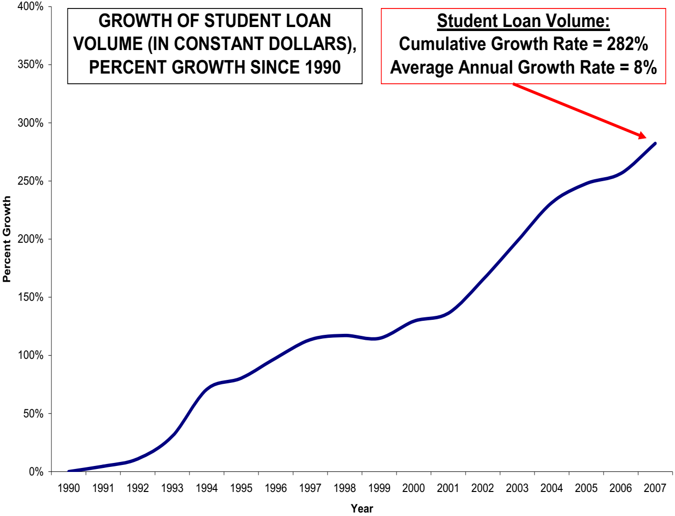
One-sided Commitment and College Enrollment

B. Ravikumar, Yaping Shan and Yuzhe Zhang

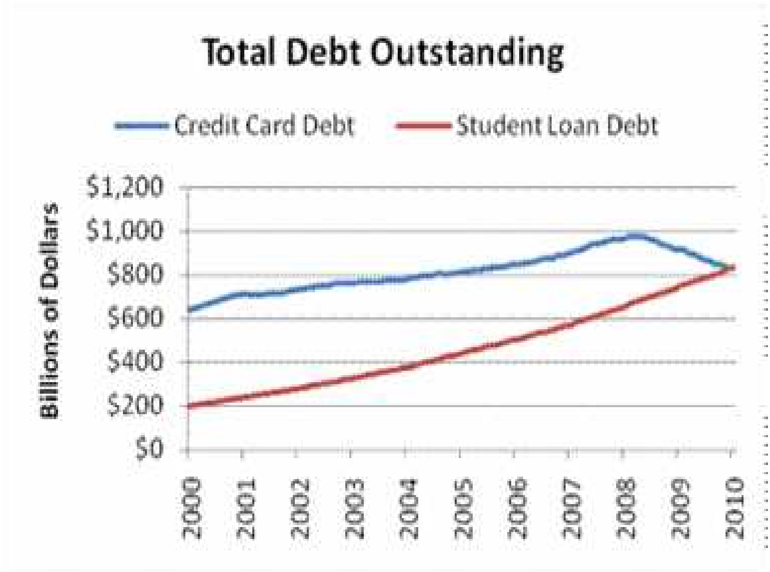
St Louis Fed, University of Iowa and Texas A&M

June 2012

- Financing college education
  - Student loan has been steadily rising, is more than credit card debt now.



Sources: U.S. Department of Education, Office of Postsecondary Education and FY2009 President's Budget.



Total Debt Outstanding
Credit Card Debt
Student Loan Debt
Billions of Dollars
$1,200
$1,000
$800
$600
$400
$200
$0
2000
2001
2002
2003
2004
2005
2006
2007
2008
2009
2010

- Financing college education
    - Student loan has been steadily rising, is more than credit card debt now.
    - Financial resources of the family have become more important in the college enrollment decision.
    - Skills and future earnings serve as poor collateral.

- Financing college education
    - Student loan has been steadily rising, is more than credit card debt now.
    - Financial resources of the family have become more important in the college enrollment decision.
    - Skills and future earnings serve as poor collateral.
- Endogenous borrowing constraints
    - Ability-enrollment correlation.

- Part 1: Principal-agent relationship
  - Borrowing over the life cycle.
  - One-sided commitment.
- Part 2: College enrollment
  - Role of life-cycle consumption smoothing.
  - Role of one-sided commitment.

Part 1: One-sided commitment: Life-cycle basics

- An agent (or a consumer, or a student) lives for T periods. Preferences are

$$\sum_{t=0}^{T} \beta^t u(c_t).$$

- His initial wealth is W.
- Earnings profile w_t has a hump shape, that is, there is a T^* such that w_t increases with t before T^* and decreases after T^*.
- There is a risk-neutral principal whose discount factor is also β.

- At time 0, the agent can sign a contract with the principal. The principal takes the wealth and income of the agent, and in exchange, the agent receives a consumption path $\{c_t; t = 0, ..., T\}$.
- The agent can choose to leave the contract.
- Default implies
  - Autarky
  - Fraction γ of his labor income seized every period.
- Participation constraint

$$\sum_{s=t}^{T} \beta^s u(c_s) \geq \sum_{s=t}^{T} \beta^s u\left((1-\gamma)w_s\right), \forall t.$$

Constrained efficient allocation

$$\max_{\{c_t;t\geq 0\}} \quad \sum_{t=0}^{T} \beta^t u(c_t),$$

$$\text{subject to} \quad \sum_{t=0}^{T} \beta^t c_t = \sum_{t=0}^{T} \beta^t w_t + W$$

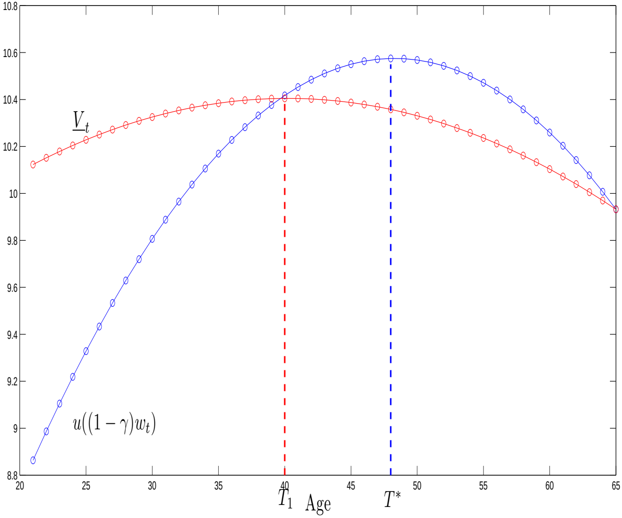
$$\sum_{s=t}^{T} \beta^s u(c_s) \geq \sum_{s=t}^{T} \beta^s u\left((1-\gamma)w_s\right), \forall t.$$

Normalized outside option

- It is useful to view the participation constraints in terms of flows.
- Normalized outside option

$$\underline{V}_t = \frac{\sum_{s=t}^{T} \beta^s u\left((1-\gamma)w_s\right)}{\sum_{s=t}^{T} \beta^s}.$$

Normalized outside option $\underline{V}(\cdot)$



First-order condition

$$(1+\beta^{-t}\lambda_0+\beta^{1-t}\lambda_1+\ldots+\lambda_t)u'(c_t)=\Phi,$$

where Φ is the multiplier for the budget constraint and λ_t is for the participation constraint (PC) in period t.

$$c_t \begin{cases} = c_{t-1}, & \text{if PC is slack at } t; \\ > c_{t-1}, & \text{if PC is binding at } t. \end{cases}$$

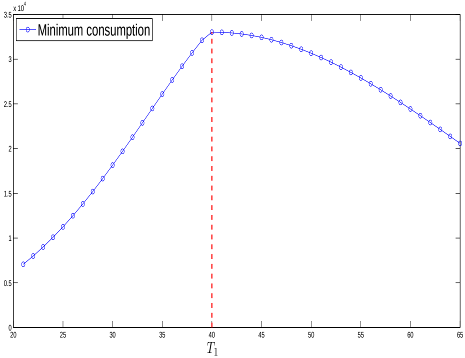
Minimum consumption $\underline{c}_t$

- Let $\underline{c}_t$ be the agent's consumption if the participation constraint binds.
- It is the minimum consumption required to prevent default.
- Efficient allocation:

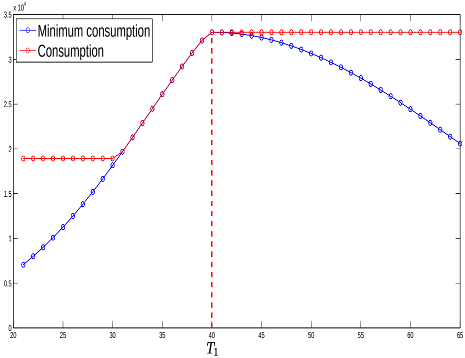
$$c_t = \max\{c_{t-1}, \underline{c}_t\}.$$

Minimum consumption $\underline{c}_t$

The minimum consumption is $(1-\gamma)w_t$ until period T_1 and $u^{-1}(\underline{V}_t)$ afterward.



Minimum consumption $\underline{c}_t$



The participation constraint is initially slack, then binds and finally becomes slack for $t \geq T_1$.

Minimum consumption before T_1

If the participation constraint binds at both t and $t+1$, then $\underline{c}_t = (1-\gamma)w_t$.

$$\begin{aligned}\sum_{s=t}^{T} \beta^s u(c_s) &= \sum_{s=t}^{T} \beta^s u((1-\gamma)w_s) \\ \sum_{s=t+1}^{T} \beta^s u(c_s) &= \sum_{s=t+1}^{T} \beta^s u((1-\gamma)w_s)\end{aligned}$$

imply that

$$u(c_t) = u((1-\gamma)w_t).$$

Implementation with endogenous borrowing constraint

- How to decentralize the constrained efficient allocation?

- Let the agent optimally borrow and save at the interest rate $r = \frac{1}{\beta} - 1$
  - Subject to a sequence of borrowing constraints.

Implementation with endogenous borrowing constraint

- Problem P:

$$\begin{aligned}
\max_{c_t; t \geq 0} \quad & \sum_{t=0}^{T} \beta^t u(c_t), \\
\text{subject to} \quad & c_t + \beta B_{t+1} = B_t + w_t \\
& B_t \geq \underline{B}_t, \forall t, \\
& B_0 = W,
\end{aligned}$$

where $\underline{B}_t$ is the endogenous borrowing constraint.

Borrowing constraint

- How to find the sequence $\underline{B}_t$?
- Construct $\underline{B}_t$ such that an agent with wealth $\underline{B}_t$ in problem P achieves the same utility as in autarky.
- This construction satisfies the participation constraint in every period.

Proposition

The borrowing constraint is initially slack, then binds and finally becomes slack for $t \geq T_1$.

Borrowing constraint

- If $t < T_1$ and $B_t = \underline{B}_t$, then the agent's participation constraint binds.
- His consumption path is $(1-\gamma)w_t, (1-\gamma)w_{t+1}, ..., (1-\gamma)w_{T_1-1}, (1-\gamma)w_{T_1}, (1-\gamma)w_{T_1}, ...$
-

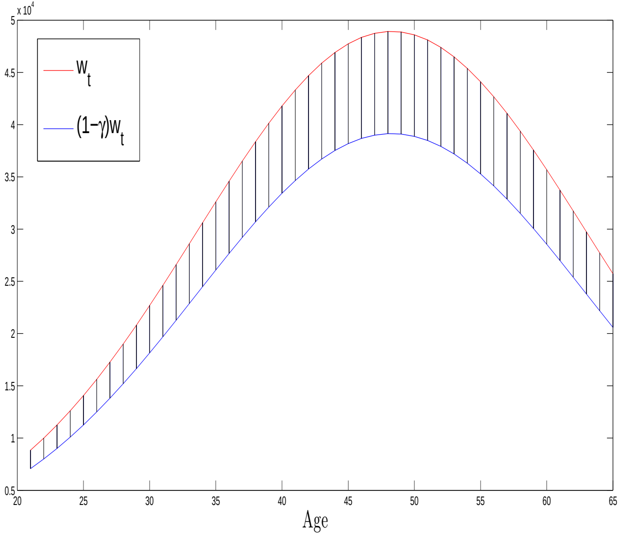
$$
\begin{aligned}
-\underline{B}_t &= \sum_{s=t}^{T} \beta^{s-t}(w_s - c_s) \\
&= \sum_{s=t}^{T} \beta^{s-t}\gamma w_t + \sum_{s=T_1}^{T} \beta^{s-t}(1-\gamma)\left(w_s - w_{T_1}\right),
\end{aligned}
$$

Borrowing constraint

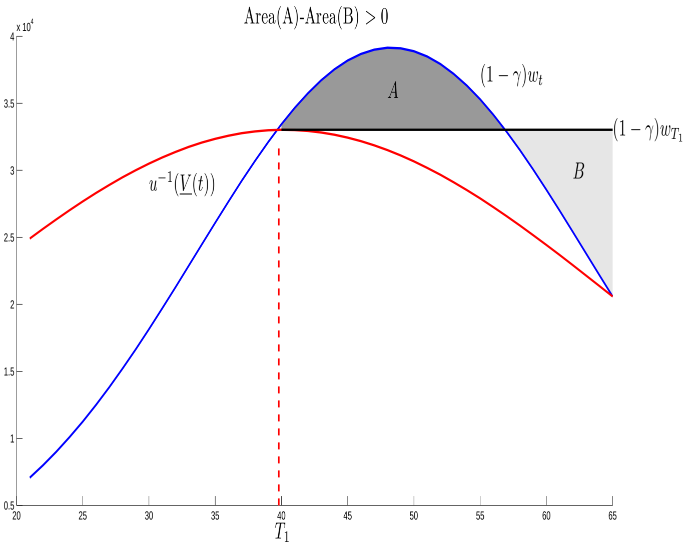
$$
\begin{aligned}
-\underline{B}_t &= \sum_{s=t}^{T} \beta^{s-t}(w_s - c_s) \\
&= \sum_{s=t}^{T} \beta^{s-t}\gamma w_t + \sum_{s=T_1}^{T} \beta^{s-t}(1-\gamma)\left(w_s - w_{T_1}\right),
\end{aligned}
$$

There are two components of income to be borrowed against:

1. penalty that can be collected after default
2. cost savings from consumption smoothing in $[T_1, T]$.



x 10^4
5
4.5
4
3.5
3
2.5
2
1.5
1
0.5
20
25
30
35
40
45
50
55
60
65
Age
w_t
(1−γ)w_t


Area(A)-Area(B) > 0
x 10^4
4
3.5
3
2.5
2
1.5
1
0.5
20
25
30
35
40
45
50
55
60
65
T_1
A
B
$(1-\gamma)w_t$
$(1-\gamma)w_{T_1}$
$u^{-1}(\underline{V}(t))$

Remarks on Borrowing constraint

- If income path is higher, the amount of borrowing is also higher.
- When $\gamma = 1$, the agent can borrow against all income. Autarky is undesirable. This is equivalent to full commitment.
- Even if $\gamma = 0$, the agent can still borrow, due to the cost savings from consumption smoothing in $[T_1, T]$.

Earnings Uncertainty

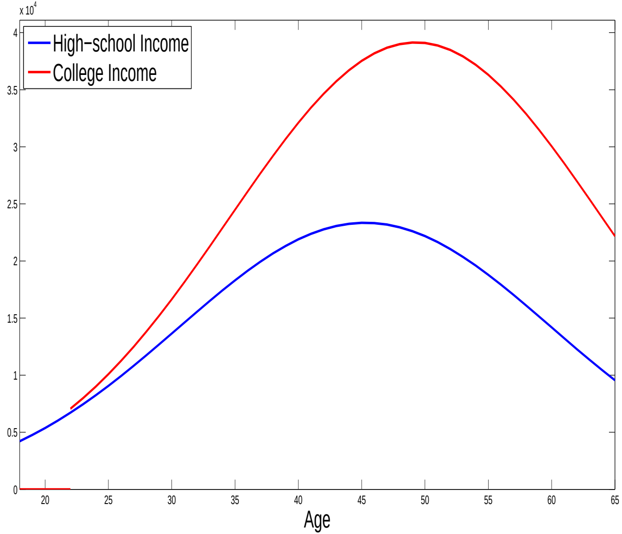
- Participation constraint depends on the realization of the income shock.
- Constrained efficient consumption depends on the history of shocks.
- The optimal contract provides insurance.

Endogenous Earnings

- Constrained efficient allocation could include human capital capital accumulation.
- Allocation in the optimal contract affects outside option.

Part 2: College enrollment under one-sided commitment

- Agents are heterogeneous in ability a and initial wealth W.
- Income depends on ability, education level and age:
  - High school income: $w_t(H)a$.
  - College income: $w_t(C)f(a)$.
- College tuition is τ.
- An agent spends one period in college.
- Agent's income is zero during college.


x 10^4
4
3.5
3
2.5
2
1.5
1
0.5
0
20
25
30
35
40
45
50
55
60
65
Age
High−school Income
College Income

Assumption

$\frac{f(a)}{a}$ *increases in* a.

- The agent compares two paths (one for college and one for high school) and chooses the one with a higher utility.
- The agent's initial wealth is $W - \tau$ if he chooses college, and W if he does not.

Full commitment

- The agent's utility relies only on the sum of discounted earnings and initial wealth.
- The agent compares the total discounted earnings under college path and high school path.
- He chooses college if and only if

$$\sum_{t=0}^{T} \beta^t w_t(H)a \leq \sum_{t=1}^{T} \beta^t w_t(C)f(a) - \tau$$

Full commitment enrollment decision

Proposition

There exists a threshold $\tilde{a}_{fc}$ such that agent with ability $a \geq \tilde{a}_{fc}$ enrolls in college and agent with ability $a < \tilde{a}_{fc}$ chooses to enter the labor market as a high school graduate.

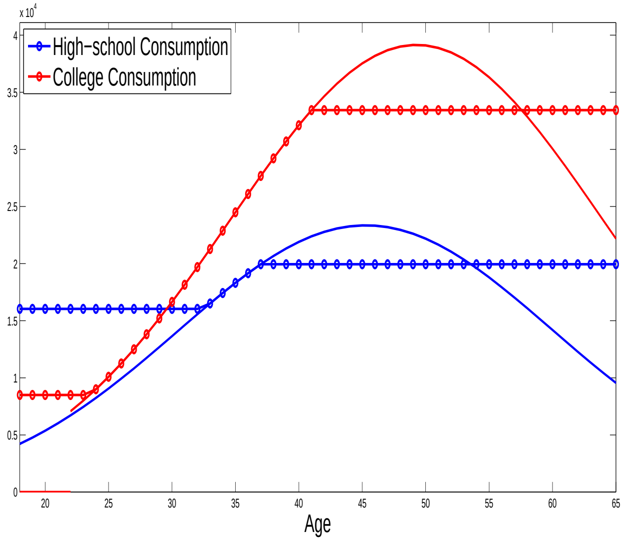
- Wealth does not enter the comparison.

One-sided commitment

Benefit and cost of college

- $+$: discounted income is higher
- $+$: college graduate may borrow more.
- $-$: tuition payment.
- $-$: consumption path is more distorted.

Unlike full commitment, comparison of discounted income alone is not sufficient.


x 10^4
High−school Consumption
College Consumption
Age
0
0.5
1
1.5
2
2.5
3
3.5
4
20
25
30
35
40
45
50
55
60
65

One-sided commitment

- Wealth enters the comparison under one-sided commitment.
- Rich agents are more likely to attend college.

Proposition

If an agent with wealth W is indifferent between college and high school, then an agent with wealth $W_1 > W$ strictly prefers college.

- High ability students are more likely to attend college, analogous to the full-commitment allocation.

Proposition

If $W \leq \tau$, then there exists a threshold $\tilde{a}(W)$ such that agent with ability $a \geq \tilde{a}(W)$ enrolls in college and agent with ability $a < \tilde{a}(W)$ chooses to enter the labor market as a high school graduate.

- Commitment friction distorts the college-enrollment decision.

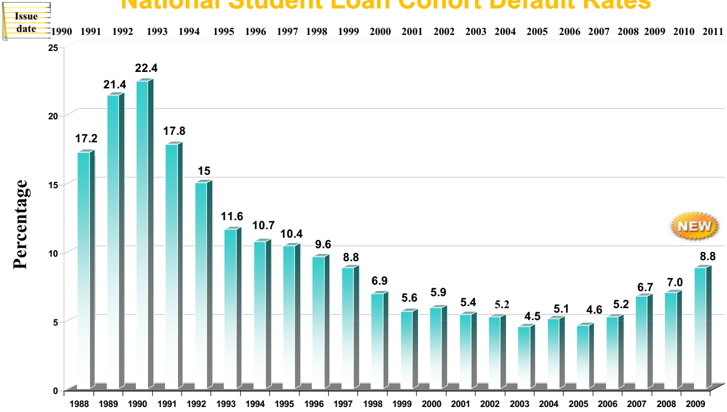
PROPOSITION

College enrollment under full commitment is greater than that under one-sided commitment, i.e., $\tilde{a}(W) > \tilde{a}_{fc}$.

Earnings Uncertainty

- It is efficient to have the repayment of student loan contingent upon the history of earnings shocks.
- The optimal contract provides insurance.
- Default might be an element of the optimal contract.

National Student Loan Cohort Default Rates



START HERE GO FURTHER FEDERAL STUDENT AID

Dropouts

- Negative signal in the first two years regarding future earnings.
- Dropout as a result of accumulated debt.

Adverse Selection - unobservable ability

- Information problems in addition to commitment problems.
- Low-ability agents could mimic high-ability agents, borrow resources for college and enroll in college.
- The optimal contract has to screen out the low-ability student by asking the agent with a low-income realization to repay the loan as well.
- Although the payment reduces insurance, it deters the low-ability agents from enrolling in college.