150) |  |  |  | $(0.285)$ | (0.451) | $(0.164)$ | (0.769) |
| Part time work (t-1) | -0.207 | 0.480 | 0.037 | 1.024 | 10 | -0.034 |  |  | 3.273 |
|  | (0.128) | (0.473) | (0.184) | (0.223) |  | (0.336) |  |  | (0.552) |
| Part time work (t-2) | 0.121 | -0.422 | 0.025 | -0.496 | 11 | 0.463 |  |  | 2.273 |
|  | (0.136) | (0.403) | (0.202) | (0.219) |  | (0.232) |  |  | (0.220) |
| Part time work (t-3) | -0.126 | 0.295 | 0.277 | -0.232 | 12 | -0.063 |  |  | 2.728 |
|  | (0.144) | (0.429) | (0.234) | (0.208) |  | (0.248) |  |  | (0.320) |
| Part time work (t-4) | -0.140 | -0.649 | 0.737 | -0.283 | 13 | -0.304 |  |  | 3.273 |
|  | (0.135) | (0.399) | (0.260) | (0.197) |  | (0.219) |  |  | (0.317) |
| Full time work ( $\mathrm{t}-1$ ) | -0.264 | -0.098 | -0.049 | 1.830 | 14 | 0.296 |  |  | 2.592 |
|  | (0.119) | (0.411) | (0.112) | (0.226) |  | (0.258) |  |  | (0.363) |
| Full time work (t-2) | 0.163 | -0.038 | 0.088 | -1.028 | 15 | -0.242 |  |  | 3.111 |
|  | (0.129) | (0.361) | (0.119) | (0.223) |  | (0.332) |  |  | (0.777) |
| Full time work (t-3) | -0.093 | $-0.045$ | $0.213$ | $-0.031$ | 16 | $0.473$ |  |  | $4.106$ |
|  | (0.135) | (0.358) | (0.133) | $(0.232)$ |  | $(0.386)$ |  |  | (1.056) |
| Full time work (t-4) | $0.138$ | -0.270 | $0.432$ |  |  |  |  |  |  |
|  | (0.122) | (0.322) | $(0.121)$ | $(0.201)$ |  |  |  |  |  |
| Age of 1st Child | -0.003 | -0.021 |  |  |  |  |  |  |  |
|  | (0.018) | (0.027) |  |  |  |  |  |  |  |
| Age of 2nd Child | $-0.003$ | $-0.014$ |  |  |  |  |  |  |  |
| Age of 3rd Child | $(0.025)$ -0.023 | $(0.031)$ -0.096 |  |  |  |  |  |  |  |
|  | (0.041) | (0.079) |  |  |  |  |  |  |  |
| Age of 4th Child | 0.076 | 0.226 |  |  |  |  |  |  |  |
|  | (0.079) | (0.109) |  |  |  |  |  |  |  |
| Time with 1st Child | -0.043 | -0.033 | $0.088$ | -0.136 |  |  |  |  |  |
|  | $(0.031)$ 0.052 | $(0.041)$ 0.072 | $(0.029)$ | (0.048) |  |  |  |  |  |
| Time with 2nd Child | (0.038) | (0.063) | (0.036) | $\begin{gathered} 0.099 \\ (0.053) \end{gathered}$ |  |  |  |  |  |
| Time with 3nd Child | 0.010 | -0.222 | 0.079 | 0.222 |  |  |  |  |  |
|  | (0.062) | (0.109) | (0.060) | (0.129) |  |  |  |  |  |
| Time with 4th Child | -0.054 | 0.771 | 0.045 | -0.494 |  |  |  |  |  |
|  | (0.092) | (0.378) | (0.171) | (0.144) | Constant | 0.450 | 4.779 |  |  |
| No. of Female Children | -0.046 | -0.056 |  |  |  | (0.819) | (1.811) |  |  |
|  | (0.066) | (0.111) |  |  | N | 23,694 | 14,740 |  |  |

Source: Data from the Family-Individual File of the Michigan Panel Study of Income Dynamics (PSID) between 1968 and 1997. Individuals choice 5 and spouse choice 4 are deterministic for male and are excluded; meaning for a married male if these choices are chosen he will remain married next period with probability one.
Table 7: Logit Coefficient of Conditional Choice Probability for Single Female (Standard Error in parenthesis; Choice 1 is the excluded class)

|  | Chioce |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Variables | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
| Black | -0.221 | -0.508 | 1.101 | 0.139 | -0.063 | -0.444 | 0.497 | 0.493 | 0.307 | -0.222 | -0.240 | -0.272 | 0.001 | 0.287 | 0.271 |
|  | (0.092) | (0.061) | (0.250) | (0.105) | (0.127) | (0.086) | (0.150) | (0.263) | (0.230) | (0.126) | (0.176) | (0.117) | (0.168) | (0.335) | (0.239) |
| High Sch. | -0.236 | 0.742 | 1.291 | -0.182 | 0.245 | 0.628 | -0.717 | 0.420 | 0.289 | -0.182 | 0.239 | 0.534 | -0.391 | -0.118 | 0.798 |
|  | (0.171) | (0.135) | (0.615) | (0.149) | (0.225) | (0.175) | (0.176) | (0.458) | (0.434) | (0.170) | (0.297) | (0.221) | (0.219) | (0.501) | (0.440) |
| Some Col. | -0.069 | 0.800 | 1.098 | -0.193 | 0.534 | 0.808 | -1.386 | 0.220 | 0.445 | -0.591 | 0.436 | 0.513 | -0.892 | -0.372 | 0.323 |
|  | (0.169) | (0.135) | (0.624) | (0.162) | (0.229) | (0.179) | (0.212) | (0.469) | (0.441) | (0.201) | (0.324) | (0.232) | (0.255) | (0.545) | (0.462) |
| College | -0.042 | 0.828 | 0.395 | -0.946 | -0.000 | 0.639 | -2.104 | -0.333 | 0.536 | -1.075 | 0.584 | 0.188 | -2.076 | -0.290 | -0.444 |
|  | (0.174) | (0.137) | (0.680) | (0.245) | (0.287) | (0.197) | (0.320) | (0.552) | (0.498) | (0.311) | (0.359) | (0.269) | (0.410) | (0.600) | (0.558) |
| Age | 0.390 | 0.517 | 1.002 | 0.209 | 0.460 | 0.186 | 0.949 | 0.861 | 0.577 | 0.026 | -0.024 | 0.097 | 0.608 | 0.727 | 0.699 |
|  | (0.039) | (0.022) | (0.178) | (0.066) | (0.082) | (0.045) | (0.151) | (0.312) | (0.207) | (0.075) | (0.106) | (0.068) | (0.142) | (0.307) | (0.217) |
| Age Sq | -0.006 | -0.008 | -0.018 | -0.003 | -0.007 | -0.003 | -0.018 | -0.017 | -0.011 | -0.000 | 0.000 | -0.002 | -0.010 | -0.014 | -0.012 |
|  | (0.001) | (0.000) | (0.003) | (0.001) | (0.001) | (0.001) | (0.003) | (0.006) | (0.004) | (0.001) | (0.002) | (0.001) | (0.003) | (0.006) | (0.004) |
| No.of kids | 1.474 | 1.222 | 3.740 | 8.462 | 8.800 | 8.238 | 3.662 | 2.272 | 9.317 | 8.270 | 8.976 | 8.207 | 3.523 | 1.798 | 2.715 |
|  | (0.442) | (0.367) | (0.910) | (0.376) | (0.492) | (0.372) | (0.456) | (0.844) | (0.767) | (0.420) | (0.584) | (0.453) | (0.432) | (0.949) | (0.630) |
| No. of kids Sq | -0.328 | -0.387 | -1.072 | -2.122 | -2.235 | -2.123 | -0.928 | -0.101 | -2.522 | -2.057 | -2.321 | -2.175 | -0.845 | -0.802 | -0.818 |
|  | (0.192) | (0.172) | (0.592) | (0.155) | (0.177) | (0.151) | (0.190) | (0.406) | (0.292) | (0.158) | (0.192) | (0.168) | (0.169) | (0.389) | (0.232) |
| Part work (t-1) | 3.812 | 3.183 | 2.165 | 1.351 | 2.764 | 2.933 | 1.081 | 1.977 | 2.885 | 1.344 | 2.508 | 2.733 | 1.555 | 2.361 | 2.891 |
|  | (0.179) | (0.142) | (0.482) | (0.219) | (0.238) | (0.219) | (0.329) | (0.468) | (0.464) | (0.250) | (0.289) | (0.265) | (0.350) | (0.529) | (0.375) |
| Part work (t-2) | 1.579 | 0.934 | 0.477 | 0.715 | 1.328 | 0.860 | 1.833 | 1.443 | 1.210 | 0.375 | 0.989 | 0.784 | 0.224 | 0.596 | 0.727 |
|  | (0.267) | (0.223) | (0.629) | (0.259) | (0.275) | (0.251) | (0.349) | (0.601) | (0.518) | (0.292) | (0.329) | (0.296) | (0.462) | (0.698) | (0.523) |
| Part work (t-3) | 0.692 | 0.447 | 0.959 | 0.266 | 0.622 | 0.239 | -0.028 | -0.851 | 0.180 | 0.151 | 0.433 | 0.301 | -0.161 | 0.470 | 0.490 |
|  | (0.299) | (0.251) | (0.517) | (0.270) | (0.293) | (0.266) | (0.414) | (1.010) | (0.504) | (0.295) | (0.339) | (0.304) | (0.525) | (0.669) | (0.565) |
| Part work (t-4) | 0.519 | 0.092 | -0.621 | -0.141 | 0.415 | 0.394 | 0.790 | -1.426 | 0.326 | -0.018 | 0.740 | 0.110 | -0.092 | 1.374 | 0.103 |
|  | (0.320) | (0.278) | (0.783) | (0.300) | (0.312) | (0.285) | (0.421) | (1.037) | (0.479) | (0.323) | (0.355) | (0.321) | (0.479) | (0.552) | (0.543) |
| Full work (t-1) | 3.950 | 5.018 | 3.334 | 1.567 | 3.399 | 5.007 | 1.214 | 2.705 | 4.195 | 1.283 | 2.675 | 4.327 | 1.214 | 3.039 | 3.561 |
|  | (0.169) | (0.104) | (0.410) | (0.208) | (0.221) | (0.186) | (0.326) | (0.412) | (0.421) | (0.256) | (0.293) | (0.227) | (0.394) | (0.551) | (0.303) |
| Full work (t-2) | 0.590 | 0.788 | 0.764 | 0.020 | 0.228 | 0.773 | 1.261 | 1.253 | 1.298 | -0.304 | 0.015 | 0.799 | 0.420 | -0.528 | 0.726 |
|  | (0.220) | (0.160) | (0.455) | (0.210) | (0.237) | (0.191) | (0.320) | (0.528) | (0.442) | (0.257) | (0.324) | (0.233) | (0.367) | (0.566) | (0.428) |
| Full work (t-3) | 0.568 | 0.605 | 0.968 | 0.048 | 0.286 | 0.536 | -0.397 | 0.947 | 0.823 | -0.030 | 0.316 | 0.296 | 0.370 | 0.551 | 0.726 |
|  | (0.273) | (0.216) | (0.461) | (0.253) | (0.281) | (0.234) | (0.395) | (0.543) | (0.407) | (0.287) | (0.334) | (0.271) | (0.414) | (0.520) | (0.480) |
| Full work (t-4) | 0.241 | 0.324 | 0.308 | 0.002 | 0.263 | 0.480 | 0.348 | -0.453 | 0.327 | 0.114 | 0.514 | 0.366 | -0.078 | 1.255 | -0.422 |
|  | (0.257) | (0.212) | (0.398) | (0.237) | (0.260) | (0.223) | (0.419) | (0.515) | (0.376) | (0.264) | (0.318) | (0.253) | (0.422) | (0.567) | (0.438) |

Source: Data from the Family-Individual File of the Michigan Panel Study of Income Dynamics (PSID) between 1968 and 1997.
Table 7 (cont'd): Logit Coefficient of Conditional Choice Probability for Single Female (Standard Error in parenthesis; Choice 1 is the excluded class)

| Variables | Chioce |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
| Age of 1st kid | -0.101 | -0.087 | -0.078 | -0.136 | -0.191 | -0.121 | -0.203 | 0.004 | -0.255 | -0.246 | -0.199 | -0.198 | -0.477 | -0.279 | -0.493 |
|  | (0.033) | (0.022) | (0.048) | (0.026) | (0.029) | (0.022) | (0.060) | (0.100) | (0.060) | (0.035) | (0.046) | (0.030) | (0.107) | (0.129) | (0.152) |
| Age of 2nd kid | 0.078 | 0.116 | 0.007 | -0.003 | 0.035 | -0.008 | 0.027 | -0.141 | 0.059 | -0.033 | -0.073 | -0.034 | -0.059 | -0.066 | 0.245 |
|  | (0.051) | (0.039) | (0.096) | (0.037) | (0.042) | (0.038) | (0.095) | (0.184) | (0.081) | (0.043) | (0.059) | (0.044) | (0.190) | (0.181) | (0.226) |
| Age of 3rd kid | 0.112 | 0.044 | -9.412 | 0.030 | 0.020 | 0.058 | -0.081 | -1.580 | 0.170 | 0.056 | 0.051 | -0.068 | -0.491 | 0.022 | 0.006 |
|  | (0.071) | (0.065) | (2.788) | (0.060) | (0.069) | (0.064) | (0.216) | (1.016) | (0.135) | (0.071) | (0.087) | (0.078) | (0.890) | (0.379) | (0.388) |
| Age of 4th kid | 1.442 | -0.686 | 5.923 | -0.188 | -0.167 | -0.087 | -2.061 | -5.752 | -3.577 | -0.293 | -0.058 | -0.047 | -3.450 | -4.869 | -6.196 |
|  | (0.384) | (0.367) | (1.672) | (0.392) | (0.396) | (0.388) | (0.749) | (2.923) | (1.663) | (0.395) | (0.408) | (0.402) | (1.634) | (1.686) | (2.968) |
| Time 1st kid | -0.066 | 0.036 | -0.247 | 0.434 | 0.450 | 0.414 | 0.388 | 0.220 | 0.399 | 0.766 | 0.735 | 0.710 | 0.865 | 0.911 | 0.745 |
|  | (0.100) | (0.069) | (0.148) | (0.064) | (0.069) | (0.065) | (0.089) | (0.131) | (0.095) | (0.071) | (0.084) | (0.072) | (0.112) | (0.196) | (0.150) |
| Time 2nd kid | 0.017 | -0.109 | -0.126 | -0.178 | -0.214 | -0.110 | -0.015 | -0.028 | -0.136 | -0.038 | 0.005 | -0.010 | 0.028 | 0.174 | -0.100 |
|  | (0.128) | (0.098) | (0.389) | (0.087) | (0.095) | (0.090) | (0.131) | (0.271) | (0.136) | (0.093) | (0.109) | (0.097) | (0.181) | (0.215) | (0.299) |
| Time 3nd kid | -0.461 | -0.153 | -0.708 | 0.440 | 0.429 | 0.371 | 0.356 | -0.492 | 0.486 | 0.431 | 0.672 | 0.716 | 0.621 | 0.635 | 0.372 |
|  | (0.272) | (0.157) | (0.797) | (0.132) | (0.149) | (0.138) | (0.208) | (0.564) | (0.227) | (0.147) | (0.161) | (0.151) | (0.642) | (0.307) | (0.411) |
| Time 4th kid | -7.249 | 2.360 | 4.533 | 3.624 | 3.838 | 3.582 | -4.556 | -3.899 | 0.868 | 3.809 | 3.674 | 3.679 | -4.155 | -3.013 | -5.094 |
|  | (2.255) | (1.388) | (1.999) | (1.388) | (1.389) | (1.385) | (1.399) | (2.457) | (1.499) | (1.387) | (1.397) | (1.389) | (1.434) | (1.285) | (2.068) |
| Female kids | -0.025 | 0.010 | -0.272 | 0.056 | -0.064 | -0.099 | -0.295 | -1.270 | -0.143 | -0.036 | -0.358 | -0.150 | -0.218 | -0.233 | 0.214 |
|  | (0.211) | (0.152) | (0.359) | (0.140) | (0.157) | (0.144) | (0.202) | (0.420) | (0.225) | (0.147) | (0.180) | (0.154) | (0.208) | (0.356) | (0.300) |
| Constant | -9.686 | -11.047 | -21.147 | -9.696 | -16.006 | -11.057 | -15.932 | -17.205 | -18.406 | -7.992 | -9.662 | -10.838 | -13.005 | -15.908 | -16.075 |
|  | (0.558) | (0.323) | (2.320) | (0.926) | (1.280) | (0.709) | (1.750) | (3.721) | (3.005) | (1.054) | (1.618) | (1.024) | (1.746) | (3.630) | (2.734) |
| N | 35,196 | 35,196 | 35,196 | 35,196 | 35,196 | 35,196 | 35,196 | 35,196 | 35,196 | 35,196 | 35,196 | 35,196 | 35,196 | 35,196 | 35,196 |

[^16]Table 8: Logit Coefficient of Conditional Choice Probability for Single Male (Standard Error in parenthesis; Choice 1 is the excluded class)

| Variables | Choice |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| Black | 0.162 | -0.392 | -11.687 | -1.034 | -0.627 | 1.080 | 0.020 | -1.085 |
|  | $(0.096)$ | $(0.061)$ | $(1.467)$ | (0.803) | (0.408) | (0.783) | (0.908) | (0.399) |
| High Sch. | -0.304 | 0.257 | -0.352 | 10.887 | 0.490 | 0.664 | 2.131 | 0.792 |
|  | (0.143) | (0.091) | (1.050) | (1.535) | (0.376) | (0.924) | (1.544) | (0.383) |
| Some Col. | -0.207 | 0.199 | -1.564 | 9.350 | 0.050 | 0.257 | 1.003 | -0.119 |
|  | (0.150) | (0.095) | (1.424) | (1.896) | (0.384) | (1.377) | (1.613) | (0.401) |
| College | -0.176 | 0.416 | -10.694 | 9.523 | 0.638 | -9.201 | -9.494 | 0.522 |
|  | (0.158) | (0.096) | (1.930) | (1.560) | (0.401) | (2.613) | (1.843) | (0.405) |
| Age | 0.747 | 0.878 | 4.777 | 0.598 | 1.231 | 0.175 | 2.905 | 1.200 |
|  | (0.070) | (0.038) | (2.284) | (0.419) | (0.194) | (0.423) | (1.173) | (0.170) |
| Age Sq | -0.013 | -0.015 | -0.066 | -0.010 | -0.020 | -0.003 | -0.040 | -0.018 |
|  | (0.001) | (0.001) | (0.032) | (0.006) | (0.003) | (0.007) | (0.017) | (0.002) |
| No.of kids | -0.344 | -0.639 | 9.951 | 8.270 | 5.007 | 13.350 | 18.071 | 5.533 |
|  | (1.133) | (0.988) | (2.536) | (2.767) | (1.070) | (1.945) | (1.833) | (1.420) |
| No. of kids Sq | -0.095 | -0.053 | -1.986 | -1.884 | -1.255 | -3.021 | -6.542 | -1.431 |
|  | (0.205) | (0.170) | (0.834) | (0.883) | (0.249) | (0.718) | (0.830) | (0.434) |
| Part work (t-1) | 4.217 | 3.321 | 3.387 | 12.425 | 3.291 | -12.458 | 3.564 | 4.093 |
|  | (0.198) | (0.154) | (1.507) | (1.619) | (1.491) | (2.129) | (1.330) | (0.842) |
| Part work (t-2) | 1.625 | 0.864 | 1.918 | -8.906 | 19.273 | 2.239 | -1.285 | 1.877 |
|  | (0.340) | (0.306) | (1.264) | (1.505) | (3.549) | (1.254) | (1.860) | (1.079) |
| Part work (t-3) | -0.070 | -0.731 | 2.332 | -2.607 | -1.128 | 0.017 | 0.551 | -0.854 |
|  | (0.405) | (0.359) | (1.106) | (1.170) | (0.901) | (1.716) | (1.581) | (0.929) |
| Part work (t-4) | 0.788 |  |  | $12.434$ | 1.755 | 1.296 | 2.003 | 1.473 |
|  | (0.446) | $(0.382)$ | (1.439) | (1.280) | (0.911) | $(1.810)$ | (1.483) | $(0.783)$ |
| Full work (t-1) | 4.397 | 5.075 | -0.887 | 10.881 | 5.668 | 0.274 | 3.195 | 4.791 |
|  | $(0.169)$ | $(0.101)$ | (1.559) | $(1.238)$ | (1.255) | $(0.994)$ | (1.357) | $(0.735)$ |
| Full work (t-2) | 0.787 | 1.079 | 2.434 | 1.101 | 19.181 | -0.119 | 0.431 | 2.194 |
|  | (0.255) | $(0.203)$ | (1.739) | (1.012) | $(3.549)$ | $(1.891)$ | (1.558) | $(0.874)$ |
| Full work (t-3) | 0.205 | 0.443 | -0.200 | -2.632 | -0.460 | -0.928 | 0.525 | -0.056 |
|  | (0.350) | (0.284) | (1.413) | (1.324) | (0.800) | (1.636) | (1.624) | (0.811) |
| Full work (t-4) | 0.741 | 0.599 | -2.839 | 8.379 | 1.543 | -1.522 | 0.705 | 1.187 |
|  | (0.338) | $(0.283)$ | $(0.981)$ | (1.460) | $(0.754)$ | (1.048) | (1.258) | $(0.650)$ |
| Age of 1st kid | 0.100 | 0.188 | 0.064 | 0.006 | 0.320 | -0.042 | 0.136 | 0.162 |
|  | (0.158) | $(0.135)$ | $(0.267)$ | (0.200) | (0.138) | $(0.185)$ | $(0.146)$ | $(0.139)$ |
| Age of 2nd kid | 0.050 | -0.063 | -0.504 | -0.029 | -0.205 | -0.302 | 0.175 | -0.168 |
|  | (0.133) | (0.123) | (0.352) | (0.170) | (0.128) | $(0.341)$ | (0.187) | (0.129) |
| Female kids | 1.402 | 1.793 | -0.329 | 1.404 | 1.247 | 1.091 | -1.029 | 1.446 |
|  | (0.831) | (0.672) | (1.864) | (0.717) | (0.667) | (0.859) | (1.291) | (0.658) |
| Constant | -14.516 | -14.955 | -94.644 | -44.118 | -46.713 | -13.193 | -68.683 | -29.242 |
|  | (0.910) | (0.481) | (40.775) | (6.828) | (0.000) | (6.150) | (21.834) | (2.969) |
| N | 35,939 | 35,939 | 35,939 | 35,939 | 35,939 | 35,939 | 35,939 | 35,939 |

Source: Data from the Family-Individual File of the Michigan Panel Study of Income Dynamics
(PSID) between 1968 and 1997.
Table 9: Logit Coefficient of Best Response Probability for Married Female (Standard Error in parenthesis; Choice 1 is the excluded class)

Table 9 (Cont'd): Logit Coefficient of Best Response Probability for Married Female (Standard Error in parenthesis; Choice 1 is the excluded class)

|  | Choice |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Individual Variables | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
| Time 3nd kid | 0.011 | -0.256 | -29.186 | -0.256 | -0.250 | -0.232 | 0.138 | -0.033 | -0.205 | -0.025 | -0.062 | -0.037 | 0.507 | 0.977 | 0.474 |
|  | (0.218) | (0.178) | (0.000) | (0.153) | (0.152) | (0.149) | (0.221) | (0.203) | (0.198) | (0.154) | (0.155) | (0.152) | (0.189) | (0.327) | (0.216) |
| Time 4th kid | 0.650 | 0.659 | -2.247 | 0.045 | 0.050 | -0.094 | -13.526 | -13.969 | -21.740 | 0.067 | 0.113 | 0.069 | -0.048 | -8.441 | -7.429 |
|  | (0.242) | (0.311) | (0.000) | (0.201) | (0.207) | (0.207) | (1.026) | (1.406) | (0.000) | (0.205) | (0.213) | (0.215) | (0.470) | (1.232) | (1.143) |
| Female kids | 0.185 | -0.162 | 0.029 | -0.072 | 0.010 | -0.082 | 0.065 | -0.334 | -0.092 | -0.100 | -0.143 | -0.131 | -0.058 | -0.082 | 0.058 |
|  | (0.211) | (0.160) | (0.321) | (0.156) | (0.157) | (0.153) | (0.191) | (0.222) | (0.175) | (0.154) | (0.157) | (0.155) | (0.171) | (0.213) | (0.202) |
| Spouse <br> Variables |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| High Sch. | 0.138 | 0.381 | 0.718 | 0.404 | 0.457 | 0.410 | 0.118 | 0.274 | 0.449 | 0.369 | 0.372 | 0.331 | 0.152 | -0.124 | 0.186 |
|  | (0.161) | (0.120) | (0.537) | (0.154) | (0.178) | (0.140) | (0.182) | (0.272) | (0.248) | (0.146) | (0.171) | (0.151) | (0.160) | (0.258) | (0.217) |
| Some Col. | 0.411 | 0.448 | 0.999 | 0.333 | 0.472 | 0.380 | 0.142 | 0.503 | 0.476 | 0.375 | 0.223 | 0.135 | 0.214 | 0.064 | 0.140 |
|  | (0.173) | (0.133) | (0.547) | (0.173) | (0.192) | (0.154) | (0.205) | (0.288) | (0.260) | (0.163) | (0.190) | (0.167) | (0.177) | (0.277) | (0.237) |
| College | 0.313 | 0.334 | 0.653 | 0.483 | 0.550 | 0.226 | 0.491 | 0.564 | 0.367 | 0.447 | 0.314 | -0.271 | 0.373 | 0.338 | -0.339 |
|  | (0.187) | (0.145) | (0.581) | (0.187) | (0.205) | (0.168) | (0.227) | (0.299) | (0.282) | (0.178) | (0.205) | (0.184) | (0.196) | (0.284) | (0.267) |
| Age | 0.099 | 0.066 | 0.224 | -0.062 | 0.007 | 0.004 | -0.019 | -0.048 | -0.033 | -0.012 | 0.099 | -0.105 | 0.003 | 0.068 | 0.109 |
|  | (0.065) | (0.047) | (0.170) | (0.066) | (0.079) | (0.058) | (0.092) | (0.104) | (0.121) | (0.066) | (0.080) | (0.066) | (0.080) | (0.133) | (0.103) |
| Age Sq | -0.002 | -0.001 | -0.005 | 0.001 | -0.001 | -0.000 | -0.000 | -0.000 | -0.000 | -0.000 | -0.002 | 0.001 | -0.001 | -0.002 | -0.003 |
|  | (0.001) | (0.001) | (0.003) | (0.001) | (0.001) | (0.001) | (0.001) | (0.002) | (0.002) | (0.001) | (0.001) | (0.001) | (0.001) | (0.002) | (0.002) |
| Part work ( $\mathrm{t}-1$ ) | -0.609 | -0.810 | -0.171 | 0.064 | -0.580 | -0.438 | 0.014 | -1.003 | 0.027 | -0.125 | -0.515 | -0.166 | -0.194 | -0.655 | -0.417 |
|  | (0.272) | (0.213) | (0.518) | (0.288) | (0.322) | (0.262) | (0.339) | (0.464) | (0.422) | (0.287) | (0.377) | (0.298) | (0.315) | (0.503) | (0.459) |
| Part work (t-2) | -0.696 | -0.514 | -0.534 | -0.365 | -0.347 | -0.323 | -0.195 | -0.189 | -0.764 | -0.325 | -0.707 | -0.281 | -0.897 | 0.031 | -1.755 |
|  | (0.316) | (0.237) | (0.540) | (0.295) | (0.327) | (0.271) | (0.370) | (0.390) | (0.433) | (0.297) | (0.372) | (0.303) | (0.377) | (0.453) | (0.602) |
| Part work (t-3) | -0.075 | -0.426 | -0.973 | -0.487 | -0.794 | -0.437 | -0.672 | -0.423 | -0.238 | -0.607 | -0.846 | -0.632 | -0.552 | -0.663 | 0.211 |
|  | (0.331) | (0.256) | (0.649) | (0.350) | (0.359) | (0.304) | (0.409) | (0.446) | (0.436) | (0.328) | (0.387) | (0.330) | (0.385) | (0.531) | (0.449) |
| Part work (t-4) | -0.142 | -0.067 | -0.228 | -0.311 | -0.046 | -0.251 | -0.375 | -0.098 | -0.647 | -0.713 | -0.426 | -0.660 | -0.009 | -0.477 | -0.761 |
|  | (0.356) | (0.275) | (0.639) | (0.337) | (0.353) | (0.304) | (0.445) | (0.476) | (0.469) | (0.338) | (0.378) | (0.334) | (0.348) | (0.541) | (0.475) |
| Full work (t-1) | -0.676 | -1.057 | -0.609 | 0.020 | -0.327 | -0.334 | 0.214 | -0.251 | 0.144 | 0.207 | 0.093 | 0.010 | 0.237 | -0.078 | 0.071 |
|  | (0.143) | (0.116) | (0.345) | (0.176) | (0.194) | (0.150) | (0.187) | (0.241) | (0.265) | (0.172) | (0.228) | (0.179) | (0.166) | (0.267) | (0.225) |
| Full work (t-2) | -0.181 | -0.205 | -0.364 | -0.032 | 0.200 | 0.110 | 0.155 | 0.067 | -0.090 | 0.327 | 0.215 | 0.305 | 0.100 | 0.145 | -0.372 |
|  | (0.168) | (0.131) | (0.337) | (0.179) | (0.195) | (0.155) | (0.192) | (0.242) | (0.233) | (0.174) | (0.225) | (0.181) | (0.179) | (0.255) | (0.220) |
| Full work (t-3) | -0.075 | -0.136 | -0.378 | 0.197 | -0.334 | 0.010 | -0.271 | -0.471 | 0.066 | -0.074 | -0.191 | -0.191 | 0.211 | -0.196 | 0.205 |
|  | (0.205) | (0.157) | (0.417) | (0.202) | (0.213) | (0.177) | (0.228) | (0.274) | (0.260) | (0.188) | (0.227) | (0.193) | (0.204) | (0.277) | (0.251) |
| Full work (t-4) | -0.175 | -0.123 | -0.293 | -0.248 | -0.058 | -0.308 | -0.269 | -0.002 | -0.339 | -0.405 | -0.360 | -0.497 | -0.481 | -0.521 | -0.731 |
|  | (0.191) | (0.144) | (0.381) | (0.181) | (0.194) | (0.161) | (0.220) | (0.261) | (0.234) | (0.171) | (0.200) | (0.173) | (0.187) | (0.263) | (0.224) |
| Time 1st kid | -0.004 | 0.021 | 0.105 | ${ }^{-0.063}$ | -0.113 | -0.106 | -0.108 | -0.080 | -0.069 | -0.204 | -0.178 | -0.173 | -0.201 | -0.358 | -0.230 |
|  | (0.099) | (0.078) | (0.129) | (0.077) | (0.077) | (0.075) | (0.089) | (0.093) | (0.081) | (0.077) | (0.078) | (0.076) | (0.081) | (0.094) | (0.086) |
| Time 2nd kid | -0.052 | 0.136 | 0.001 | ${ }^{-0.116}$ | -0.026 | -0.043 | -0.042 | -0.044 | -0.132 | -0.050 | -0.072 | -0.041 | -0.046 | -0.024 | -0.039 |
|  | (0.168) | (0.107) | (0.178) | (0.106) | (0.105) | (0.104) | (0.144) | (0.132) | (0.127) | (0.105) | (0.106) | (0.104) | (0.117) | (0.143) | (0.128) |
| Time 3nd kid | -0.132 | -0.009 | -14.406 | -0.327 | -0.345 | -0.344 | -0.352 | -0.150 | -0.542 | -0.376 | -0.329 | -0.399 | -0.534 | -0.569 | -0.299 |
|  | (0.226) | (0.161) | (0.000) | (0.160) | (0.160) | (0.156) | (0.207) | (0.360) | (0.219) | (0.158) | (0.159) | (0.158) | (0.195) | (0.308) | (0.229) |
| Time 4th kid | -0.218 | -0.521 | 0.038 | -0.018 | 0.078 | 0.273 | -5.067 | -5.680 | -12.044 | 0.147 | 0.170 | 0.002 | 1.106 | -1.434 | -2.344 |
|  | (0.433) | (0.276) | (0.000) | (0.256) | (0.253) | (0.246) | (0.916) | (0.908) | (0.000) | (0.244) | (0.249) | (0.246) | (0.382) | (0.648) | (0.638) |

Source: Data from the Family-Individual File of the Michigan Panel Study of Income Dynamics (PSID) between 1968 and 1997.
Table 9 (cont'd): Logit Coefficient of Best Response Probability for Married Female

| Spouse Choice | Choice |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
| 2 | 2.120 | 1.517 | 1.664 | 0.164 | 1.811 | 0.952 | -0.013 | 1.611 | 0.628 | 0.369 | 1.539 | 1.392 | 0.039 | 2.087 | 0.775 |
|  | (0.264) | (0.223) | (0.939) | (0.323) | (0.345) | (0.299) | (0.366) | (0.523) | (0.623) | (0.312) | (0.434) | (0.342) | (0.366) | (0.615) | (0.786) |
| 3 | 1.224 | 1.712 | 1.903 | 0.420 | 1.300 | 1.395 | -0.003 | 1.110 | 1.061 | 0.551 | 1.432 | 1.395 | 0.366 | 1.119 | 1.370 |
|  | (0.198) | (0.147) | (0.743) | (0.195) | (0.249) | (0.188) | (0.214) | (0.421) | (0.397) | (0.196) | (0.310) | (0.232) | (0.223) | (0.529) | (0.530) |
| 4 | -9.888 | 2.112 | -6.142 | 3.262 | 3.432 | 3.784 | 3.417 | -11.592 | 3.577 | 3.378 | 3.663 | 3.949 | 4.718 | 3.710 | 4.771 |
|  | (1.078) | (1.134) | (1.315) | (1.124) | (1.154) | (1.111) | (1.108) | (1.135) | (1.247) | (1.120) | (1.205) | (1.137) | (1.084) | (1.543) | (1.314) |
| 5 | -10.517 | 1.996 | 5.948 | 2.406 | 4.893 | 3.885 | 3.018 | 5.511 | 3.911 | 3.366 | 5.164 | 4.051 | 3.788 | 3.833 | 4.984 |
|  | (0.916) | (1.027) | (1.323) | (1.019) | (0.985) | (0.986) | (0.991) | (1.044) | (1.152) | (0.971) | (1.006) | (1.018) | (0.941) | (1.462) | (1.195) |
| 6 | 1.689 | 2.720 | 4.202 | 3.066 | 4.224 | 4.483 | 3.009 | 4.522 | 4.145 | 3.638 | 4.703 | 4.682 | 4.008 | 5.108 | 5.593 |
|  | (0.543) | (0.427) | (0.902) | (0.448) | (0.473) | (0.441) | (0.460) | (0.589) | (0.568) | (0.446) | (0.507) | (0.462) | (0.453) | (0.663) | (0.663) |
| 7 | 0.081 | 0.407 | 3.095 | -0.040 | 0.406 | 0.985 | 1.031 | -10.111 | 1.782 | 0.922 | 0.754 | 1.251 | 1.076 | 2.274 | 2.736 |
|  | (0.826) | (0.517) | (0.990) | (0.601) | (0.660) | (0.537) | (0.625) | (0.600) | (0.685) | (0.585) | (0.738) | (0.593) | (0.669) | (0.906) | (0.852) |
| 8 | -10.054 | 1.744 | 3.784 | 1.065 | 2.941 | 2.167 | -13.517 | 3.711 | 2.645 | 1.856 | 3.787 | 2.679 | 2.586 | 2.397 | 3.500 |
|  | (0.729) | (0.797) | (1.276) | (0.899) | (0.862) | (0.832) | (0.780) | (0.964) | (0.997) | (0.857) | (0.888) | (0.875) | (0.884) | (1.398) | (1.103) |
| 9 | 0.712 | 1.190 | 3.321 | 1.495 | 2.686 | 3.203 | 1.889 | 3.331 | 3.061 | 2.691 | 3.732 | 4.033 | 3.093 | 4.343 | 5.177 |
|  | (0.489) | (0.338) | (0.857) | (0.368) | (0.394) | (0.350) | (0.391) | (0.526) | (0.502) | (0.358) | (0.431) | (0.375) | (0.364) | (0.609) | $(0.599)$ |
| Constant | -1.843 | -3.304 | -18.480 | -1.846 | -4.269 | -3.091 | -6.185 | -8.347 | -10.911 | -2.498 | -3.544 | -0.720 | -6.742 | -4.432 | -2.633 |
|  | (0.928) | (0.680) | (3.105) | (0.956) | (1.060) | (0.833) | (1.440) | (1.940) | (1.944) | (0.946) | (1.195) | (0.926) | (1.333) | (1.998) | (1.606) |
| N | 26,834 | 26,834 | 26,834 | 26,834 | 26,834 | 26,834 | 26,834 | 26,834 | 26,834 | 26,834 | 26,834 | 26,834 | 26,834 | 26,834 | 26,834 |

Table 10: Logit Coefficient of Best Response for Married Male
(Standard Error in parenthesis; Choice 1 is the excluded class)


Source: Data from the Family-Individual File of the Michigan Panel Study of Income Dynamics (PSID) between 1968 and 1997.

Table 10 (Cont'd): Logit Coefficient of Best Response for Married Male (Standard Error in parenthesis; Choice 1 is the excluded class)


Source: Data from the Family-Individual File of the Michigan Panel Study of Income Dynamics (PSID) between 1968 and 1997.

Table 11: GMM Estimates of Utility Function and Discount Factors (Standard Errors in Parenthesis; Choice 1 is the Excluded Class)

| Utility of Leisure |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Choice | Female |  |  | (1) | Choice | Male |  | (2) |
|  | Labor <br> Market <br> Work | Child <br> Birth | Parental Time |  |  | Labor <br> Market <br> Work | Parental Time |  |
| 2 | Part time | None | None | -2.98 | 2 | Part Time | None | 0.23 |
|  |  |  |  | $(0.02)$ 0.48 |  |  |  | (0.01) |
| 3 | Full Time | None | None | $(3.40 \mathrm{e}-3)$ | 3 | Full Time | None | $\begin{gathered} -0.24 \\ (0.01) \end{gathered}$ |
| 4 | Full Time | Yes | None | -9.82 | 4 | None | Low | -3.63 |
| 5 | None | None | Low | $(0.04)$ 0.09 | 5 | Part Time | Low | $\xrightarrow{(0.02)}$ |
|  |  |  |  | (0.01) |  |  |  | (0.01) |
| 6 | Part Time | None | Low | $-0.55$ | 6 | Full Time | Low | $0.59$ |
| 7 | Full Time | None | Low | -0.57 | 7 | None | High | 0.08 |
| 8 | None | Yes | Low | (3.1e-34) | 8 | Part Time | High | (0.01) |
|  |  |  |  | (0.02) |  |  |  | (0.01) |
| 9 | Part Time | Yes | Low | $\begin{gathered} -4.27 \\ (0.02) \end{gathered}$ | 9 | Full Time | High | $0.04$ |
| 10 | Full Time | Yes | Low | -1.19 |  |  |  |  |
|  |  |  |  | (0.02) |  |  |  |  |
| 11 | None | None | High | -0.10 |  |  |  |  |
| 12 | Part Time | None | High | ${ }_{1.10}$ |  |  |  |  |
|  |  |  |  | (0.01) |  |  |  |  |
| 13 | Full Time | None | High | 0.62 |  |  |  |  |
| 14 |  | Yes | High | (0.01) |  |  |  |  |
| 14 | None |  |  | $\begin{gathered} -0.27 \\ (0.02) \end{gathered}$ |  |  |  |  |
| 15 | Part Time | Yes | High | -2.38 |  |  |  |  |
|  |  |  |  | (0.02) |  |  |  |  |
| 16 | Full Time | Yes | High | $\begin{array}{r} -1.83 \\ (0.02) \\ \hline \end{array}$ |  |  |  |  |
| Discount Factors |  |  |  |  |  |  |  |  |
| Intertemporal |  |  | $\beta$ | $0.0 .85$ |  |  |  |  |
| Intergenerational |  |  | $\lambda$ | 0.90 |  |  |  |  |
| Number Children |  |  |  | (1.0E-5) |  |  |  |  |
|  |  |  | $\nu$ | $\begin{gathered} 0.10 \\ (1.3 \mathrm{E}-7) \\ \hline \end{gathered}$ |  |  |  |  |
| Utility of Earnings and Net Cost of Children |  |  |  |  |  |  |  |  |
| Married own earnings |  |  |  | $\begin{gathered} \hline 0.31 \\ (1.0 \mathrm{e}-3) \end{gathered}$ | Married | own earning |  | $\begin{aligned} & \hline 0.22 \\ & (2.0 \mathrm{e}-3) \end{aligned}$ |
| Married Spouse earnings |  |  |  | -0.03 | Married | Spouse earn |  | -0.14 |
| Married number of children |  |  |  | ( ${ }^{(7.00-4)}$ | Married | number of c | ildren | $(1.0 \mathrm{e}-3)$ -0.29 |
| Single earnings |  |  |  | (2.0e-3) |  |  |  | (2.0e-3) |
|  |  |  |  | 0.29 | Single | rnings |  | 0.03 |
| Single number of children |  |  |  | ${ }_{(1.00-3)}^{(0.22)}$ | Single number of children |  |  | $(8.0 \mathrm{e}-4)$ 0.12 |
|  |  |  |  | (2.0e-3) |  |  |  | (2.0e-3) |
| N |  |  |  | 50,514 |  |  |  |  |

Table 12: OLS Estimates of Aggregated Return to Parental Time Investment Dependent Variable: $\log \left(\frac{\left(N_{\sigma T}\right)^{1-v}}{N_{\sigma}} \bar{V}_{N \sigma}\left(x_{T}\right)\right)$ (Standard Errors in Parenthesis)

| Variables | Baseline Model |  |
| :---: | :---: | :---: |
|  | Black | White |
| Number Children | 0.458 | 0.645 |
|  | (0.020) | (0.012) |
| Number Children Squared | -0.054 | -0.071 |
|  | (0.003) | (0.002) |
| Number of Female Children | 1.081 | 0.515 |
|  | (0.007) | (0.004) |
| Number of Female Children Squared | -0.160 | -0.066 |
|  | (0.002) | (0.001) |
| Mother: High School | 0.053 | 0.046 |
|  | (0.007) | (0.004) |
| Mother: Some College | 0.025 | 0.025 |
|  | (0.007) | (0.004) |
| Mother: College | 0.074 | 0.072 |
|  | (0.007) | (0.004) |
| Father: High School | 0.064 | 0.061 |
|  | (0.007) | (0.004) |
| Father : Some College | 0.125 | 0.116 |
|  | (0.007) | (0.004) |
| Father : College | 0.193 | 0.177 |
|  | (0.007) | (0.004) |
| Mother's Time Investment (per child) | 0.082 | 0.073 |
|  | (0.003) | (0.002) |
| x Number of Children | 0.002 | 0.002 |
|  | (0.001) | (0.001) |
| x Number Female Children | -0.005 | -0.005 |
|  | (0.001) | (0.000) |
| Father's Time Investment (per child) | 0.053 | 0.049 |
|  | (0.003) | (0.002) |
| x Number of Children | -0.000 | 0.000 |
|  | (0.001) | (0.001) |
| x Number Female Children | 0.001 | -0.000 |
|  | (0.001) | (0.000) |
| Constant | 6.683 | 7.807 |
|  | (0.033) | (0.020) |
| N | 6,720 | 6,720 |
| R-squared | 0.948 | 0.96 |

Table 13: OLS Estimates of Quantity-Quality tradeoff
Dependent Variable: $1-2 \log \left(\frac{\left(N_{\sigma T}\right)^{1-v}}{N_{\sigma T}} \bar{V}_{N \sigma}\left(x_{T}\right)\right), 3-4 \log \left(\frac{\left(\bar{V}_{N \sigma}\left(x_{T}\right)\right.}{N_{\sigma T}}\right)$,
(Standard Errors in Parenthesis)

| (Standard Errors in Parenthesis) |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | (1) | (2) | (3) | (4) |
| VARIABLES | Total Discounted Value | Total Discounted Value | Average Value | Average Value |
|  | White | Black | White | Black |
| numc | $\begin{gathered} 0.8893^{* * *} \\ (0.032) \end{gathered}$ | $\begin{gathered} 0.9956^{* * *} \\ (0.053) \end{gathered}$ | $\begin{gathered} -0.1934^{* * *} \\ (0.023) \end{gathered}$ | $\begin{gathered} -0.3572^{* * *} \\ (0.029) \end{gathered}$ |
| numc2 | $\begin{gathered} -0.0932^{* * *} \\ (0.006) \end{gathered}$ | $\begin{gathered} -0.1070^{* * *} \\ (0.010) \end{gathered}$ | $\begin{gathered} 0.0165^{* * *} \\ (0.004) \end{gathered}$ | $\begin{gathered} 0.0339^{* * *} \\ (0.005) \end{gathered}$ |
| nfemalec |  |  | $\begin{gathered} 0.4908^{* * *} \\ (0.010) \end{gathered}$ | $\begin{gathered} 1.0621^{* * *} \\ (0.011) \end{gathered}$ |
| nfemalec2 |  |  | $\begin{gathered} -0.0660^{* * *} \\ (0.003) \end{gathered}$ | $\begin{gathered} -0.1598^{* * *} \\ (0.003) \end{gathered}$ |
| HSM | $\begin{gathered} 0.0462^{* * *} \\ (0.015) \end{gathered}$ | $\begin{gathered} 0.0530^{* *} \\ (0.025) \end{gathered}$ | $\begin{gathered} 0.0462^{* * *} \\ (0.011) \end{gathered}$ | $\begin{gathered} 0.0530^{* * *} \\ (0.013) \end{gathered}$ |
| SCM | $\begin{gathered} 0.0253^{*} \\ (0.015) \end{gathered}$ | $\begin{aligned} & 0.0251 \\ & (0.025) \end{aligned}$ | $\begin{gathered} 0.0253^{* *} \\ (0.011) \end{gathered}$ | $\begin{gathered} 0.0251^{*} \\ (0.013) \end{gathered}$ |
| COLM | $\begin{gathered} 0.0719^{* * *} \\ (0.015) \end{gathered}$ | $\begin{gathered} 0.0739 * * * \\ (0.025) \end{gathered}$ | $\begin{gathered} 0.0719^{* * *} \\ (0.011) \end{gathered}$ | $\begin{gathered} 0.0739^{* * *} \\ (0.013) \end{gathered}$ |
| HSF | $\begin{gathered} 0.0615^{* * *} \\ (0.015) \end{gathered}$ | $\begin{gathered} 0.0636^{* *} \\ (0.025) \end{gathered}$ | $\begin{gathered} 0.0615^{* * *} \\ (0.011) \end{gathered}$ | $\begin{gathered} 0.0636^{* * *} \\ (0.013) \end{gathered}$ |
| SCF | $\begin{gathered} 0.1162^{* * *} \\ (0.015) \end{gathered}$ | $\begin{gathered} 0.1247^{* * *} \\ (0.025) \end{gathered}$ | $\begin{gathered} 0.1162^{* * *} \\ (0.011) \end{gathered}$ | $\begin{gathered} 0.1247^{* * *} \\ (0.013) \end{gathered}$ |
| COLF | $\begin{gathered} 0.1768^{* * *} \\ (0.015) \end{gathered}$ | $\begin{gathered} 0.1929^{* * *} \\ (0.025) \end{gathered}$ | $\begin{gathered} 0.1768^{* * *} \\ (0.011) \end{gathered}$ | $\begin{gathered} 0.1929^{* * *} \\ (0.013) \end{gathered}$ |
| Constant | $\begin{gathered} 8.3139 * * * \\ (0.042) \end{gathered}$ | $\begin{gathered} 7.2456^{* * *} \\ (0.071) \end{gathered}$ | $\begin{gathered} 9.0618^{* * *} \\ (0.029) \end{gathered}$ | $\begin{gathered} 7.9934^{* * *} \\ (0.038) \end{gathered}$ |
| Observations $R^{2}$ | $6,720$ | $6,720$ | $6,720$ | $6,720$ |
| $R^{2}$ | $0.500$ | 0.301 | 0.522 | 0.746 |
| Robust standard errors in parentheses ${ }^{* * *} \mathrm{p}<0.01,{ }^{* *} \mathrm{p}<0.05,{ }^{*} \mathrm{p}<0.1$ |  |  |  |  |



Figure 1: Parental Time Densities by Marital Status, Gender and Race


Figure 2: Parental Time Densities by Own Education, Spouse's Education and Number of Children


Figure 3: Parental Time Densities by Labor Supply and Education


Figure 4: Parental Time Densities by Labor Supply and Education

Change in average quality as a function of number of girls- 4 children families


Figure 5: Parental Time Densities by Labor Supply and Education


[^0]:    ${ }^{*}$ We thank the seminar participants of the applied micro workshop in NYU, Washington U. in St. Louis, UNC-Chapel Hill, and the participants of New Directions in Applied Microeconomics, La Pietra 2011, NBER summer workshop Macro Perspectives, 2011, the Conference in honor of John Kennan, May 2012, and Conference on Early Childhood Development, UCL June 2012.

[^1]:    ${ }^{1}$ See surveys by Carneiro and Heckman (2003) and Almond and Currie (2011) for evidence that cognitive and noncognitive skills are largly determined early in life.
    ${ }^{2}$ As demonstrated by Bernheim and Bagwell (1988), introducing marriages can lead to complicated links between dynasties. In order to circumvent this problem, dynasties in our model are anonymous, in the sense that the parents' utility depend on the child's realized type (education, labor market skill, and spouse's educations) as opposed to the individual identity of a child and the child's spouse.

[^2]:    ${ }^{3}$ Echevarria and Merlo (1999) estimate implications of dynastic model with endogenous fertility in which household allocation is determined by a Nash bargaining solution in a model with no divorce and marriage.
    ${ }^{4}$ As demonstrated in Alvarez (1999), relaxing several other assumptions in Becker and Barro (1989) can lead to intergenerational persistence in outcomes. For example, if previous generations transfer change the marginal costs of raising children, transfers depend on wealth. In our model, parental time investment affects children education outcome and their spouse's education which affect the marginal costs of raising children.

[^3]:    ${ }^{5}$ Most studied that use parental time data focus on mothers' time. Several papers find no significant impact of fathers time on long-term outcomes of children (see Haveman and Wolfe 1995 among others). These results may be different from our findings partially do to selection problem. When we estimate the effect of fathers time on children' outcomes without accounting for selection, the effect is negative. Del Boca et al. (2010) control for selection, and also find a positive effect of fathers' time on child's quality, using test score data, and they also find that it is lower than the effect of mothers' time.

[^4]:    ${ }^{6}$ See also Bernal (2008) which estimates a dynamic model in which mothers choose child care and labor supply and quantifies the impact of these choices on children's cognitive skills, and does not use data on time spent with children.
    ${ }^{7}$ See Carniero, Heckman and Materov (2000), Fryer and Levitt (2004) and Todd and Wolpin (2007). For example Fryer and Levitt (2004) argues that test scores outcomes may be too sensitive and are not a sufficient statistics to approximate the differences between blacks and white when they are older. Thus using long-term measures like we do is important.

[^5]:    ${ }^{8}$ We only observe the amount of time parents spent in total and the number of young children in the household, thus we are unable to model parents response to child unobserved ability.

[^6]:    ${ }^{9}$ After we estimate the model, we verify that the parameters estimated are in the range where equilibrium exists.

[^7]:    ${ }^{10}$ This version is preiminary. We are currently adding interaction term of income and number of children. Similarly, the interaction term of income and number of children in the utility function captures differences in the net costs/benefits from children of households with different income levels. These differences can be due to child care and other expenditures which can also reduce or increase the non-pecuniary benefits from children.

[^8]:    ${ }^{11}$ We will include an interaction term of each of the spouses income and number of children in the utility function to captures differences in the differences in expenditures and net cost/benefit of children in households with different income levels.

[^9]:    ${ }^{12}$ Level of education $E d_{\sigma}$ is a discrete random variable in the model where it can take 4 different values for: less than high school (LHS), high school (HS), some college (SC) and college (COL).

[^10]:    ${ }^{13}$ In many cases, an equilibrium exists, but a pure strategies equilibrium does not.
    Similar issues with existence arise in adverse selection models, see Gayle and Golan (2012).
    ${ }^{14}$ This equilibrium selection assumption is similar to to the assumptions used in the empirical literature on the estimation of dynamic incomplete information games.

[^11]:    ${ }^{15}$ Note that $T_{3}$ does not have to be $39(17$ to 55$)$ one can use less that 39 period in the final estimation. Reducing the number period in the final step will increase the computation speed of the estimator and the estimator will still be consistent but less efficient.

[^12]:    ${ }^{16}$ See Altug and Miller (1988) and Gayle and Golan (2012) for similar treatment of the skill.

[^13]:    ${ }^{17}$ These results are also consistent with part time jobs being more diffferent than full time jobs, for males more than for females.

[^14]:    ${ }^{18}$ We assume that females can not have children after the age of 45 in our empirical implementation, so this assumption is more restrictive for males who are significantly older than their spouse.
    ${ }^{19}$ In general, this may not hold in equilibrium because, as noted in Hill and Stafford (1974), when parents make the time allocation decisions, they take into account the differential effect of time on the different children which affect this trade-off and may adjust hours so the average quality does not decline.

[^15]:    ${ }^{20}$ The data limitation imposes the restriction that an additional hour is divided equally among all children under the age of 5 in the household. However, in addition to increasing the aggregate number of hours in order to increase the hours spent with each child, parents can also control the timing of birth which allows them to space children.

[^16]:    Source: Data from the Family-Individual File of the Michigan Panel Study of Income Dynamics (PSID) between 1968 and 1997.

