Human Capital Formation in Childhood and Adolescence

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Evolution of Inequality in USA

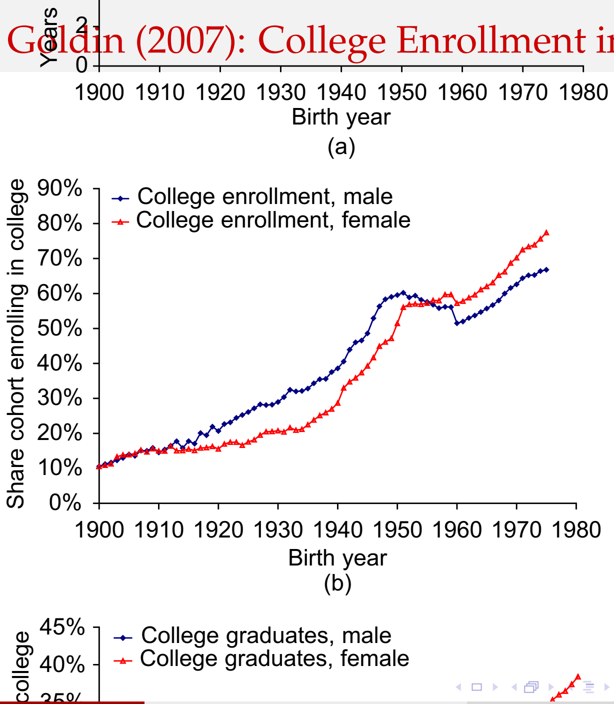
Katz and Goldin (2007): College Graduation in USA

Figure 1

College Graduation Rates (by 35 years) for Men and Women: Cohorts Born from 1876 to 1975

Sources: 1940 to 2000 Census of Population Integrated Public Use Micro-data Samples (IPUMS).

Katz and Goldin (2007): College Enrollment in USA



Katz and Goldin (2007): College Graduation Conditional on Enrollment in USA

quartile of the family income distribution, completion rates rose slightly from 67.4 to 71% between those starting college in the early 1980s and those starting in the early 1990s, while the college completion rates fell for students from other income groups (Bowen, Chingos, and McPherson (2009)). Indeed, for 1992 high school seniors who enrolled in college, the difference in college completion rates between the students

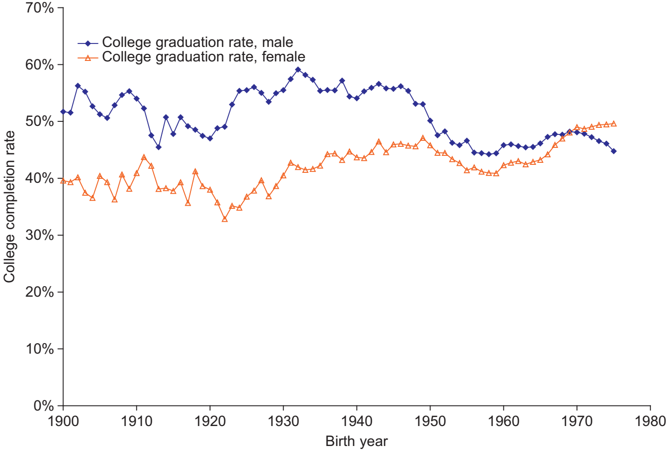
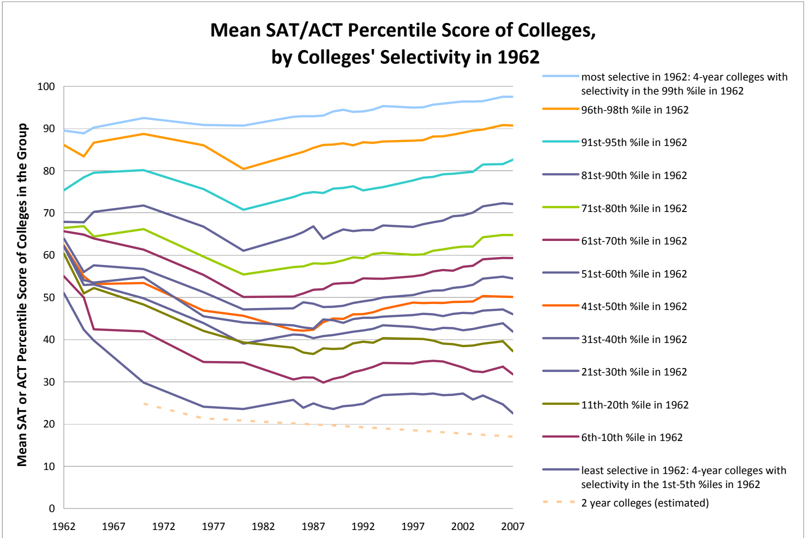


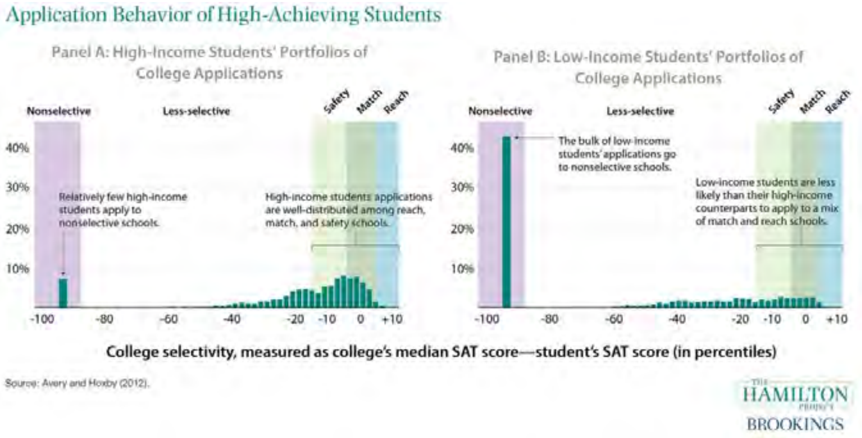
Figure 8.4 Share of College Entrants Receiving BA Degree.

Notes: The completion rate presented in this figure represents the ratio of the number of college degree recipients (Fig. 8.3c) to the number of individuals with at least some college (Fig. 8.3b). See Fig. 8.3 for

Hoxby (2009): Segmented Markets in Higher Education



Transition to College



USA vs OECD

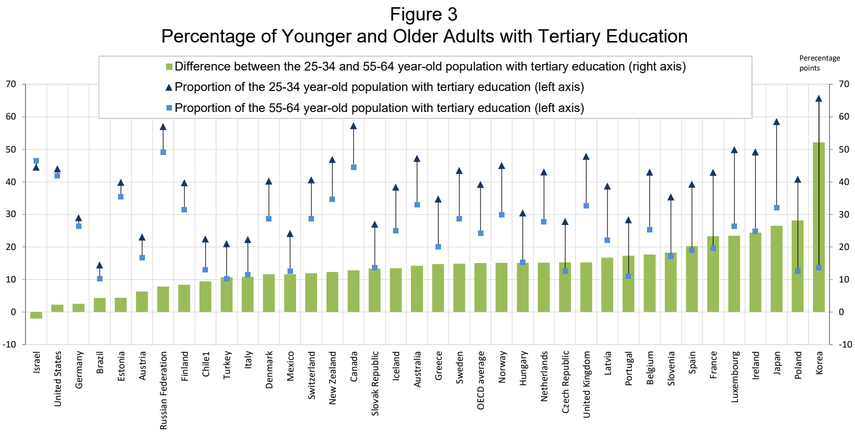


Figure 3
Percentage of Younger and Older Adults with Tertiary Education
Difference between the 25-34 and 55-64 year-old population with tertiary education (right axis)
Proportion of the 25-34 year-old population with tertiary education (left axis)
Proportion of the 55-64 year-old population with tertiary education (left axis)
Perecentage points
70
60
50
40
30
20
10
0
-10
Israel
United States
Germany
Brazil
Estonia
Austria
Russian Federation
Finland
Chile1
Turkey
Italy
Denmark
Mexico
Switzerland
New Zealand
Canada
Slovak Republic
Iceland
Australia
Greece
Sweden
OECD average
Norway
Hungary
Netherlands
Czech Republic
United Kingdom
Latvia
Portugal
Belgium
Slovenia
Spain
France
Luxembourg
Ireland
Japan
Poland
Korea

Returns and Stocks of Skilled/Unskilled Labor

- Let L_S and L_U denote, respectively, skilled and unskilled labor.
- Let w_S and w_U denote, respectively, skilled and unskilled wage rates.
- Consider the following problem:
- $$\min w_S L_S + w_U L_U$$

  subject to the technology of skill formation:

  $$Y = \left[\gamma L_S^{\phi} + (1-\gamma) L_U^{\phi}\right]^{\frac{1}{\phi}}$$

  where $\gamma \in [0,1]$ and $\phi \leq 1$.

Returns and Stocks of Skilled/Unskilled Labor

- Taking first-order conditions:

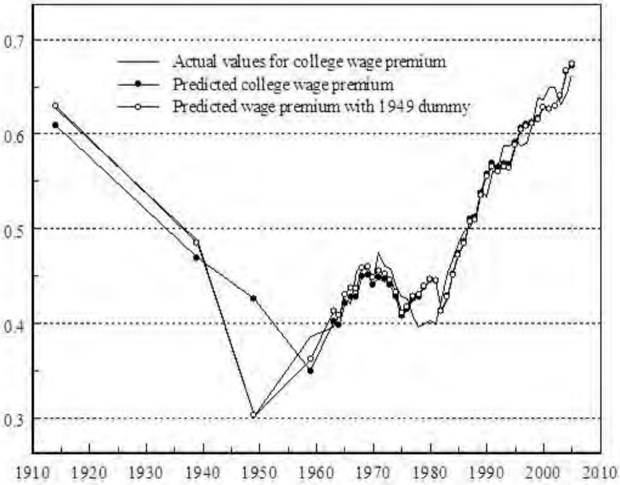
$$w_S = \lambda \left[\gamma L_S^{\phi} + (1-\gamma) L_U^{\phi}\right]^{\frac{1-\phi}{\phi}} \gamma L_S^{\phi-1}$$

$$w_U = \lambda \left[\gamma L_S^{\phi} + (1-\gamma) L_U^{\phi}\right]^{\frac{1-\phi}{\phi}} (1-\gamma) L_U^{\phi-1}$$

which yields:

$$\ln \frac{w_S}{w_U} = \ln \frac{\gamma}{1-\gamma} + (\phi - 1) \ln \frac{L_S}{L_U}$$

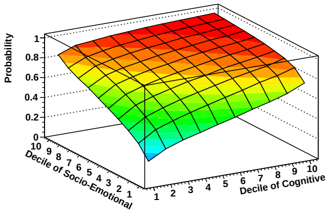
Katz and Goldin (2007): Model vs Data

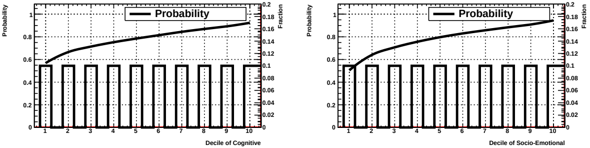


Plan

- Inequality in skills and inequality in adult socio-economic outcomes.
- Inequality in investments and inequality in skills.
- Increasing inequality in skills.
- Increasing inequality in investments.
- Evidence from RCTs.

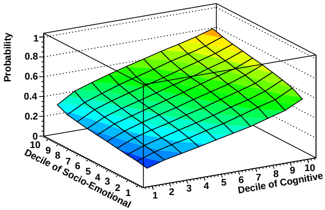
Figure 1: The Probability of Educational Decisions, by Endowment Levels, Dropping from Secondary School vs. Graduating

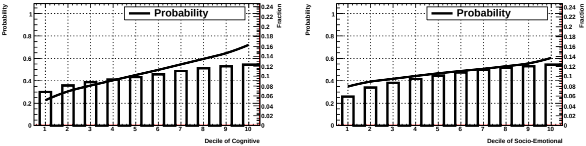




Source: Heckman, Humphries, Urzua, and Veramendi (2011).

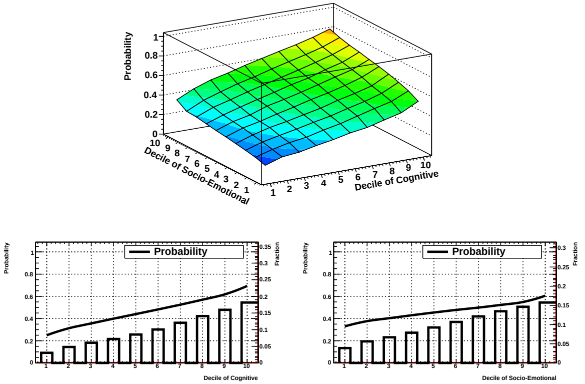
Figure 2: The Probability of Educational Decisions, by Endowment Levels, **HS Graduate** vs. College Enrollment





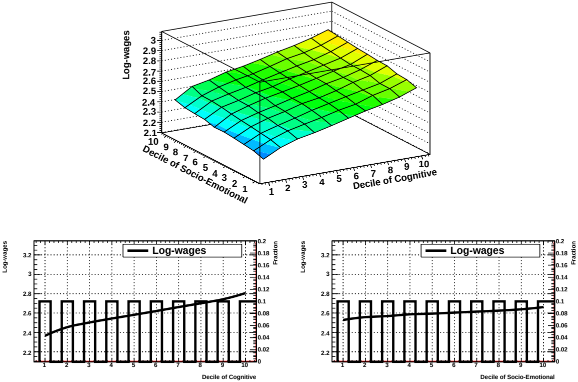
Source: Heckman, Humphries, Urzua, and Veramendi (2011).

Figure 3: The Probability of Educational Decisions, by Endowment Levels, **Some College** vs. **4-year college degree**



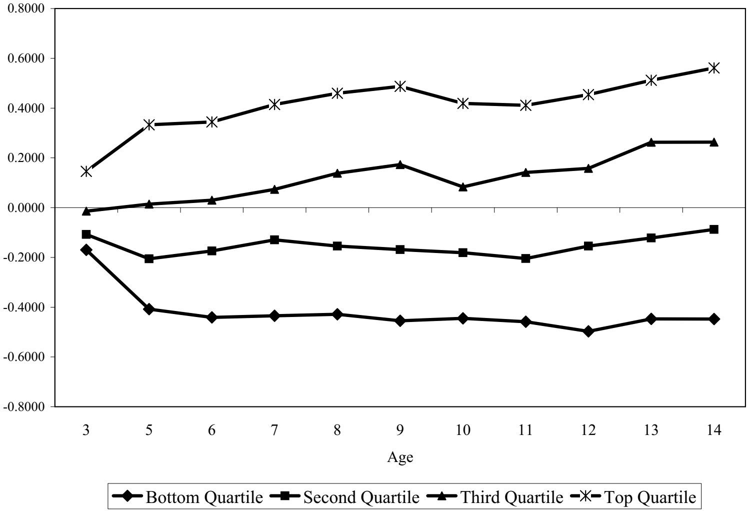
Source: Heckman, Humphries, Urzua, and Veramendi (2011).

Figure 4: The Effect of Cognitive and Socio-emotional endowments, (log) Wages



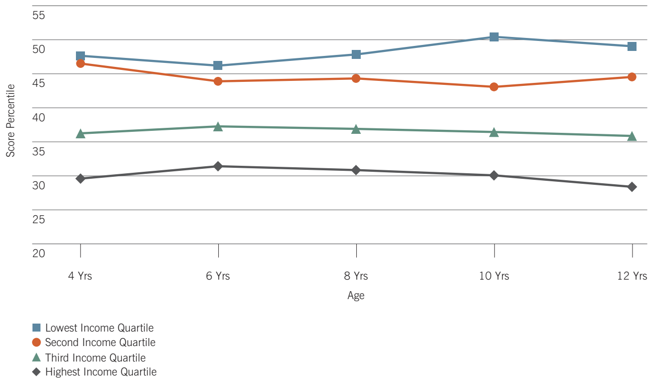
Source: Heckman, Humphries, Urzua, and Veramendi (2011).

The Gaps in Skill Open Up at Early Ages: Carneiro and Heckman (2002).



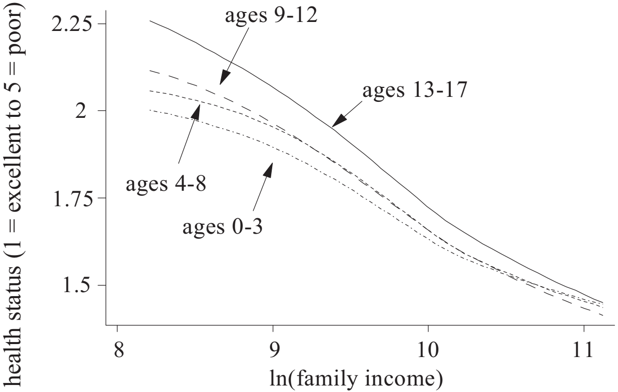
erage percentile rank on anti-social behavior score, by income quartile

Average Percentile Rank on Anti-Social Behavior Score, by Income Quartile



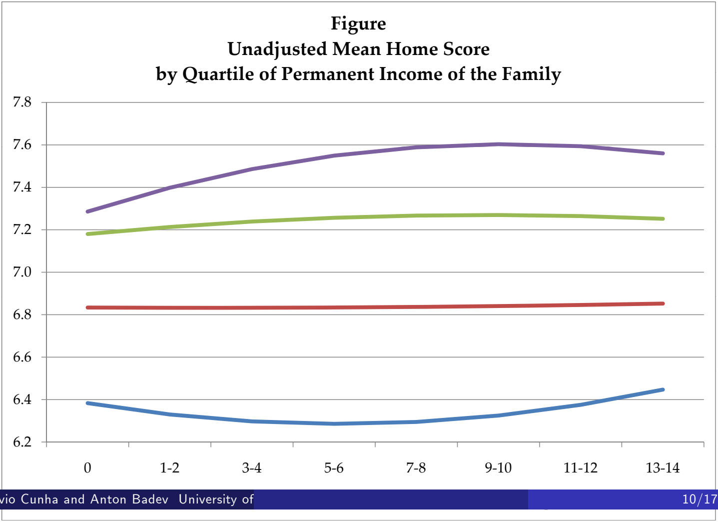
Inequality in Health as Children Age

Health and income for children and adults, U.S. National Health Interview Survey 1986-1995.*



* From Case, A., Lubotsky, D. & Paxson, C. (2002), American Economic Review, Vol. 92, 1308-1334.

Inequality in Skills are Partially the Result of Inequality in Investments: Cunha (2007)



Inequality in Investments as Children Age

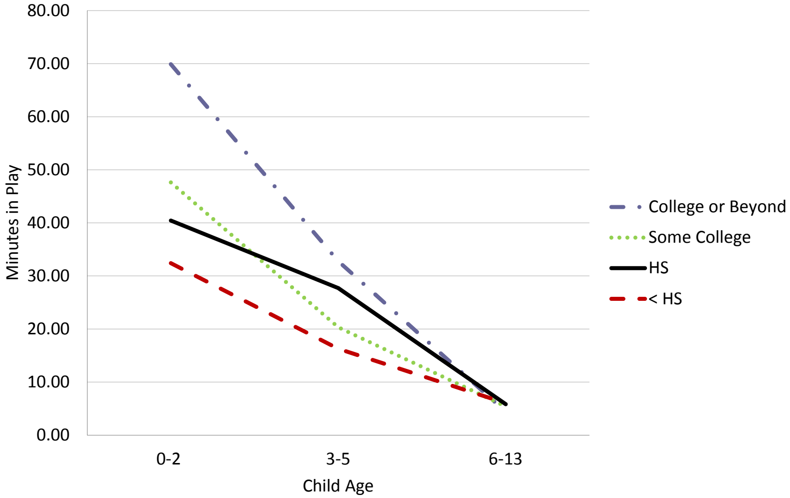
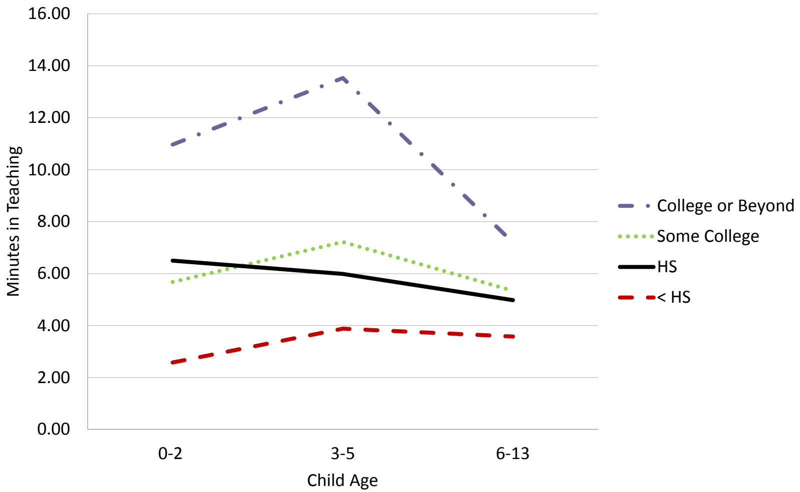


Figure 3: Education gradient in play time. Source: Kalil, A., Ryan, R., & Corey, M. (2012). Diverging
destinies: Maternal education and the developmental gradient in time with children. *Demography*, 49

Inequality in Investments as Children Age



Inequality in Investments as Children Age

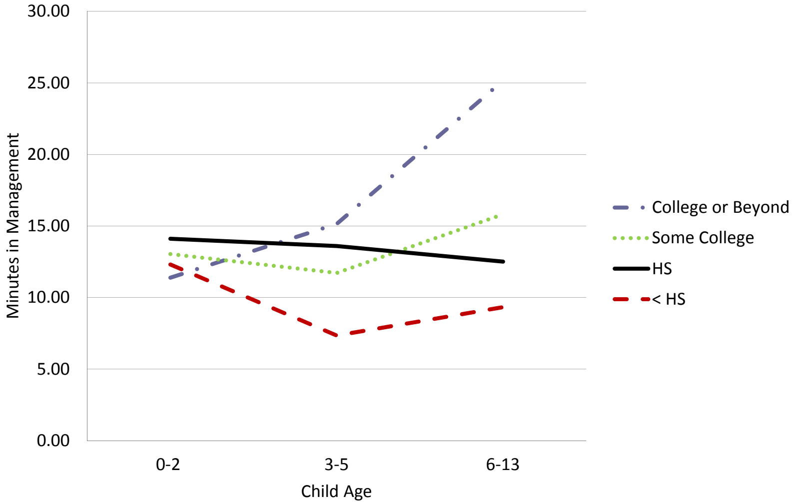
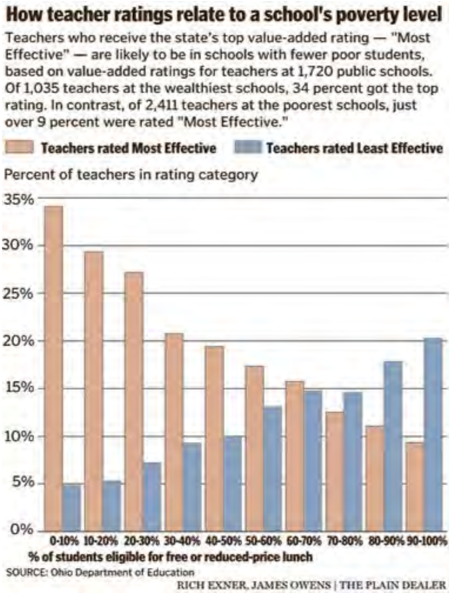
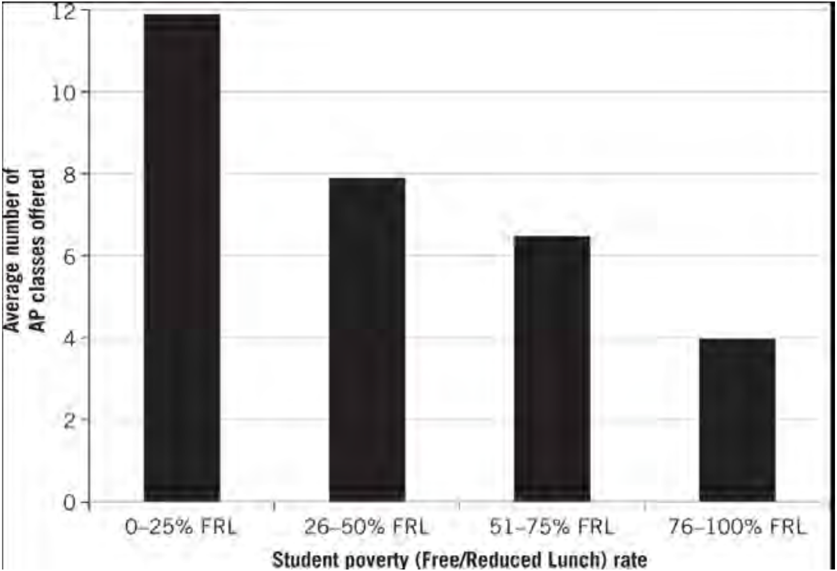
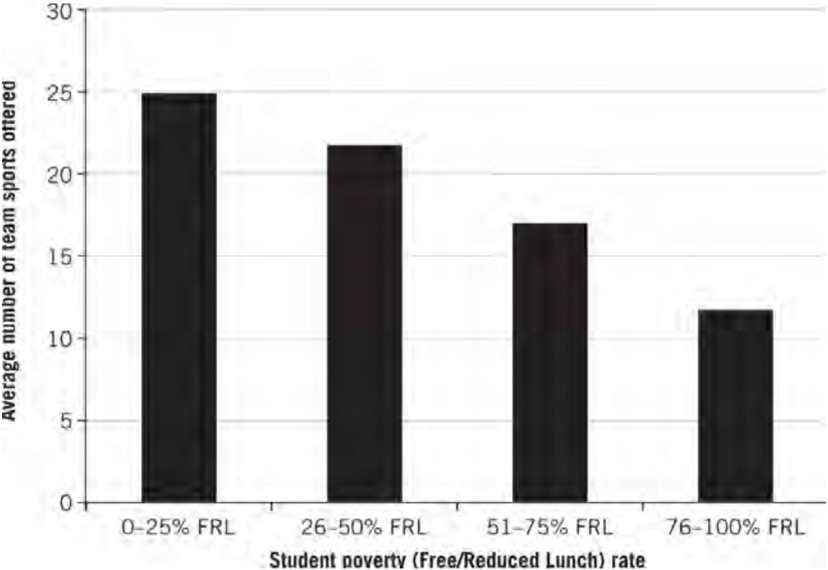


Figure 5: Education gradient in management time. Source: Kalil, A., Ryan, R., & Corey, M. (2012).


How teacher ratings relate to a school's poverty level
Teachers who receive the state's top value-added rating — "Most Effective" — are likely to be in schools with fewer poor students, based on value-added ratings for teachers at 1,720 public schools. Of 1,035 teachers at the wealthiest schools, 34 percent got the top rating. In contrast, of 2,411 teachers at the poorest schools, just over 9 percent were rated "Most Effective."
Teachers rated Most Effective
Teachers rated Least Effective
Percent of teachers in rating category
35%
30%
25%
20%
15%
10%
5%
0%
0-10%
10-20%
20-30%
30-40%
40-50%
50-60%
60-70%
70-80%
80-90%
90-100%
% of students eligible for free or reduced-price lunch
SOURCE: Ohio Department of Education
RICH EXNER, JAMES OWENS | THE PLAIN DEALER



Average number of AP classes offered
12
10
8
6
4
2
0
0–25% FRL
26–50% FRL
51–75% FRL
76–100% FRL
Student poverty (Free/Reduced Lunch) rate


Average number of team sports offered
30
25
20
15
10
5
0
0–25% FRL
26–50% FRL
51–75% FRL
76–100% FRL
Student poverty (Free/Reduced Lunch) rate

Inequality in Cognitive Skills Over Time

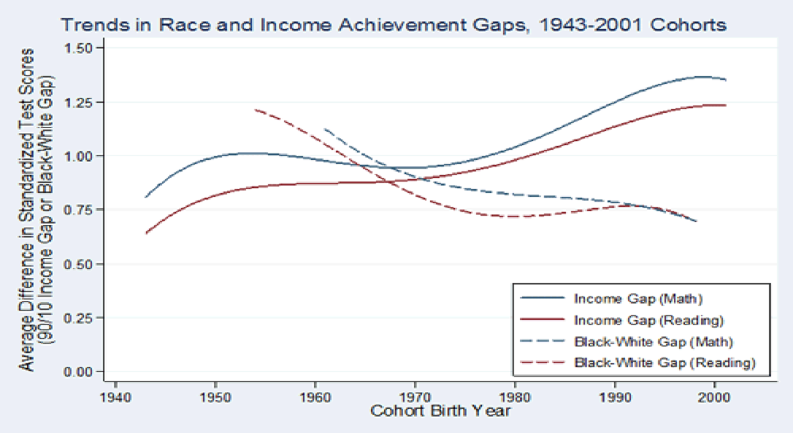
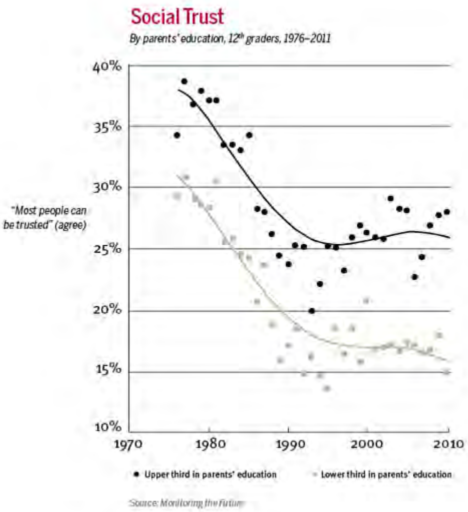
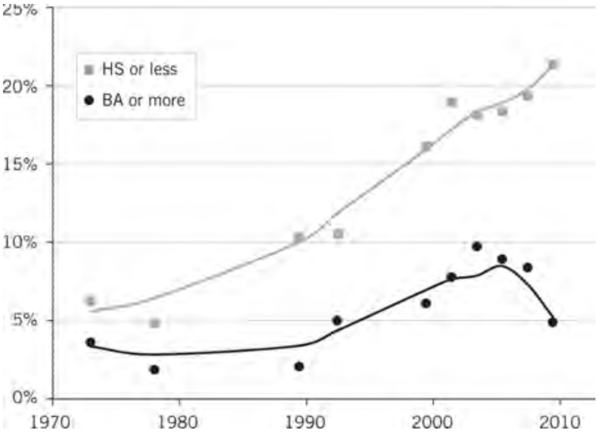


Figure 1: Trends in race and income achievement gaps, 1943-2001 Cohorts. Source: Reardon, S. (2011). The widening academic achievement gap between the rich and the poor: New evidence and possible explanations. In G. Duncan & R. Murnane (Eds.). *Whither Opportunity? Rising Inequality, Schools, and Children's Life Chances* (pp. 91-116). New York: Russell Sage Foundation and Spencer Foundation.

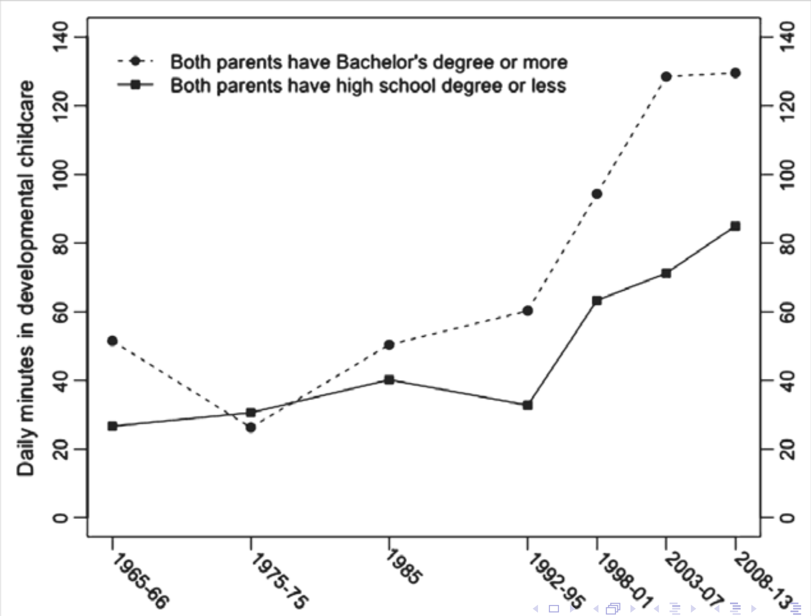
Inequality in Noncognitive Skills Over Time



Inequality in Health Over Time



Inequality in Investments Over Time



Inequality in Investments Over Time

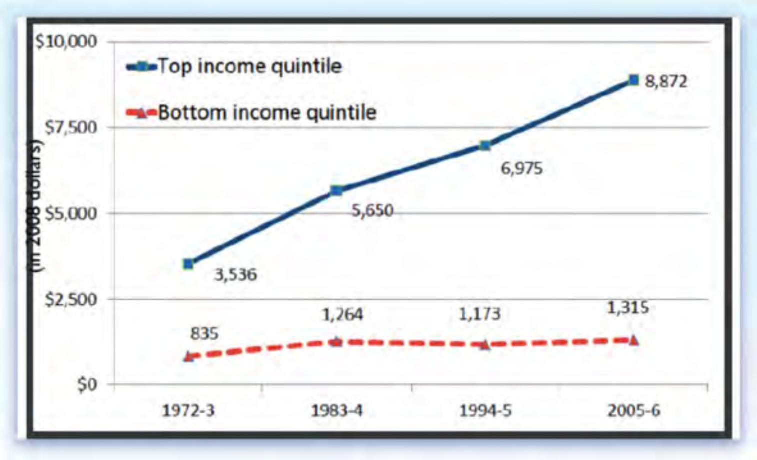
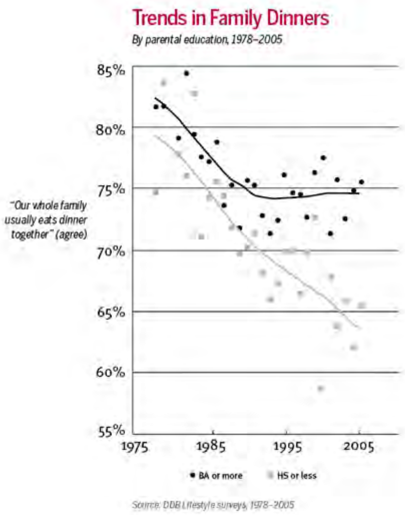
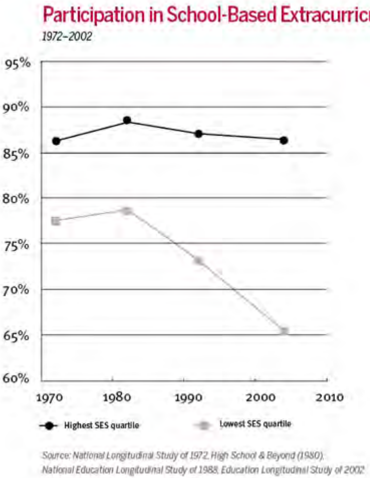


Figure 7: Enrichment expenditures on children, 1972-2006 (in $2008). Source: Kornrich, S., & Furstenberg, F. (2013). Investing in children: Changes in spending on children, 1972 to 2007. *Demography*, *50*, 1-23.

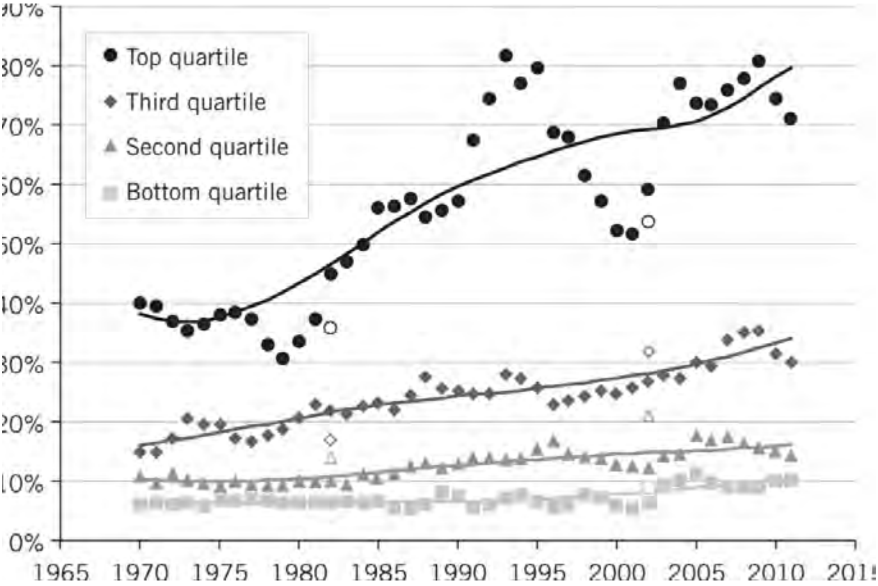
Inequality in Investments Over Time



Inequality in Investments Over Time



Increasing Inequality in College Attendance



Evidence from RCTs in Early Childhood and Adolescence

- Early interventions:
  - Perry Preschool Program
  - Abecedarian
  - Infant Health and Development Program (IHDP)

Early Childhood Education

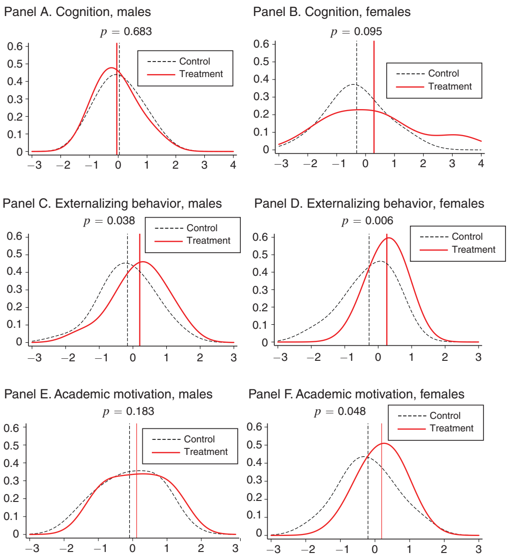


Figure 5. Kernel Densities of Factor Scores

In PPP and ABC, and for early education programs in general, non-cognitive skills are not typically followed in the long term.

Table 7: Life-Cycle Outcomes, PPP and ABC

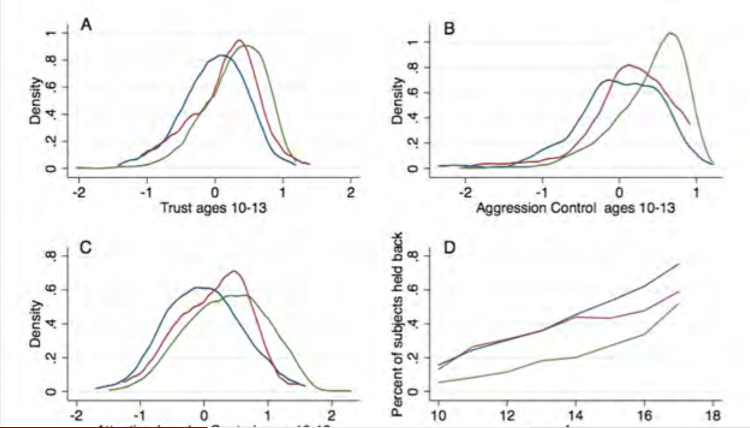
	PPP			ABC		
	Age	Female	Male	Age	Female	Male
Cognition and Education						
Adult IQ	-	-	-	21[c]	10.275	2.588
	-	-	-		**(0.005)**	(0.130)
High School Graduation	19[a]	0.56	0.02	21[c]	0.238	0.176
		(0.000)	(0.416)		**(0.090)**	**(0.100)**
Economic						
Employed	40[a]	-0.01	.29	30[c]	0.147	0.302
		(0.615)	**(0.011)**		(0.135)	**(0.005)**
Yearly Labor Income, 2014 USD	40[a]	$6,166	$8,213	30[c]	$3,578	$17,214
		(0.224)	(0.150)		**(0.000)**	(0.110)
HI by Employer	40[a]	0.129	0.206	31[b]	0.043	0.296
		(0.055)	(0.103)		(0.512)	**(0.035)**
Ever on Welfare	18–27[a]	-0.27	0.03	30[c]	0.006	-0.062
		(0.049)	(0.590)		(0.517)	**(0.000)**
Crime						
No. of Arrests[d]	≤40[a]	-2.77	-4.88	≤34[c]	-5.061	-6.834
		(0.041)	**(0.036)**		**(0.051)**	(0.187)
No. of Non-Juv. Arrests	≤40[a]	-2.45	-4.85	≤34[c]	-4.531	-6.031
One-sided permutation		**(0.051)**	**(0.025)**		**(0.061)**	(0.181)
Lifestyle						
Self-reported Drug User	-	-	-	30[c]	0.031	-0.438
-	-	-	-		(0.590)	**(0.030)**
Not a Daily Smoker	27[a]	0.111	0.119	-	-	-
		(0.110)	**(0.089)**	-	-	-
Not a Daily Smoker	40[a]	0.067	0.194	-	-	-
		(0.206)	**(0.010)**	-	-	-
Physical Activity	40[a]	0.330	0.090	21[b]	0.249	0.084
		(0.002)	(0.545)		**(0.004)**	(0.866)
Health						
Obesity (BMI >30)	-	-	-	30–34[c]	0.221	-0.292
	-	-	-		(0.920)	**(0.060)**
Hypertension I	-	-	-	30–34[c]	0.096	0.339
	-	-	-		(0.380)	**(0.010)**

Evidence from RCTs in Adolescence

- Becoming-A-Man (B.A.M.) Study:
  - Student training: Learning how to "read" the context to employ the "appropriate" reaction.
- Montreal Longitudinal Study
  - Parent training: Improve monitoring and positive reinforcement; implement non-punitive discipline; and how to better cope with crisis.
  - Child training: Teaching social skills to reduce aggressive behavior (including how to manage anger-inducing situations).

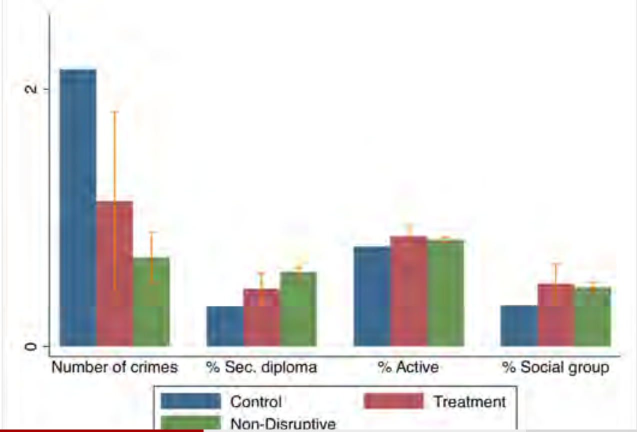
MELS: Algan et al (2014)

Figure 1. Non-cognitive skills and school performance during adolescence. A, B and C show distributions for non-cognitive skills measured in early adolescence for the control, treatment and non-disruptive groups (the non-disruptive boys being those who were not disruptive in kindergarten and did not participate in the experiment as treatment or control: they serve as a normative population baseline). Kolmorgorov-Smirnov test for equality of Treatment and Control distributions gives p-value of 0.003 for Trust, 0.036 for Aggression Control, and 0.023 for Attention-Impulse Control. D shows the increasing gap in the percent of subjects held back at each age. P-value from χ^2 test between Treatment and Control groups is 0.60 at age 10 and 0.01 at age 17.



MELS: Algan et al (2014)

Figure 2. Young Adult Outcomes. As young adults, treatment subjects commit fewer crimes, are more likely to graduate from secondary school, are more likely to be active fulltime in school or work, and are more likely to belong to a social or civic group. The intervention closed part or all of the gap between boys ranked as disruptive in kindergarten but not treated (the control group) and the non-disruptive boys (who represent the normative population). Raw differences are significant for secondary diploma (p-value=0.04) and group membership (p-value=0.05), conditional differences (controlling for group imbalances) are significant for number of crimes (p-value=0.09) and percent active fulltime (p-value=0.03).



Rest of Presentation

- Equation that describes skill formation process.
- Identification and estimation of key parameters of the equation.
- Constraints: Decision maker preference and information set
- Identification and estimation of subjective information set.

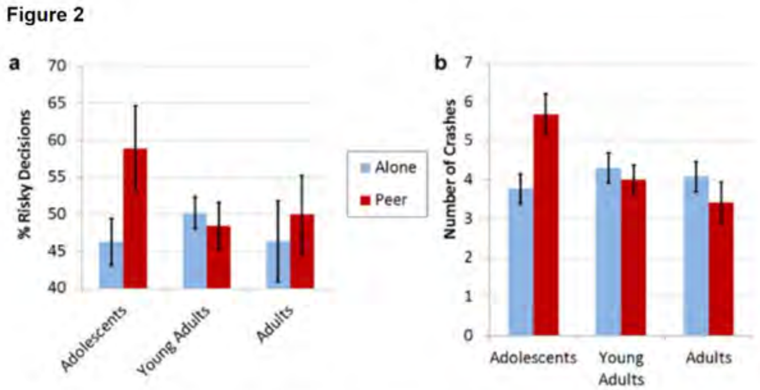
Skills Developed in Early Childhood

- **Early development**:
  - Development of language and cognitive skills
  - Development of **executive functions**:
    - Working memory;
    - Inhibitory control;
    - Cognitive flexibility.

- **Adolescent development**:

  It seems like people accept you more if you're, like, a dangerous driver or something. If there is a line of cars going down the road and the other lane is clear and you pass eight cars at once, everybody likes that. . . . If my friends are with me in the car, or if there are a lot of people in the line, I would do it, but if I'm by myself and I didn't know anybody, then I wouldn't do it. That's no fun. — Anonymous teenager, as quoted in The Culture of Adolescent Risk-Taking (Lightfoot, 1997, p. 10)

Adolescence and Risk Taking



Differential susceptibility of adolescents to peer influences on Stoplight task performance

Mean (a) percentage of risky decisions and (b) number of crashes for adolescent, young adult, and adult participants when playing the Stoplight driving game either alone or with a peer audience. Error bars indicate the standard error of the mean.

Skills Developed in Adolescence

- **Adolescent development**:
  - Fast development of the reward system potentialized by the influence of peers.
  - Slow development of emotional intelligence: self regulation:
    - Patience for reflection and thoughtfulness;
    - Comfort with ambiguity and change;
    - Ability to say no to impulsive urges.

Technology of Skill Formation

- We formalize the notion that human capital accumulation is one in which we produce different types of skills at different stages of the lifecycle.
- This notion leads to a technology of skill formation that is described by two parameters:
  - Self-productivity of skills: I learn how inhibit control early on, that helps me learn how to "read" the context before choosing an action when adolescent.
  - Dynamic complementarity: The returns to the development of "reading" context are higher for the children that have learned how to inhibit control early on (and vice-versa).

Optimal Investments in Early Childhood and Adolescence

- Let $h_{i,0}$ and $x_{i,e}$ denote, respectively, human capital at birth and investment during early childhood.
- Let $h_{i,a}$ denote the human capital at beginning of adolescence. Assume that:

$$h_{i,a} = \left[\gamma_e x_{i,e}^{\phi_e} + (1 - \gamma_e) \, h_{i,0}^{\phi_e} \right]^{\frac{1}{\phi_e}}$$

Optimal Investments in Early Childhood and Adolescence

- Let $x_{i,a}$ denote investment during adolescence.
- Let $\overline{h}_i$ denote the human capital at beginning of adulthood. Assume that:

$$\overline{h}_i = \left[\gamma_a x_{i,a}^{\phi_a} + (1 - \gamma_a)\, h_{i,a}^{\phi_a}\right]^{\frac{1}{\phi_a}}$$

Optimal Investments in Early Childhood and Adolescence

- Apply recursion and assume $\phi_e = \phi_a = \phi$:

$$\overline{h} = \left\{ \gamma_a x_{i,a}^{\phi} + (1 - \gamma_a)\, \gamma_e x_{i,e}^{\phi} + (1 - \gamma_a)\,(1 - \gamma_e)\, h_{i,0}^{\phi} \right\}^{\frac{1}{\phi}}$$

- Note that:
  - The parameter $1 - \gamma_a$ captures self-productivity.
  - The parameter ϕ captures dynamic complementarity or substitutability.

Optimal Investments in Early Childhood and Adolescence

- The problem of the parent:

$$\min \; x_{i,e} + \frac{1}{1+r} x_{i,a}$$

subject to the technology of skill formation:

$$\overline{h} = \left\{ \gamma_a x_{i,a}^{\phi} + (1-\gamma_a)\, \gamma_e x_{i,e}^{\phi} + (1-\gamma_a)\,(1-\gamma_e)\, h_{i,0}^{\phi} \right\}^{\frac{1}{\phi}}$$

where $\gamma_a \in [0,1]$, $\gamma_e \in [0,1]$, and $\phi \leq 1$.

Boundary Solution when $\phi = 1$

- In this case:

$$\overline{h} = \gamma_a x_{i,a} + (1 - \gamma_a)\,\gamma_e x_{i,e} + (1 - \gamma_a)\,(1 - \gamma_e)\,h_{i,0}$$

- Two investment strategies: Invest early and produce $(1 - \gamma_a)\,\gamma_e$ units of human capital per unit of investment.
- Save in physical assets early and invest $1 + r$ late and produce $(1 + r)\,\gamma_a$ units of human capital.
- Should invest all early if, and only if:

$$\frac{(1 - \gamma_a)\,\gamma_e}{\gamma_a} > 1 + r$$

Boundary Solution when $\phi \to -\infty$

- In this case:

$$\overline{h}_i = \min \{x_{i,a}, x_{i,e}, h_{i,0}\}$$

- The solution to this problem is $x_{i,a} = x_{i,e} = h_{i,0}$ regardless of r.

Interior Solution when $-\infty < \phi < 1$

- The solution to this problem is characterized by the following ratio:

$$\ln \frac{x_{i,e}}{x_{i,a}} = \frac{1}{1-\phi} \ln \left[\frac{(1-\gamma_a)\,\gamma_e}{\gamma_a} \right] + \frac{1}{1-\phi} \ln \left(\frac{1}{1+r} \right)$$

Dual Side of Dynamic Complementarity

- Returns to late investments are higher for the individuals that have high early investments.
- BUT: Returns to early investments are higher for the individuals who will also have high late investments.
- In other words, if the child will not receive high late investments, then the impacts of early investments will be diminished.

Estimating the Technology of Skill Formation

- Return to the recursive formulation of the technology of skill formation:

$$h_{i,t+1} = \left[\gamma_t x_{i,t}^{\phi_t} + (1-\gamma_t)\, h_{i,t}^{\phi_t}\right]^{\frac{\rho_t}{\phi_t}} e^{\eta_{i,t+1}}$$

- Consider (simplified version of) the Kmenta (1967) approximation:

$$\ln h_{i,t+1} = \psi_{t,1} \ln x_{i,t} + \psi_{t,2} \ln h_{i,t} + \psi_{t,3} \ln x_{i,t} \ln h_{i,t} + \eta_{i,t+1}$$

- Where: $\psi_{t,1} = \gamma_t \phi_t$, $\psi_{t,2} = (1-\gamma_t)\phi_t$, and $\psi_{t,3} = \frac{1}{2}\rho_t \phi_t \gamma_t\,(1-\gamma_t)$.
- Possible to decompose $\eta_{i,t+1}$ into permanent and temporary shocks, but not going to do it today.

Estimating the Technology of Skill Formation

- To simplify the math, I will use a simpler version of the Kmenta approximation:

$$\ln h_{i,t+1} = \psi_{t,1} \ln x_{i,t} + \psi_{t,2} \ln h_{i,t} + \psi_{t,3} \ln x_{i,t} \ln h_{i,t} + \eta_{i,t+1}$$

  for $i = 1, ..., I$ and $t = 1, ..., T$.
- I will illustrate three problems in the estimation of the technology of skill formation:
  - Problem 1: data on measures of human capital have no cardinality: anchoring.
  - Problem 2: data on measures of human capital and investment have measurement error: latent factors.
  - Problem 3: data on investment is endogenous: instruments.

Estimating the Technology of Skill Formation

- To simplify the math, I will use a simpler version of the Kmenta approximation:

$$\ln h_{i,t+1} = \psi_{t,1} \ln x_{i,t} + \psi_{t,2} \ln h_{i,t} + \psi_{t,3} \ln x_{i,t} \ln h_{i,t} + \eta_{i,t+1}$$

  for $i = 1, ..., I$ and $t = 1, ..., T$.
- I will illustrate three problems in the estimation of the technology of skill formation:
  - **Problem 1: data on measures of human capital have no cardinality: anchoring.**
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Problem 1: Cardinality of Human Capital

- The notion of a production function implies that inputs and output have a well-defined metric.
- You put a units of investments and b units of current-period human capital and you produce x units of next-period human capital.
- Usually units of investments are time (e.g., hours per day) or money (e.g., dollars per month).
- What is the unit of human capital?

Table

Type of scale	Description	Possible statements	Allowed operators	Example
Nominal	Describes qualitative attributes	Identity, countable	=, ≠	Binary variable denoting gender
Ordinal	Describes objects that can be ordered in terms of "greater", "less", or "equal"	Identity, countable, less than/greater than relations	=, ≠, ≤, ≥	Utility levels, test scores, percentile scores
Interval (cardinal)	Describes objects that can be placed in equally spaced units without a true zero point.	Identity, countable, less than/greater than relations, equality of differences	=, ≠, ≤, ≥, +, -	Educational attainment, dates
Ratio (cardinal)	Describes objects that can be placed in equally spaced units that have a true zero point.	Identity, countable, less than/greater than relations, equality of differences, equality of ratios, true zero	=, ≠, ≤, ≥, +, -, ×, ÷	Earnings, length, age

Problem 1: Cardinality of Human Capital

- Let's approach this problem in the following way. Suppose that we have data on labor income, Y_i, at some point in adulthood (e.g., when the individual is 45 years old).
- We can "anchor" human capital at age t before adulthood, $t = 1, ..., T$, through the equation:

$$\ln Y_i = \ln h_{i,t} + \nu_{i,t}$$

- Now $\ln h_{i,t}$ is cardinal. Assume that $\ln h_{i,t} \sim N\left(\mu_h, \sigma^2_{h,t}\right)$, $\nu_{i,t} \sim N\left(0, \sigma^2_{\nu,t}\right)$.
- Note that $\ln Y_i \sim N\left(\mu_h, \sigma^2_{h,t} + \sigma^2_{\nu,t}\right)$

Problem 1: Cardinality of Human Capital

- Now, we have data on scores in standardized tests $M_{i,t,j}$ for $j = 1, ..., J$.
- Assume that the relationship between $M_{i,t,j}$ and $\ln h_{i,t}$ is:

$$M_{i,t,j} = \alpha_{t,j} + \beta_{t,j} \ln h_{i,t} + \varepsilon_{i,t,j}$$

  where $\varepsilon_{i,t,j} \sim N\left(0, \sigma^2_{t,j}\right)$ is measurement error.
- Therefore, we have that $M_{i,t,j} \sim N\left(\alpha_{t,j} + \beta_{t,j}\mu_h, \beta^2_{t,j}\sigma^2_{t,h} + \sigma^2_{t,j}\right)$.
- In particular, note that $M_{i,t,j} | \ln h_{i,t} \sim N\left(\alpha_{t,j} + \beta_{t,j} \ln h_{i,t}, \sigma^2_{t,j}\right)$.

Problem 1: Cardinality of Human Capital

- Solution: We need to transform at least one of the test scores at t so that the transformed measure has cardinality.
- Define $\tilde{m}_{i,t,1} = E\left(\ln Y_i \mid M_{i,t,1}\right)$ and $s_{t,1} = \frac{\beta_{t,1}^2 \sigma_{t,h}^2}{\beta_{t,1}^2 \sigma_{t,h}^2 + \sigma_{t,j}^2}$
- Use the fact that $\ln Y_i$ and $M_{i,t,1}$ are jointly normal to conclude that:

$$\tilde{m}_{i,t,1} = \left(1 - s_{t,1}\right) \mu_h + s_{t,1}\left(M_{i,t,1} - \alpha_{t,1}\right).$$

Problem 1: Cardinality of Human Capital

- Given that:

$$\tilde{m}_{i,t,1} = (1 - s_{t,1})\,\mu_h + s_{t,1}\,(M_{i,t,1} - \alpha_{t,1})$$

- and that:

$$M_{i,t,j} = \alpha_{t,j} + \beta_{t,j} \ln h_{i,t} + \varepsilon_{i,t,1}$$

- We conclude that:

$$\tilde{m}_{i,t,1} = (1 - s_{t,1})\,\mu_h + s_{t,1} \ln h_{i,t} + \frac{s_{t,1}}{\beta_{t,1}} \varepsilon_{i,t,1}$$

- We need to estimate $s_{t,1}$.

Problem 1: Cardinality of Human Capital

- We need to estimate $s_{t,1}$, but we don't observe $\ln h_{i,t}$. We do observe $\ln Y_i = \ln h_{i,t} + \nu_{i,t}$, so

$$\tilde{m}_{i,t,1} = (1 - s_{t,1})\, \mu_h + s_{t,1} \ln Y_i + \frac{s_{t,1}}{\beta_{t,1}} \varepsilon_{i,t,1} - s_{t,1} \nu_{i,t}$$

- Clearly, we can't use OLS because $\ln Y_i$ is correlated with $\nu_{i,t}$.
- We need an instrument. In particular, we need something that is correlated with $\ln Y_i$ (through $\ln h_{i,t}$), but not correlated with $\varepsilon_{i,t,1}$ or $\nu_{i,t}$.
- We have a few candidates:
  - Investment at period $t-1$.
  - Determinants of investment at period $t-1$ (e.g., random assignment to control or treatment arms of intervention).
  - If nothing else, then $\tilde{m}^*_{i,\tau,1}$ which is leave-one-out estimator of $\tilde{m}_{i,\tau,1}$ where $\tau \neq t$

Problem 1: Cardinality of Human Capital

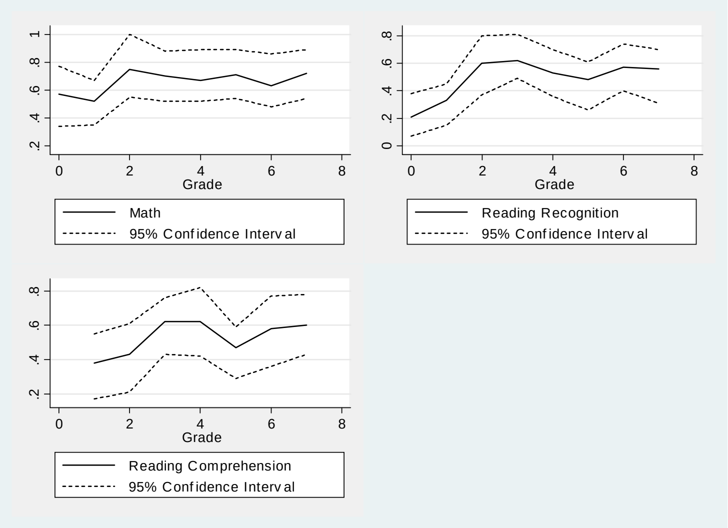
- Use one of these instruments to identify $s_{t,1}$ and define $m_{i,t,1} = \frac{\tilde{m}_{i,t,1}}{s_{t,1}}$

$$m_{i,t,1} = \frac{(1 - s_{t,1})}{s_{t,1}} \mu_h + \ln h_{i,t} + \frac{1}{\beta_{t,1}} \varepsilon_{i,t,1}$$

- Now we have a rescaled score that has a cardinal scale.

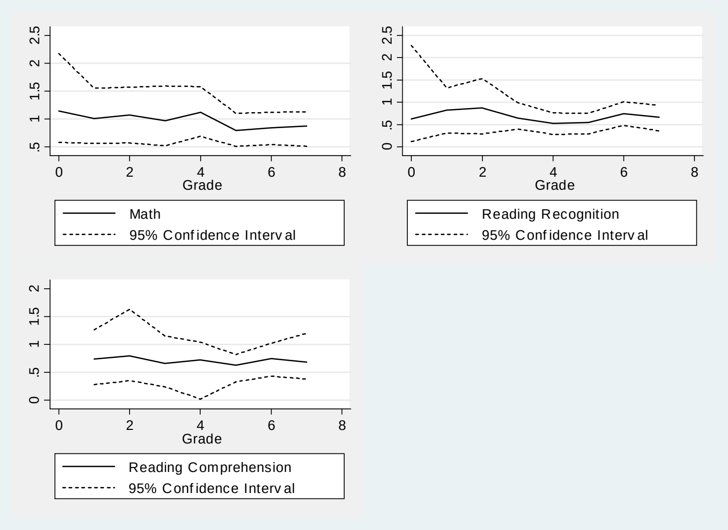
Applications of Anchoring: Bond & Lang (2018)

Figure 1: Raw Difference in Expected White Grade Completion conditional on Test Score



Applications of Anchoring: Bond & Lang (2018)

Figure 2: Measurement Error Adjusted Difference in Achievement in Units of Predicted White Education



Estimating the Technology of Skill Formation

- To simplify the math, I will use a simpler version of the Kmenta approximation:

$$\ln h_{i,t+1} = \psi_{t,1} \ln x_{i,t} + \psi_{t,2} \ln h_{i,t} + \psi_{t,3} \ln x_{i,t} \ln h_{i,t} + \eta_{i,t+1}$$

  for $i = 1, ..., I$ and $t = 1, ..., T$.
- I will illustrate three problems in the estimation of the technology of skill formation:
  - Problem 1: data on measures of human capital have no cardinality: anchoring.
  - **Problem 2: data on measures of human capital and investment have measurement error: latent factors**.
  - Problem 3: data on investment is endogenous: instruments.

Problem 2: Latent Factors

- At every age t we have J test scores and at least one of which (e.g., the first) is anchored:

$$m_{i,t,1} = \frac{(1-s_{t,1})}{s_{t,1}}\mu_h + \ln h_{i,t} + \frac{1}{\beta_{t,1}}\varepsilon_{i,t,1}$$

$$m_{i,t,j} = \alpha_{t,j} + \beta_{t,j} \ln h_{i,t} + \varepsilon_{i,t,j}$$

- At every age t we have J measures of investments:

$$p_{i,t,j} = \delta_{t,j} + \kappa_{t,j} \ln x_{i,t} + \xi_{i,t,j}$$

Problem 2: Latent Factors

- Rewrite in vector form:

$$\boldsymbol{m}_{i,t} = \boldsymbol{\alpha}_t + \boldsymbol{\beta}_t \ln h_{i,t} + \boldsymbol{\varepsilon}_{i,t}$$

- At every age t we have J measures of investments:

$$\boldsymbol{p}_{i,t} = \boldsymbol{\delta}_t + \boldsymbol{\kappa}_t \ln x_{i,t} + \boldsymbol{\xi}_{i,t}$$

Problem 2: Latent Factors

- Estimate $\boldsymbol{\alpha}_t$, $\boldsymbol{\beta}_t$, $\boldsymbol{\delta}_t$, $\boldsymbol{\kappa}_t$, matrix Σ_ϵ and matrix Σ_ξ to predict Bartlett scores:

$$\ln h_{i,t}^B = \left[\boldsymbol{\beta}_t^{'}\Sigma_\epsilon^{-1}\boldsymbol{\beta}_t\right]^{-1}\left[\boldsymbol{\beta}_t^{'}\Sigma_\epsilon^{-1}\left(\boldsymbol{m}_{i,t} - \boldsymbol{\alpha}_t\right)\right]$$

$$\ln x_{i,t}^B = \left[\boldsymbol{\kappa}_t^{'}\Sigma_\xi^{-1}\boldsymbol{\kappa}_t\right]^{-1}\left[\boldsymbol{\kappa}_t^{'}\Sigma_\xi^{-1}\left(\boldsymbol{p}_{i,t} - \boldsymbol{\delta}_t\right)\right]$$

Problem 2: Latent Factors

- Estimate $\boldsymbol{\alpha}_t$, $\boldsymbol{\beta}_t$, $\boldsymbol{\delta}_t$, $\boldsymbol{\kappa}_t$, matrix Σ_ϵ and matrix Σ_ξ to predict Bartlett scores:

$$\ln h_{i,t}^B = \ln h_{i,t} + \left[\boldsymbol{\beta}_t^{'}\Sigma_\epsilon^{-1}\boldsymbol{\beta}_t\right]^{-1}\left[\boldsymbol{\beta}_t^{'}\Sigma_\epsilon^{-1}\boldsymbol{\varepsilon}_{i,t}\right]$$

$$\ln x_{i,t}^B = \ln x_{i,t} + \left[\boldsymbol{\kappa}_t^{'}\Sigma_\xi^{-1}\boldsymbol{\kappa}_t\right]^{-1}\left[\boldsymbol{\kappa}_t^{'}\Sigma_\xi^{-1}\boldsymbol{\xi}_{i,t}\right]$$

Problem 2: Latent Factors

- Note that:

$$\ln h_{i,t}^{B} = \ln h_{i,t} + \tilde{\varepsilon}_{i,t}$$

$$\ln x_{i,t}^{B} = \ln x_{i,t} + \tilde{\xi}_{i,t}$$

- Note that $\tilde{\varepsilon}_{i,t} \sim N\left(0, \left[\beta_t^{'} \Sigma_{\epsilon}^{-1} \beta_t\right]^{-1}\right)$ and $\tilde{\xi}_{i,t} \sim N\left(0, \left[\kappa_t^{'} \Sigma_{\xi}^{-1} \kappa_t\right]^{-1}\right)$ and the variances are known.
- Using factor scores directly will not work because factor scores inherit measurement error (attenuation bias).
- However, bias is a function of $\left[\beta_t^{'} \Sigma_{\epsilon}^{-1} \beta_t\right]^{-1}$ and $\left[\kappa_t^{'} \Sigma_{\xi}^{-1} \kappa_t\right]^{-1}$ which are known. Therefore, we can account for the bias.

Problem 2: Latent Factors

- Define

$$h_t = \{\ln h_{i,t}\}_{i=1}^{I}$$
$$w_t = \{(\ln h_{i,t}, \ln x_{i,t}, \ln h_{i,t} \times \ln x_{i,t})\}_{i=1}^{I}$$
$$\boldsymbol{\gamma_t} = (\gamma_{t,1}, \gamma_{t,2}, \gamma_{t,3})$$

- Rewrite:

$$h_{t+1} = w_t \boldsymbol{\gamma_t} + \eta_{t+1}$$

- Let $\hat{\boldsymbol{\gamma}}_{\boldsymbol{t}}$ denote the infeasible OLS estimator that uses h and w (assumed to be exogenous).

$$\hat{\boldsymbol{\gamma}}_{\boldsymbol{t}} = \left(w_t^T w_t\right)^{-1} \left(w_t^T h_{t+1}\right)$$

- Easy to show that $\hat{\boldsymbol{\gamma}}_{\boldsymbol{t}}$ is consistent.

Problem 2: Latent Factors

- Let $\hat{\gamma}^{\boldsymbol{B}}$ denote the OLS estimator that uses Bartlett scores h^B and w^B (assumed to be exogenous).

$$\hat{\gamma}_{\boldsymbol{t}}{}^{\boldsymbol{B}} = \left[\left(w_t^B \right)^T w_t^B \right]^{-1} \left[\left(w_t^B \right)^T h_{t+1}^B \right]$$

- Note that w^B is error-ridden measure of w, so standard attenuation bias arises.
- Difference: attenuation bias is a function of variance of measurement error.
- The bias arises because of matrix $\left[\left(w_t^B \right)^T w_t^B \right]$.

Problem 2: Latent Factors

- The matrices $\left[w_t^T w_t\right]$ and $\left[\left(w_t^B\right)^T w_t^B\right]$ are symmetric with the following elements:

Element	plim $\left[w_t^T w_t\right]$	plim $\left[\left(w_t^B\right)^T w_t^B\right]$
$(1,1)$	$E\left(x_t^2\right)$	$E\left(x_t^2\right)+\boldsymbol{Var}\left(\boldsymbol{\xi_t}\right)$
$(1,2)$	$E\left(x_t h_t\right)$	$E\left(x_t h_t\right)$
$(1,3)$	$E\left(x_t^2 h_t\right)$	$E\left(x_t^2 h_t\right)+\boldsymbol{E}\left(\boldsymbol{h_t}\right)\boldsymbol{Var}\left(\boldsymbol{\xi_t}\right)$
$(2,2)$	$E\left(h_t^2\right)$	$E\left(h_t^2\right)+\boldsymbol{Var}\left(\boldsymbol{\varepsilon_t}\right)$
$(2,3)$	$E\left(x_t h_t^2\right)$	$E\left(x_t h_t^2\right)+\boldsymbol{E}\left(\boldsymbol{x_t}\right)\boldsymbol{Var}\left(\boldsymbol{\varepsilon_t}\right)$
$(3,3)$	$E\left(x_t^2 h_t^2\right)$	$E\left(x_t^2 h_t^2\right)+\boldsymbol{\Delta}$

where

$$\boldsymbol{\Delta}=\boldsymbol{E}\left(\boldsymbol{x_t^2}\right)\boldsymbol{Var}\left(\boldsymbol{\varepsilon_t}\right)+\boldsymbol{E}\left(\boldsymbol{h_t^2}\right)\boldsymbol{Var}\left(\boldsymbol{\xi_t}\right)+\boldsymbol{Var}\left(\boldsymbol{\xi_t}\right)+\boldsymbol{Var}\left(\boldsymbol{\varepsilon_t}\right)$$

Problem 2: Latent Factors

- Define matrix $A = \left(w_t^B\right)^T w_t^B - B$ where

$$B = \begin{bmatrix} \boldsymbol{Var}\left(\xi_t\right) & 0 & \boldsymbol{E}\left(h_t\right)\boldsymbol{Var}\left(\xi_t\right) \\ & \boldsymbol{Var}\left(\varepsilon_t\right) & \boldsymbol{E}\left(x_t\right)\boldsymbol{Var}\left(\varepsilon_t\right) \\ & & \boldsymbol{\Delta} \end{bmatrix}$$

- Feasible estimator $\hat{\gamma}^{\boldsymbol{A}}$ is consistent:

$$\hat{\gamma}^{\boldsymbol{A}} = \left[\left(w_t^B\right)^T w_t^B - B\right]^{-1}\left[\left(w_t^B\right)^T h_{t+1}^B\right]$$

Figure 3
Share of Residual Variance in Measurements of Cognitive Skills Due to the Variance of Cognitive Factor (Signal) and Due to the Variance of Measurement Error (Noise)

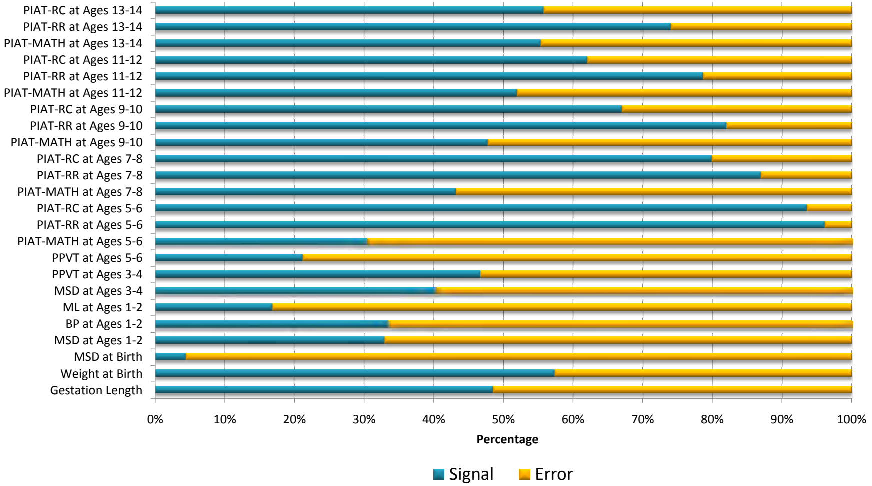


Figure 4A

Share of Residual Variance in Measurements of Noncognitive Skills Due to the Variance of Noncognitive Factor (Signal) and Due to the Variance of Measurement Error (Noise)

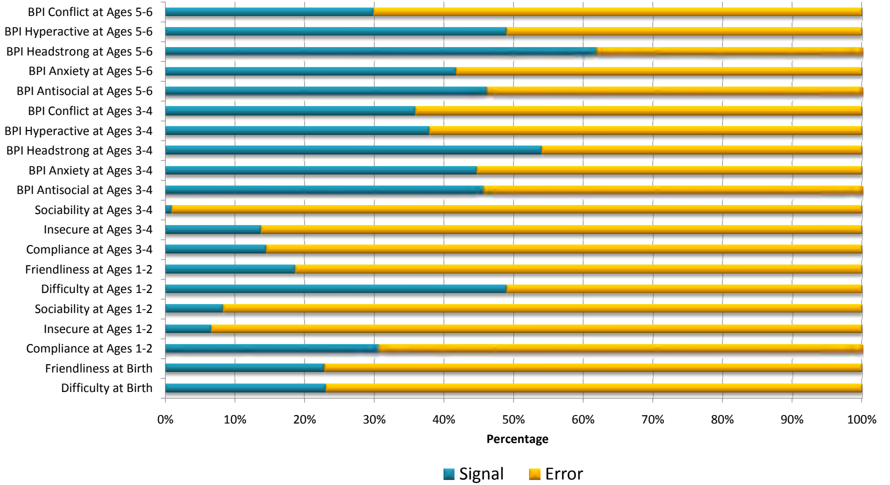


Figure 4B
Share of Residual Variance in Measurements of Noncognitive Skills Due to the Variance of Noncognitive Factor (Signal) and Due to the Variance of Measurement Error (Noise)

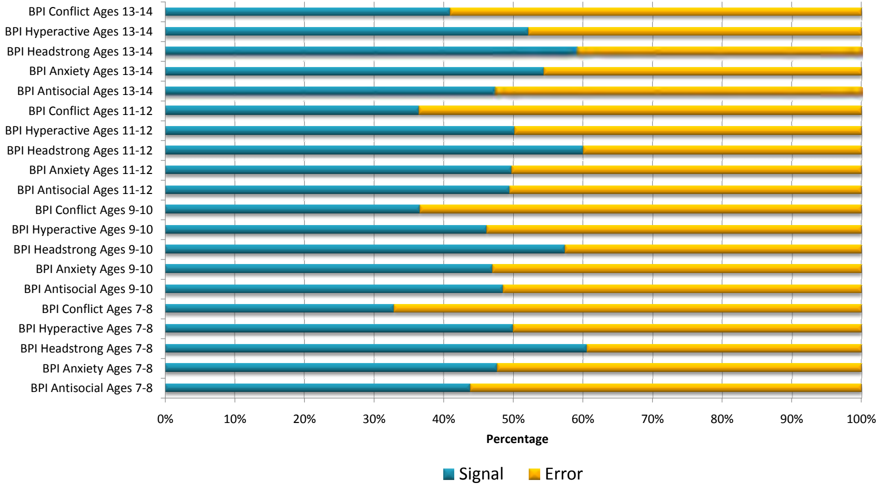
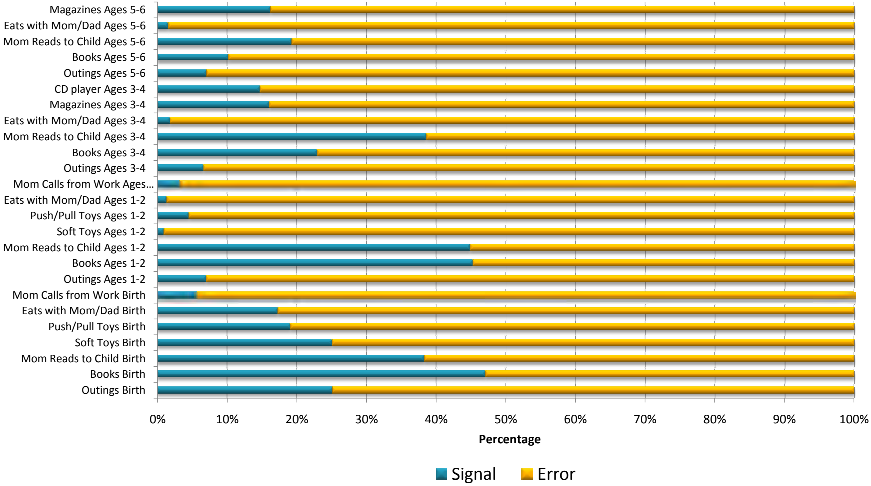


Figure 5A
Share of Residual Variance in Measurements of Investments Due to the Variance of Investment Factor (Signal) and Due to the Variance of Measurement Error (Noise)



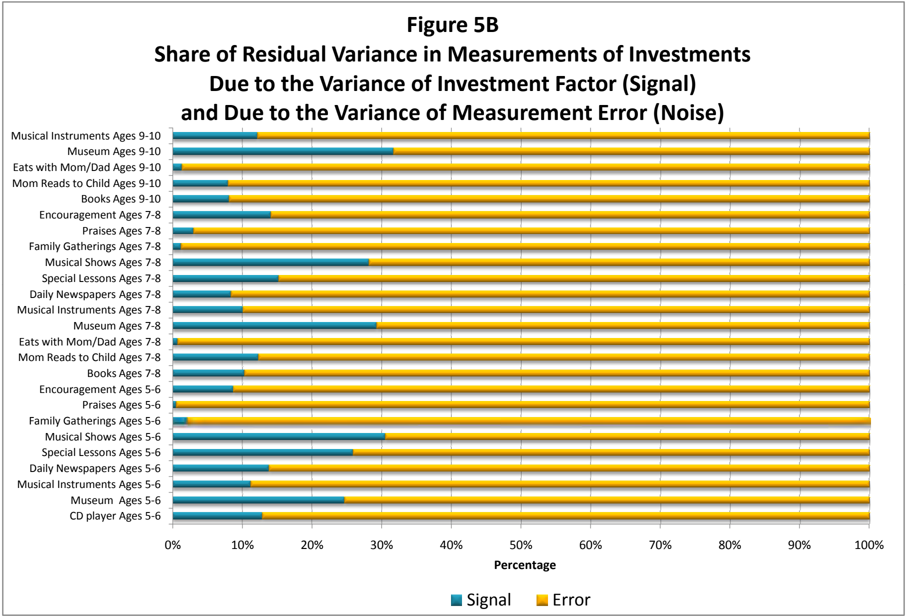
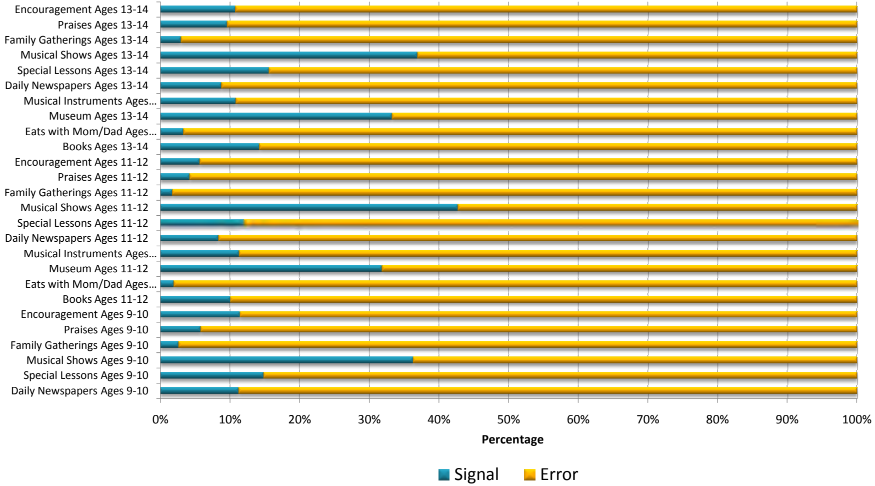


Figure 5B
Share of Residual Variance in Measurements of Investments
Due to the Variance of Investment Factor (Signal)
and Due to the Variance of Measurement Error (Noise)
Musical Instruments Ages 9-10
Museum Ages 9-10
Eats with Mom/Dad Ages 9-10
Mom Reads to Child Ages 9-10
Books Ages 9-10
Encouragement Ages 7-8
Praises Ages 7-8
Family Gatherings Ages 7-8
Musical Shows Ages 7-8
Special Lessons Ages 7-8
Daily Newspapers Ages 7-8
Musical Instruments Ages 7-8
Museum Ages 7-8
Eats with Mom/Dad Ages 7-8
Mom Reads to Child Ages 7-8
Books Ages 7-8
Encouragement Ages 5-6
Praises Ages 5-6
Family Gatherings Ages 5-6
Musical Shows Ages 5-6
Special Lessons Ages 5-6
Daily Newspapers Ages 5-6
Musical Instruments Ages 5-6
Museum Ages 5-6
CD player Ages 5-6
0%
10%
20%
30%
40%
50%
60%
70%
80%
90%
100%
Percentage
Signal
Error

Figure 5C
Share of Residual Variance in Measurements of Investments Due to the Variance of Investment Factor (Signal) and Due to the Variance of Measurement Error (Noise)



Estimating the Technology of Skill Formation

- To simplify the math, I will use a simpler version of the Kmenta approximation:

$$\ln h_{i,t+1} = \psi_{t,1} \ln x_{i,t} + \psi_{t,2} \ln h_{i,t} + \psi_{t,3} \ln x_{i,t} \ln h_{i,t} + \eta_{i,t+1}$$

  for $i = 1, ..., I$ and $t = 1, ..., T$.
- I will illustrate three problems in the estimation of the technology of skill formation:
  - Problem 1: data on measures of human capital have no cardinality: anchoring.
  - Problem 2: data on measures of human capital and investment have measurement error: latent factors.
  - **Problem 3: data on investment is endogenous: instruments.**

Problem 3: Instruments

- Note that:

$$\ln h_{i,t+1} = \psi_{t,1} \ln x_{i,t} + \psi_{t,2} \ln h_{i,t} + \psi_{t,3} \ln x_{i,t} \ln h_{i,t} + \eta_{i,t+1}$$

$$\ln x_{i,t} = z_{i,t} + v_{i,t}$$

- Here $z_{i,t}$ is the instrument.
- Valid instruments address not only endogeneity ($\ln x_{i,t}$ correlated with η_{t+1}) but also problems created by measurement error in $\ln x_{i,t}^{B}$.
- Instrument does not address bias due to measurement error in $\ln h_{i,t}^{B}$ unless we have a specific instrument for $\ln h_{i,t}$.

Estimates of the Technology of Skill Formation

Table V
The Technology for Cognitive and Noncognitive Skill Formation
Estimated Along With Investment Equation With Linear Anchoring on Educational Attainment (Years of Schooling); Factors Normally Distributed
Panel A: Technology of Cognitive Skill Formation (Next Period Cognitive Skills)

		First Stage Parameters		Second Stage Parameters
Current Period Cognitive Skills (Self-Productivity)	$\gamma_{1,C,1}$	0.426	$\gamma_{2,C,1}$	0.901
		(0.03)		(0.01)
Current Period Noncognitive Skills (Cross-Productivity)	$\gamma_{1,C,2}$	0.127	$\gamma_{2,C,2}$	0.014
		(0.04)		(0.01)
Current Period Investments	$\gamma_{1,C,3}$	0.322	$\gamma_{2,C,3}$	0.024
		(0.04)		(0.01)
Parental Cognitive Skills	$\gamma_{1,C,4}$	0.059	$\gamma_{2,C,4}$	0.062
		(0.02)		(0.01)
Parental Noncognitive Skills	$\gamma_{1,C,5}$	0.066	$\gamma_{2,C,5}$	0.000
		(0.04)		(0.01)
Complementarity Parameter	$\phi_{1,C}$	0.748	$\phi_{2,C}$	-1.207
		(0.25)		(0.16)
Implied Elasticity Parameter	$1/(1-\phi_{1,C})$	3.968	$1/(1-\phi_{2,C})$	0.453
Variance of Shocks $\eta_{C,t}$	$\delta^2_{1.C}$	0.159	$\delta^2_{2.C}$	0.092
		(0.01)		(0.00)

Panel B: Technology of Noncognitive Skill Formation (Next Period Noncognitive Skills)

		First Stage Parameters		Second Stage Parameters

Estimates of the Technology of Skill Formation

	$\gamma_{1,C,5}$	(0.04)	$\gamma_{2,C,5}$	(0.01)
Complementarity Parameter	$\phi_{1,C}$	0.748	$\phi_{2,C}$	-1.207
		(0.25)		(0.16)
Implied Elasticity Parameter	$1/(1-\phi_{1,C})$	3.968	$1/(1-\phi_{2,C})$	0.453
Variance of Shocks $\eta_{C,t}$	$\delta^2_{1,C}$	0.159	$\delta^2_{2,C}$	0.092
		(0.01)		(0.00)

Panel B: Technology of Noncognitive Skill Formation (Next Period Noncognitive Skills)

		First Stage Parameters		Second Stage Parameters
Current Period Cognitive Skills (Cross-Productivity)	$\gamma_{1,N,1}$	0.000	$\gamma_{2,N,1}$	0.000
		(0.02)		(0.01)
Current Period Noncognitive Skills (Self-Productivity)	$\gamma_{1,N,2}$	0.712	$\gamma_{2,N,2}$	0.868
		(0.03)		(0.01)
Current Period Investments	$\gamma_{1,N,3}$	0.195	$\gamma_{2,N,3}$	0.121
		(0.03)		(0.03)
Parental Cognitive Skills	$\gamma_{1,N,4}$	0.000	$\gamma_{2,N,4}$	0.000
		(0.01)		(0.01)
Parental Noncognitive Skills	$\gamma_{1,N,5}$	0.093	$\gamma_{2,N,5}$	0.011
		(0.03)		(0.02)
Complementarity Parameter	$\phi_{1,N}$	0.017	$\phi_{2,N}$	-0.323
		(0.27)		(0.21)
Elasticity Parameter	$1/(1-\phi_{1,N})$	1.017	$1/(1-\phi_{2,N})$	0.756
Variance of Shocks $\eta_{N,t}$	$\delta^2_{1,N}$	0.170	$\delta^2_{2,N}$	0.104
		(0.01)		(0.00)

Note: Standard errors in parenthesis.

Interpretation of Findings: Maximizing Average Education

- Suppose that H children are born, $h = 1, ..., H$.
- These children represent draws from the distribution of initial conditions $F(\theta_{c,1,h}, \theta_{n,1,h}, \theta_{c,p}, \theta_{n,p}, \pi)$.
- We want to allocate finite resources B across these children for early and late investments.
- Formally:

$$S^* = \max \frac{1}{H} \left[\sum_{h=1}^{H} S(\theta_{c,3}, \theta_{n,3}, \pi_h) \right]$$

subject to the technologies for the formation of cognitive and noncognitive skills as well as:

$$\textstyle\sum_{h=1}^{H} (x_{1,h} + x_{2,h}) = B$$

Interpretation of Findings: Minimizing Average Crime

- Another possibility is to minimize aggregate crime (average crime per individual).
- This will lead to different optimal ratios as crime is more sensitive to changes in noncognitive skills.
- Relative to cognitive skills, noncognitive skills are more malleable at later ages.

maximizing aggregate education (left) and minimizing aggregate crime (right) (other endowments held at mean levels).

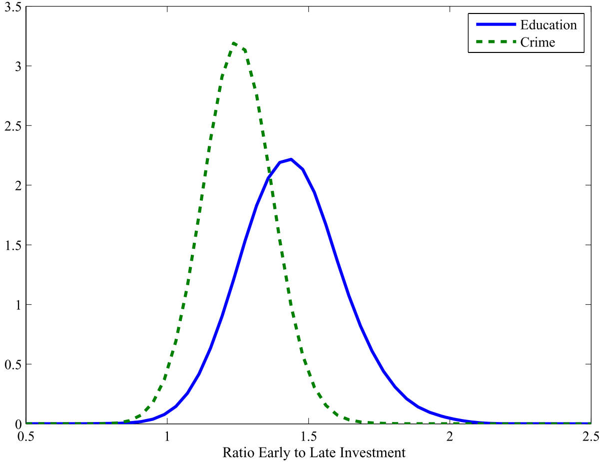


FIGURE 6.—Densities of ratio of early to late investments maximizing aggregate education versus minimizing aggregate crime.

Hart and Risley (1995): Children's Vocabulary Size

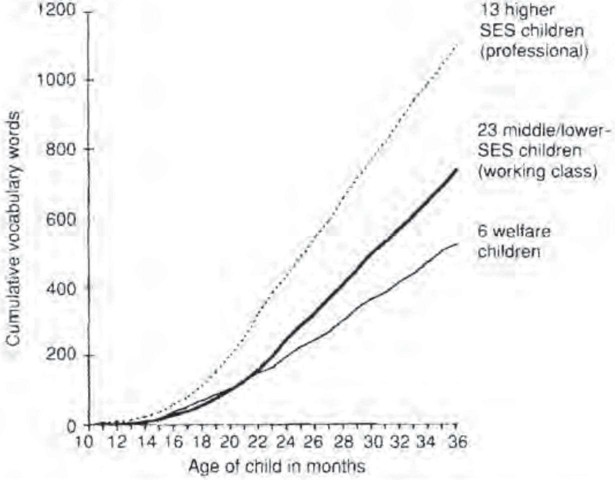
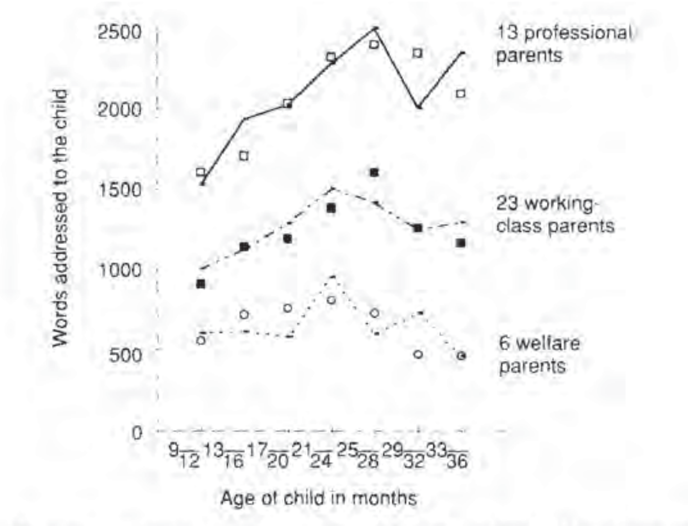


Figure 2. The widening gap we saw in the vocabulary growth of children from professional, working-class, and welfare families across their first 3

Hart and Risley (1995): Adult Words per Hour



Extending the Theory: Preferences

- Preferences are represented by the following utility function:

$$U\left(c, h_1, h_1^R\right) = \ln c + \alpha \ln h_1 + \beta 1\left(\ln h_1 \leq \ln h_R\right)$$

- Where:
  - c is consumption;
  - h_1 is the child's human capital at the end of the early childhood period;
  - h_R is the parent's reference point for the child's human capital level at the end of the early childhood period.
  - From the point of view of the parent, $\ln h_R \sim N\left(\mu_R, \sigma_R^2\right)$.

Theory: Budget Constraint

- I assume that parents cannot borrow or save:

$$c + px = y$$

- Where:
  - p is the relative price of the investment good;
  - x is the investment good;
  - y is the family income during the early childhood period.

Theory: Technology of Skill Formation

- I assume that the child's human capital at the end of the early childhood period is determined according to:

$$\ln h_1 = \gamma_0 + \gamma_1 \ln h_0 + \gamma_2 \ln x + \nu$$

- Where:
  - h_0 is the child's human capital at birth;
  - ν is a shock that is unanticipated by the parent and unobserved by the economist.
  - From the point of view of the parent, $\gamma_k \sim N\left(\mu_k, \sigma_k^2\right)$.

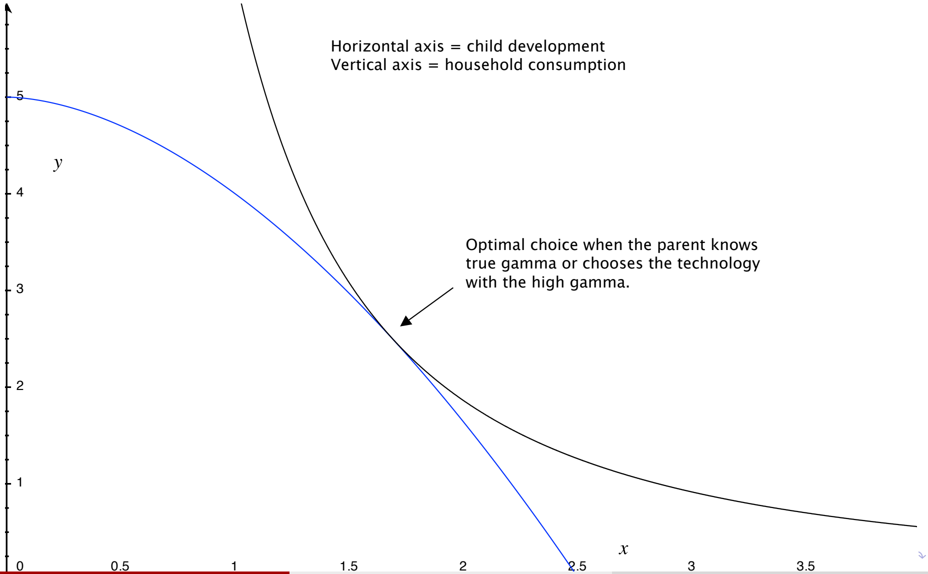
Theory: Parent's Information Set

- The parent's information set:

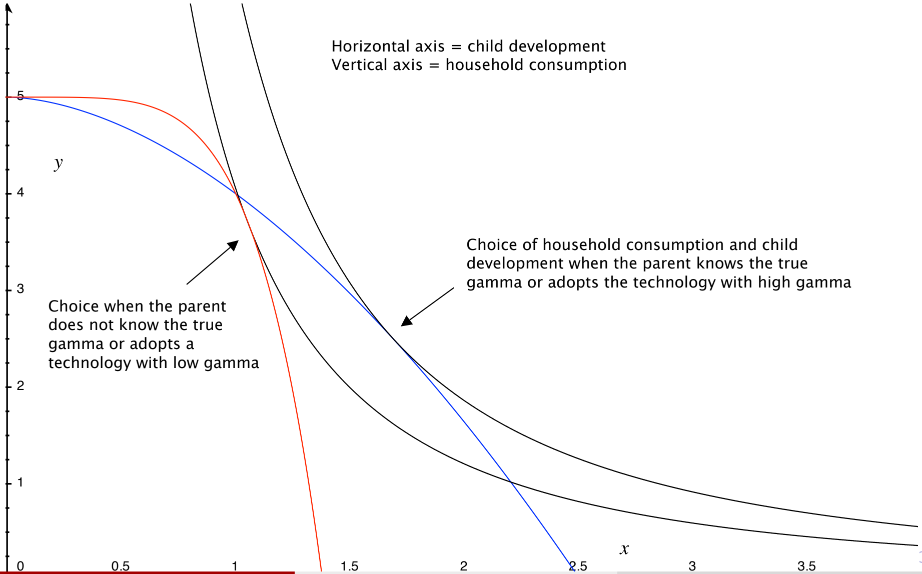
$$\Omega = \left\{ p, y, h_0, \epsilon, \Phi\left(\mu_R, \sigma_R^2\right), \left[\Phi\left(\mu_k, \sigma_k^2\right)\right]_{k=0}^{3} \right\}$$

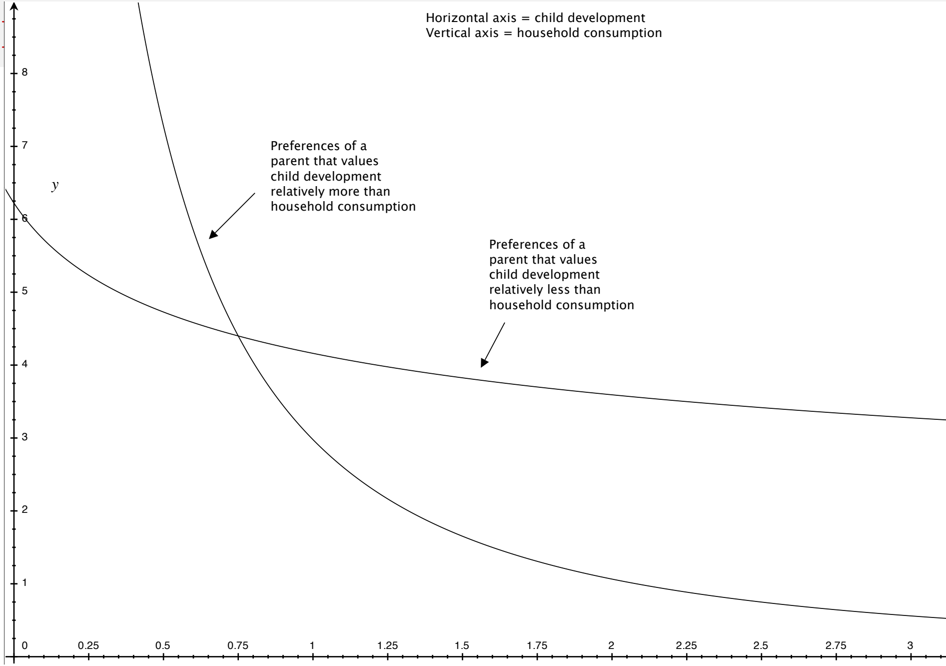
- Note that from the point of view of the parent:
  - $\Phi\left(\mu_R, \sigma_R^2\right)$ is the parent's perceived distribution of $\ln h_R$.
  - $\Phi\left(\mu_k, \sigma_k^2\right)$ is the parent's perceived distribution of γ_k.
- We do not impose any a priori restrictions on the parameters of these distributions.

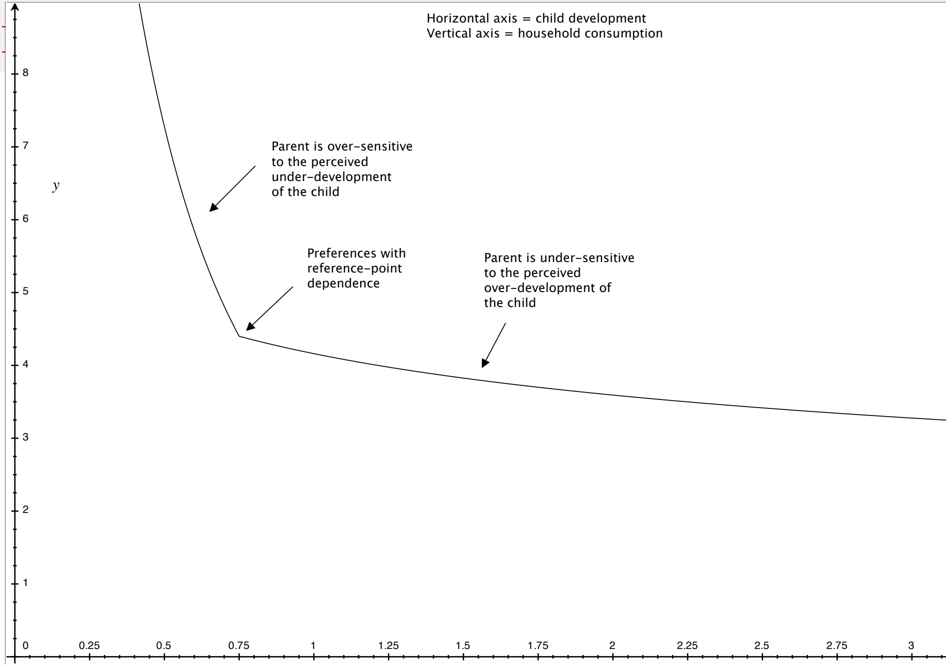
Typical Textbook Model



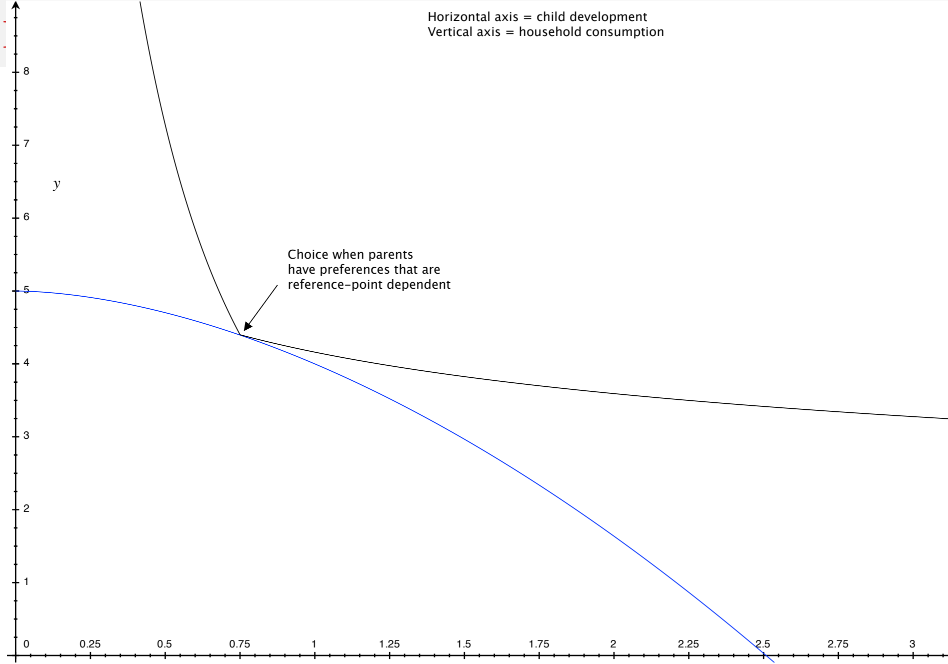
Introducing Heterogeneity in Beliefs



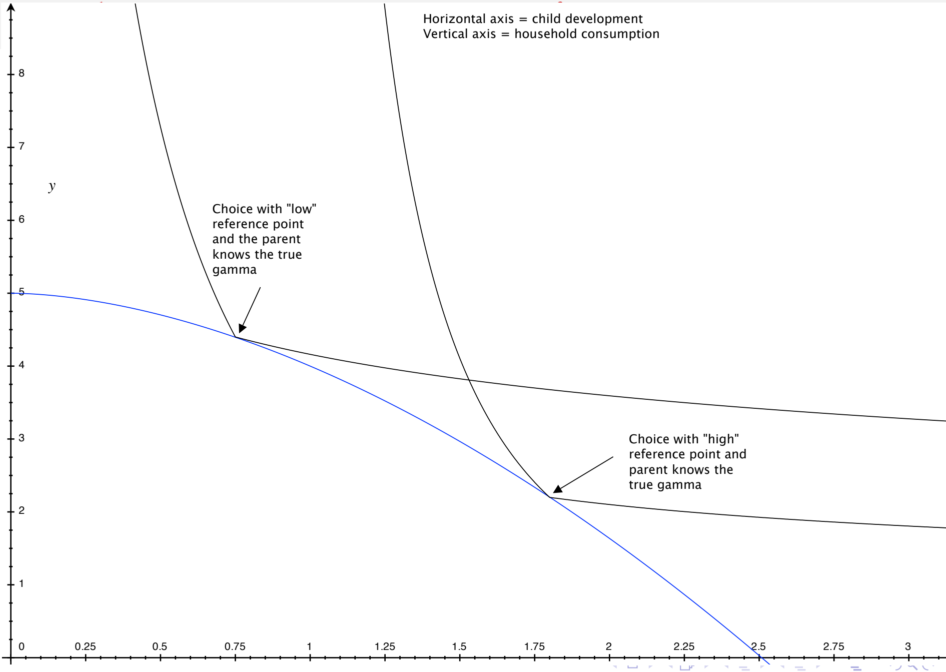

Horizontal axis = child development
Vertical axis = household consumption
Preferences of a parent that values child development relatively more than household consumption
Preferences of a parent that values child development relatively less than household consumption
y
8
7
6
5
4
3
2
1
0
0.25
0.5
0.75
1
1.25
1.5
1.75
2
2.25
2.5
2.75
3

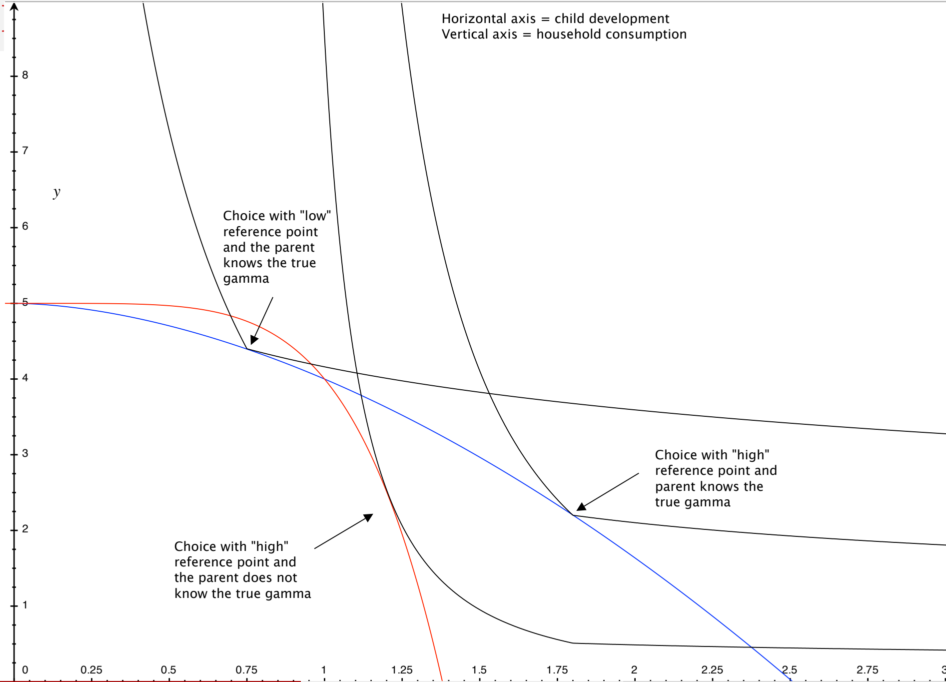


Horizontal axis = child development
Vertical axis = household consumption
Parent is over–sensitive to the perceived under–development of the child
Preferences with reference–point dependence
Parent is under–sensitive to the perceived over–development of the child
y
8
7
6
5
4
3
2
1
0
0.25
0.5
0.75
1
1.25
1.5
1.75
2
2.25
2.5
2.75
3



Horizontal axis = child development
Vertical axis = household consumption
y
Choice when parents have preferences that are reference-point dependent
8
7
6
5
4
3
2
1
0
0.25
0.5
0.75
1
1.25
1.5
1.75
2
2.25
2.5
2.75
3


Horizontal axis = child development
Vertical axis = household consumption
y
Choice with "low" reference point and the parent knows the true gamma
Choice with "high" reference point and parent knows the true gamma
8
7
6
5
4
3
2
1
0
0.25
0.5
0.75
1
1.25
1.5
1.75
2
2.25
2.5
2.75
3


Horizontal axis = child development
Vertical axis = household consumption
y
Choice with "low" reference point and the parent knows the true gamma
Choice with "high" reference point and parent knows the true gamma
Choice with "high" reference point and the parent does not know the true gamma
8
7
6
5
4
3
2
1
0
0.25
0.5
0.75
1
1.25
1.5
1.75
2
2.25
2.5
2.75
3

Three Papers

- Can we elicit maternal subjective expectations?
  - **Cunha, Elo, and Culhane (2013, revised 2017)**.
- Does home visitation affect maternal subjective expectations?
  - **Attanasio, Cunha, and Jervis (2018).**
- Do reference points affect parental investments in children?
  - **Wang, Puentes, Behrman, and Cunha (2018)**: Use RCT to see if reference points affect children's height by age 2 years.

Cunha, Elo, and Culhane (2013): Project Timeline

- Philadelphia Human Development (PHD) Study.
  - Round 1: Elicit maternal subjective expectations during 2nd trimester of 1st pregnancy.
  - Round 2: Measure maternal investments when child is 9-12 months old.
  - Round 3: Measure child development when child is 22-26 months old.
  - **Round 4: RCT about language development when child is 28-32 months old.**

Defining Subjective Expectation

- The technology of skill formation is:

$$\ln h_{i,1} = \psi_0 + \psi_1 \ln h_{0,i} + \psi_2 \ln x_i + \psi_3 \ln h_{0,i} \ln x_i + \nu_i$$

- Let Ψ_i denote the mother's information set.
- Let $E\left(\psi_j \middle| h_{0,i}, x_i, \Psi_i\right) = \mu_{i,j}$ and assume that $E\left(\nu_i \middle| \Psi_i\right) = 0$.
- From the point of view of the mother:

$$E\left(\ln h_{i,1} \middle| h_{0,i}, x_i, \Psi_i\right) = \mu_{i,0} + \mu_{i,1} \ln h_{0,i} + \mu_{i,2} \ln x_i + \mu_{i,3} \ln h_{0,i} \ln x_i$$

Model: Preferences and budget constraint

- Consider a simple static model. Parent's utility is:

$$u\left(c_i, h_{i,1}; \alpha_{i,1}, \alpha_{i,2}\right) = \ln c_i + \alpha_{i,1} \ln h_{i,1} + \alpha_{i,2} \ln x_i$$

- Budget constraint is:

$$c_i + px_i = y_i.$$

Model

- The problem of the mother is to maximize expected utility subject to the mother's information set, the budget constraint, and the technology of skill formation.
- The solution is

$$x_i = \left[\frac{\alpha_{i,1} \left(\mu_{i,2} + \mu_{i,3} \ln h_{0,i} \right) + \alpha_{i,2}}{1 + \alpha_{i,1} \left(\mu_{i,2} + \mu_{i,3} \ln h_{0,i} \right) + \alpha_{i,2}} \right] \frac{y_i}{p}$$

- Clearly, we cannot separately identify α_i from $\mu_{i,\gamma}$ if we only observe x_i, y_i, and p.

Eliciting subjective expectations: Steps

- Measure actual child development: MSD and Item Response Theory (IRT).
- Develop the survey instrument to elicit beliefs $E\left[\ln h_{i,1} \middle| h_0, x, \psi_i\right]$:
  - Reword MSD items.
  - Create hypothetical scenarios of h_0 and x.
- Estimate beliefs from answers allowing for error in responses.

SECTION 3: MOTOR AND SOCIAL DEVELOPMENT

PART H: (22 MONTHS - 3 YEARS, 11 MONTHS)

MOTHER/GUARDIAN:

If ______________________ is **at least 22 months** old, but **not yet 4 years** old, please answer these 15 questions.
Child's Name

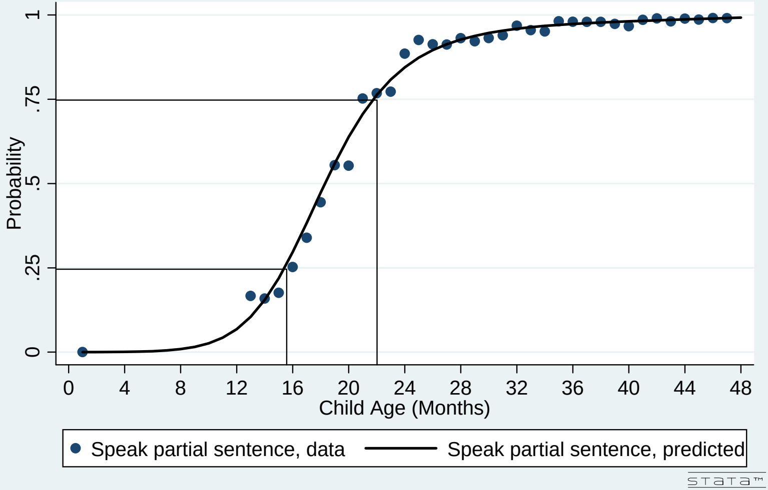
1.	Has your child ever let someone know, without crying, that wearing wet (soiled) pants or diapers bothered him/her?	YES.... 1 NO..... 0	72/
2.	Has your child ever spoken a partial sentence of 3 words or more?	YES.... 1 NO..... 0	73/
3.	Has your child ever walked upstairs by himself/herself without holding on to a rail?	YES.... 1 NO..... 0	74/
4.	Has your child ever washed and dried his/her hands without any help except for turning the water on and off?	YES.... 1 NO..... 0	75/
5.	Has your child ever counted 3 objects correctly?	YES.... 1 NO..... 0	76/

Eliciting beliefs: Item response theory

- Let $d^*_{i,j} = b_{0,j} + b_{1,j} \left(\ln a_i + \frac{b_{2,j}}{b_{1,j}} \theta_i \right) + \eta_{i,j}$
- We observe $d_{i,j} = 1$ if $d^*_{i,j} \geq 0$ and $d_{i,j} = 0$, otherwise.
- Measure of (log of) human capital: $\ln h_i = \ln a_i + \frac{b_{2,j}}{b_{1,j}} \theta_i$.
- In this sense, θ_i is deviation from typical development for age.

Figure 4

Probability as a Function of Child's Age



Eliciting beliefs: Changing wording of the MSD Instrument

- In order to measure $E\left[\ln h_{i,1} \middle| h_0, x, \psi_i\right]$, we take the tasks from the MSD Scale, but instead of asking: *"Has your child ever spoken a partial sentence with three words or more?"*, we ask:
- **Method 1**: **How likely is it that a baby will speak a partial sentence with three words or more by age 24 months?**
- **Method 2: What is the youngest and oldest age a baby learns to speak a partial sentence with three words or more?**

Eliciting beliefs: Scenarios of human capital and investments

- We consider four scenarios:
  - Scenario 1: Child is healthy at birth (e.g., normal gestation, birth weight, and birth length) and investment is high (e.g., six hours per day).
  - Scenario 2: Child is healthy at birth and investment is low (e.g., two hours per day).
  - Scenario 3: Child is not healthy at birth (e.g., premature, low birth weight, and small at birth) and investment is high.
  - Scenario 4: Child is not healthy at birth and investment is low.
- Scenarios are described to survey respondents through a video.

Method 1: Transforming probabilities into mean beliefs

- **Method 1**: **How likely is it that a baby will speak a partial sentence with three words or more by age 24 months?**
- Let's say that when investment is high – that is, when $x = \overline{x}$ – the mother states that there is a 75% chance that the child will learn how to speak a partial sentence with three words or more.
- And when investment is low– that is, when $x = \underline{x}$ – the mother states that there is a 25% chance that the child will learn how to speak a partial sentence with three words or more.
- We convert this probability statement into an age-equivalent statement using the NHANES data.

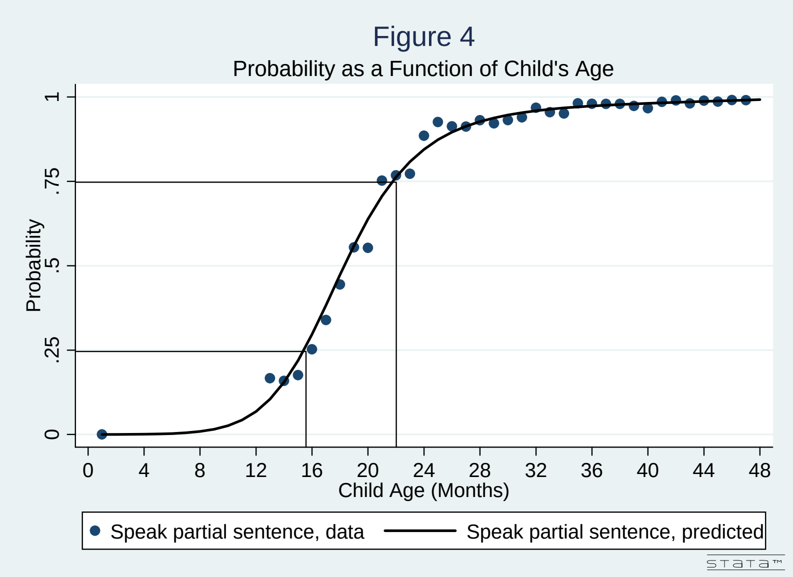


Figure 4
Probability as a Function of Child's Age
Probability
0
.25
.5
.75
1
0
4
8
12
16
20
24
28
32
36
40
44
48
Child Age (Months)
Speak partial sentence, data
Speak partial sentence, predicted
stata™

Method 2: Transforming age ranges into probabilies

- **Method 2: What is the youngest and oldest age a baby learns to speak a partial sentence with three words or more?**
- Let's say that when investment is high, so that $x = \overline{x}$, the mother states that the youngest and oldest ages a baby will learn how to speak a sentence with three words or more are, respectively, 18 and 28 months.
- And when investment is low, so that $x = \underline{x}$, the mother states that the ages are 20 and 30 months.
- We need to transform the age ranges into probabilities. We use the age ranges to estimate a mother-specific IRT model.

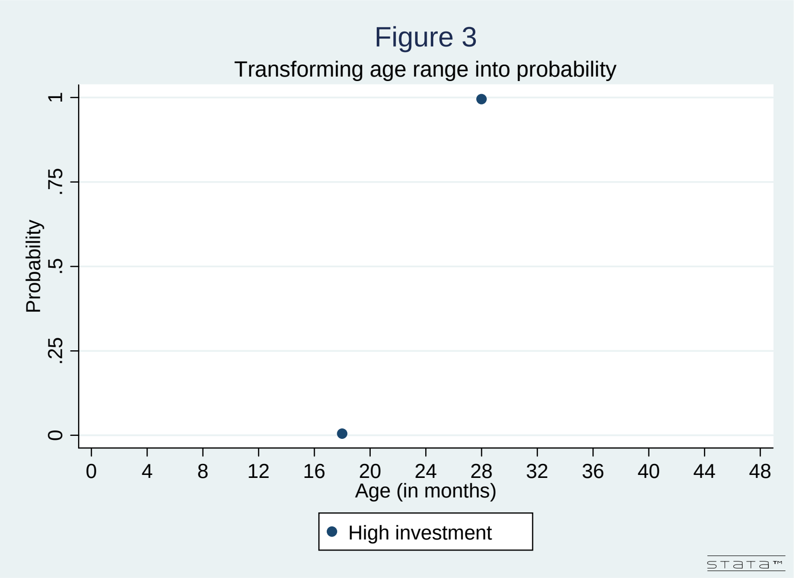


Figure 3
Transforming age range into probability
Probability
0
.25
.5
.75
1
0
4
8
12
16
20
24
28
32
36
40
44
48
Age (in months)
High investment
stata™

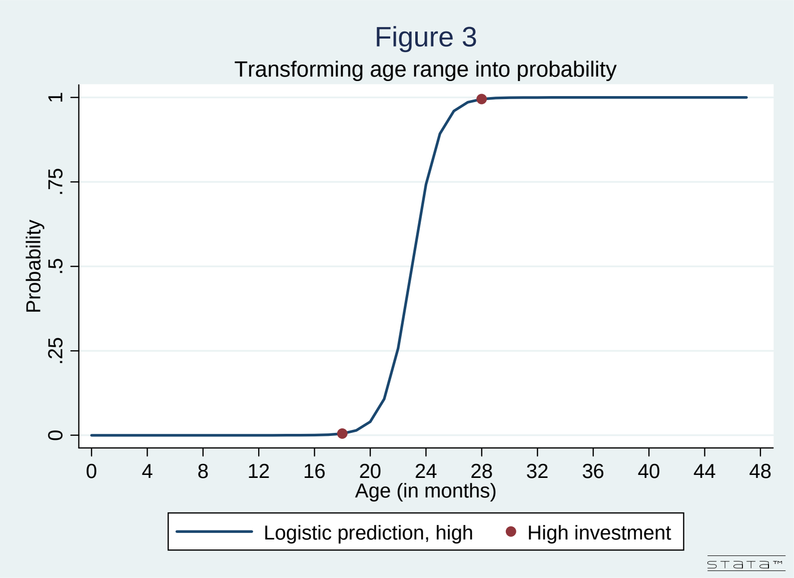


Figure 3
Transforming age range into probability
Probability
0
.25
.5
.75
1
0
4
8
12
16
20
24
28
32
36
40
44
48
Age (in months)
Logistic prediction, high
High investment
stata™

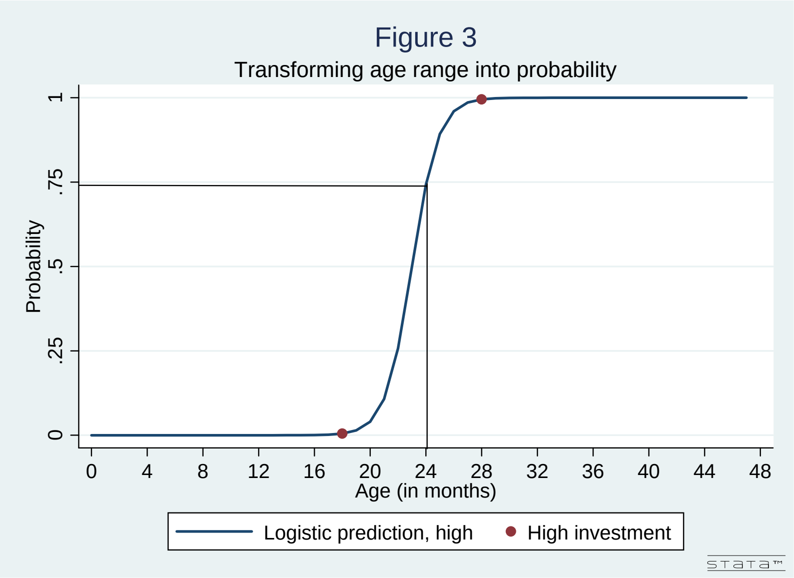


Figure 3
Transforming age range into probability
Probability
0
.25
.5
.75
1
0
4
8
12
16
20
24
28
32
36
40
44
48
Age (in months)
Logistic prediction, high
High investment
stata™

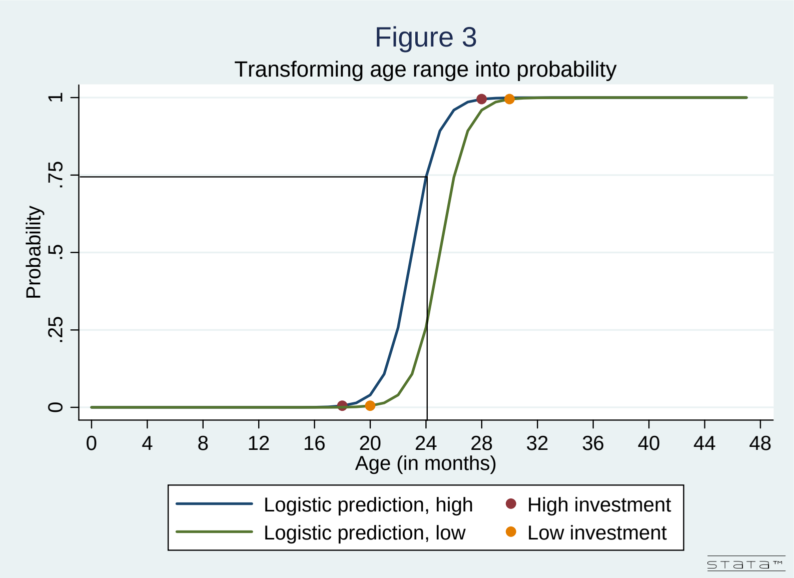


Figure 3
Transforming age range into probability
Probability
0
.25
.5
.75
1
0
4
8
12
16
20
24
28
32
36
40
44
48
Age (in months)
Logistic prediction, high
Logistic prediction, low
High investment
Low investment
stata™

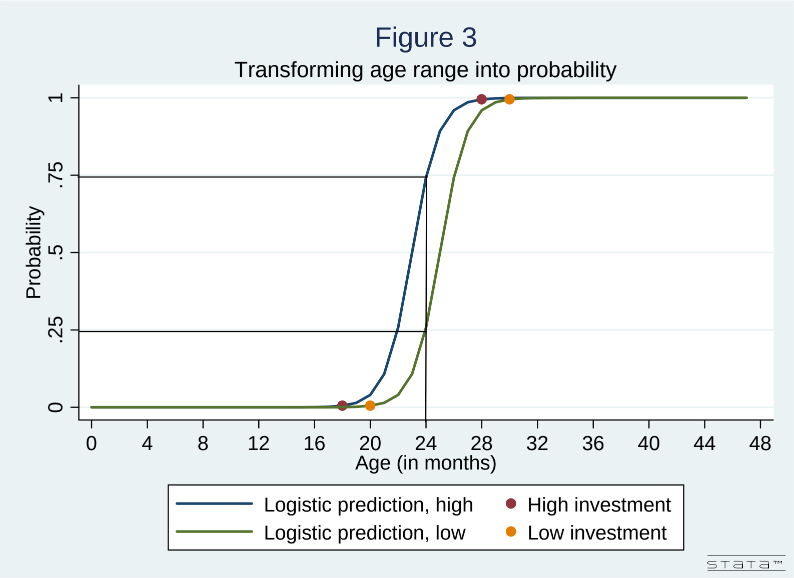


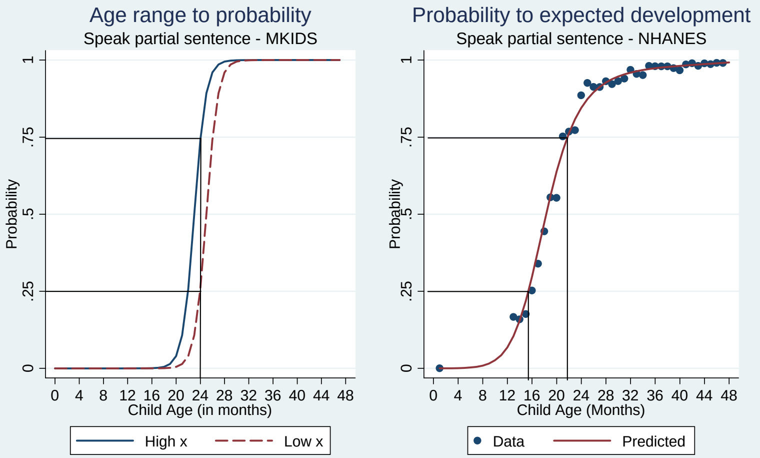
Figure 3
Transforming age range into probability
Probability
0
.25
.5
.75
1
0
4
8
12
16
20
24
28
32
36
40
44
48
Age (in months)
Logistic prediction, high
Logistic prediction, low
High investment
Low investment
stata™

Method 2: Transforming probabilities into mean beliefs

- **Method 2: Given scenario for h_0 and x, how likely is it that a baby will speak a partial sentence with three words or more by age 24 months?**
- Given maternal supplied age range and the logistic assumption, we conclude that when $x = \overline{x}$, the mother believes that there is a 75% chance that the child will learn how to speak a partial sentence with three words or more.
- Analogously, when $x = \underline{x}$, the mother believes that there is a 25% chance that the child will learn how to speak a partial sentence with three words or more.
- We convert this probability statement into an age-equivalent statement using the NHANES data.

Figure 3

Expected development for two levels of investments (x)



Recovering mean beliefs: Measurement error model

- Let $\ln q^L_{i,j,k}$ denote an error-ridden measure of $E\left[\ln h_{i,1} \middle| h_{0,k}, x_k, \psi_i\right]$ generated by "how likely" questions:

$$\ln q^L_{i,j,k} = E\left[\ln h_{i,1} \middle| h_{0,k}, x_k, \psi_i\right] + \epsilon^L_{i,j,k}.$$

- Let $\ln q^A_{i,j,k}$ denote an error-ridden measure of $E\left[\ln h_{i,1} \middle| h_{0,k}, x_k, \psi_i\right]$ generated by "age range" questions:

$$\ln q^A_{i,j,k} = E\left[\ln h_{i,1} \middle| h_{0,k}, x_k, \psi_i\right] + \epsilon^A_{i,j,k}.$$

- For each scenario, we have multiple measures of the same underlying latent variable.

Recovering mean beliefs:

- Use technology of skill formation, and the mother's information set, to obtain:

$$\ln q^L_{i,j,k} = \mu_{i,0} + \mu_{i,1} \ln h_{0,k} + \mu_{i,2} \ln x_k + \mu_{i,3} \ln h_{0,k} \ln x_k + \epsilon^L_{i,j,k}$$

$$\ln q^A_{i,j,k} = \mu_{i,0} + \mu_{i,1} \ln h_{0,k} + \mu_{i,2} \ln x_k + \mu_{i,3} \ln h_{0,k} \ln x_k + \epsilon^A_{i,j,k}.$$

- We have a factor model where:
  - $\mu_i = (\mu_{i,0}, \mu_{i,1}, \mu_{i,2}, \mu_{i,3})$ are the latent factors;
  - $\lambda_k = (1, h_{0,k}, \ln x_k, \ln h_{0,k} \ln x_k)$ are the factor loadings;
  - $\epsilon_{i,j,k} = \left(\epsilon^L_{i,j,k}, \epsilon^A_{i,j,k}\right)$ are the uniquenesses.

Eliciting beliefs: Intuitive explanation

- Let $E\left[\ln h_{i,1} \middle| h_0, h, \Psi_i\right]$ denote maternal expectation of child development at age 24 months conditional on the child's intial level of human capital, investments, and the mother's information set.
- Assume, for now, technology is Cobb-Douglas.
- Suppose we measure $E\left[\ln h_{i,1} \middle| h_0, x, \Psi_i\right]$ at two different levels of investments:

$$E\left[\ln h_{i,1} \middle| h_0, \overline{x}, \Psi_i\right] = \mu_{i,0} + \mu_{i,1} \ln h_0 + \mu_{i,2} \ln \overline{x}$$

$$E\left[\ln h_{i,1} \middle| h_0, \underline{x}, \Psi_i\right] = \mu_{i,0} + \mu_{i,1} \ln h_0 + \mu_{i,2} \ln \underline{x}$$

- Subtracting and re-organizing terms:

$$\mu_{i,2} = \frac{E\left[\ln h_{i,1} \middle| h_0, \overline{x}, \Psi_i\right] - E\left[\ln h_{i,1} \middle| h_0, \underline{x}, \Psi_i\right]}{\ln \overline{x} - \ln \underline{x}}$$

Important issue

- We could use only one MSD item to elicit beliefs.
- But, if we use more items, we can relax assumptions about measurement error.
- And, we can check for consistency in answers.

Figure 5

Comparing answers across scenarios

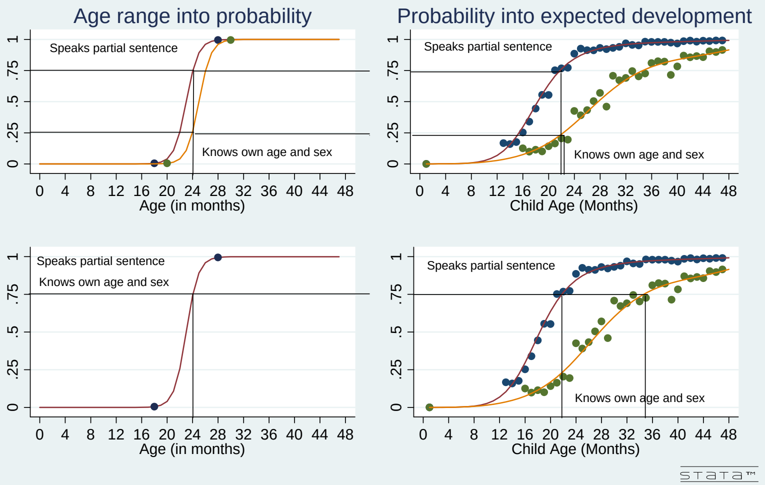


Table 5

Correlation between MSE and demographic characteristics of PHD Study Participants

VARIABLES	Standardized $\mu_{i,\psi,1}$	Standardized $\mu_{i,\psi,2}$	Standardized $\mu_{i,\psi,3}$
Dummies for household income (y)			
$\$25,000\ per\ year \leq y < \$55,000\ per\ year$	0.2243	0.3452	0.1908
	(0.1003)	(0.0928)	(0.1027)
$\$55,000\ per\ year \leq y < \$105,000\ per\ year$	-0.1701	0.3662	-0.2460
	(0.1265)	(0.1209)	(0.1135)
$y \geq \$105,000\ per\ year$	-0.5060	0.4694	-0.5276
	(0.1278)	(0.1405)	(0.1203)
Constant	-0.2746	-0.5133	0.0514
	(0.1581)	(0.1758)	(0.1664)
Observations	822	822	822
R-Squared	0.0709	0.0641	0.0900

Robust standard errors in parentheses.

Table 6

Correlation between the HOME Score and MSE

VARIABLES	Both	Both	How Likely Only	How Likely Only	Age Range Only	Age Range Only
	Dependent variable: Standardized HOME Score					
Standardized μ_1	-0.0237	-0.0015	-0.0946	-0.0577	-0.0136	0.0306
	(0.0813)	(0.0740)	(0.0799)	(0.0742)	(0.0585)	(0.0530)
Standardized μ_2	0.1667	0.1141	0.1185	0.0980	0.1699	0.0834
	(0.0449)	(0.0385)	(0.0435)	(0.0395)	(0.0446)	(0.0383)
Standardized μ_3	-0.0856	0.0096	-0.0401	0.0344	-0.0581	-0.0137
	(0.0673)	(0.0611)	(0.0662)	(0.0618)	(0.0479)	(0.0422)
Demographic characteristics included?*	No	Yes	No	Yes	No	Yes
Observations	687	687	687	687	687	687
R-squared	0.0369	0.2695	0.0343	0.2706	0.0314	0.2655

Robust standard errors in parentheses.

*Note: The following variables describe demographic characteristics: A dummy variable that takes the value one if the mother's year of birth is between 1978 and 1987 and zero otherwise; a dummy variable that takes the value one if the mother's year of birth is between 1988 and 1997 and zero otherwise; a dummy variable that takes the value one if the mother is Hispanic and zero otherwise; a dummy variable that takes the value one if the mother is non-Hispanic black and zero otherwise;

Attanasio, Cunha, and Jervis (2018)

- Attanasio and colleagues have adapted an influential home visitation program from the Jamaica.
- Gertler et al (2014) follow up participants when they were in the early 20s and find positive impacts of the program on educational attainment and labor market outcomes.
- Attanasio et al (2015) report positive impact of the program on cognitive development, socio-emotional development, and parental investment in children.
- Question: Why has investment increased?

Project Timeline

- Baseline:
  - Measure h_0: BSID (cognitive, receptive language, expressive language) and MacArthur-Bates Language Scale.
  - Intervention assignment: $d_i \in \{0, 1\}$.
- First follow up:
  - Measure h_1: BSID (cognitive, receptive language, expressive language) and MacArthur-Bates Language Scale.
  - Measure x_1: Family Care Indicators (materials, activities) and time diary for child.
- Second follow up:
  - Measure h_2: BSID (cognitive, receptive language, expressive language) and MacArthur-Bates Language Scale.
  - Measure x_2: Family Care Indicators (materials, activities) and time diary for child.
  - Measure μ: maternal beliefs

Elicitation of Maternal Beliefs

- We would like to elicit parental beliefs about the parameters of the technology of skill formation:

$$E\left[\ln h_{i,1} \middle| h_0, x_1, \Omega\right] = \mu_0 + \mu_1 \ln h_0 + \mu_2 \ln x_1$$

- Approach proposed in Cunha, Elo, and Culhane (2013):

Steps

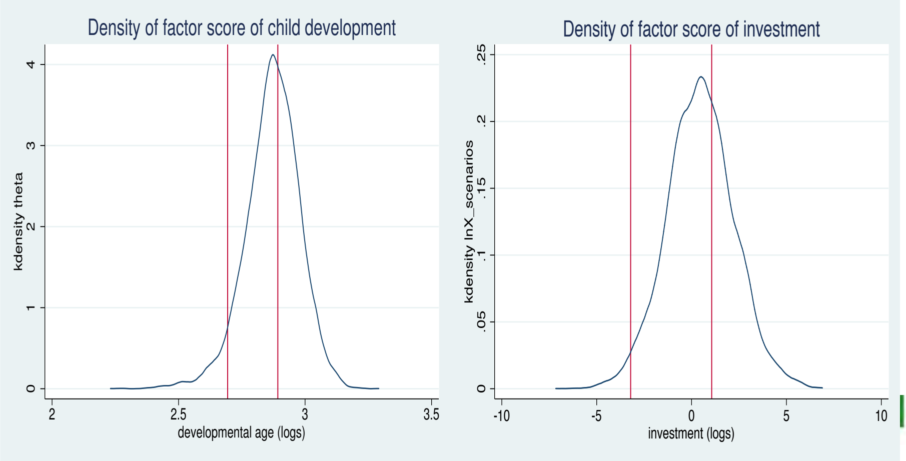
- Step 3: Choose items for the elicitation instrument. Choose 9 words from MacArthur Bates
  - 3 words are "easy" (α is high): $w^e = (w_1^e, w_2^e, w_3^e)$.
  - 3 words are "moderate" (α is average): $w^m = (w_1^m, w_2^m, w_3^m)$.
  - 3 words are "hard" (α is low): $w^h = \left(w_1^h, w_2^h, w_3^h\right)$.
- These 9 words are the items in the elicitation instrument: $W = \left(w^e, w^m, w^h\right)$.

Steps

- Step 4: Choose scenarios for human capital at baseline and investments between beginning and end of the program:
  - Scenario 1: h_0 is low (at baseline, child understands only "easy" words) and x_1 is low (few materials, few activities): $s_1 = \left(h_0^L, x_1^L\right)$.
  - Scenario 2: h_0 is low and x_1 is high (lots of materials, lots of activities): $s_2 = \left(h_0^L, x_1^H\right)$.
  - Scenario 3: h_0 is high (at baseline, child understands "easy" and "difficult" words) and x_1 is low: $s_3 = \left(h_0^H, x_1^L\right)$.
  - Scenario 4: h_0 is high and x_1 is high: $s_1 = \left(h_0^H, x_1^H\right)$.

establish what part of the relevant domains are spanned by the scenarios.

All the chosen words had relatively high loading factors' β's. Easy words had lo[w] intercepts (α's), hard words had high α's and medium words medium αs.

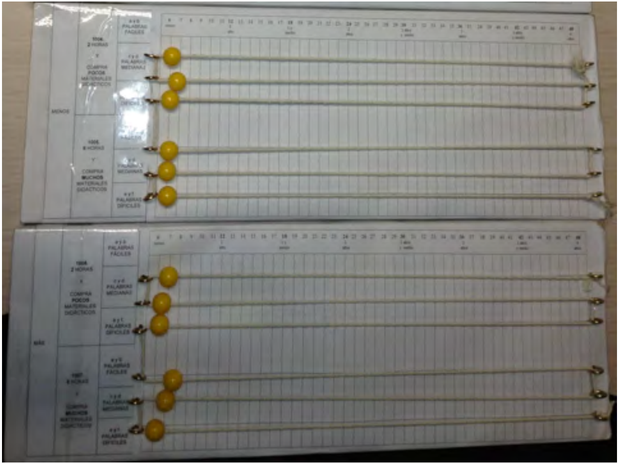


High' and 'Low' levels of maternal investment

Steps

- Step 5: Elicitation instrument is based on MacArthur Bates CDI.
  - In order to measure $E\left[\ln h_{i,1} \middle| h_0, x\right]$, we select words, but instead of asking: *"Has your child ever spoken word X?"*, we ask:
  - **Suppose [describe scenario for h_0 and x]. At what age do you think the child will speak words w^d?**
  - The index d denotes the difficulty of the words.
- For every combination of scenarios of $\ln h_0$ and $\ln x$, parents answer three questions.
- Let $a_{i,d,s}$ denote the age reported by mother i answer for word difficulty level d when scenario was s.
- For each parent, we have 12 answers. Why so many? Measurement error.

ting Beliefs - Instrument



ing Beliefs - Instrument

Transforming age answers into mean beliefs

- Step 6: Now we go from $a_{i,d,s}$ to $E\left[\ln h_{i,1} \middle| h_0, x_1, \Omega\right]$.
- Easier to explain with the following example.

Transforming age answers into mean beliefs

Table

From maternal answers to model variables

	Scenario 1: h_0^L, x_1^L			Scenario 4: h_0^H, x_1^H		
	Easy words	Medium words	Hard words	Easy words	Medium words	Hard words
Maternal answer	27	30	33	23	26	28
Typical age children learn words	22	24	26	22	24	25
Difference (developmental delay)	5	6	7	1	2	3
Chronological age (36 months)	36	36	36	36	36	36
Developmental age	31	30	29	35	34	33
Error-ridden measure of $E(\ln h_1 \mid h_0^s, x_1^s, \Omega)$	3.43	3.40	3.37	3.56	3.53	3.50

Estimating beliefs: Intuitive explanation

- Let $E\left[\ln h_{i,1} \middle| h_0, x\right]$ denote maternal expectation of child development at follow-up conditional on the child's intial level of human capital, investments, and the mother's information set.
- Let $h_{i,d,s}$ denote the error-ridden maternal report of $E\left[\ln h_{i,1} \middle| h_0, x\right]$. Define the measurement error as $\eta_{i,d,s}$:

$$h_{i,d,s} = E\left[\ln h_{i,1} \middle| h_0, x\right] + \eta_{i,d,s}$$

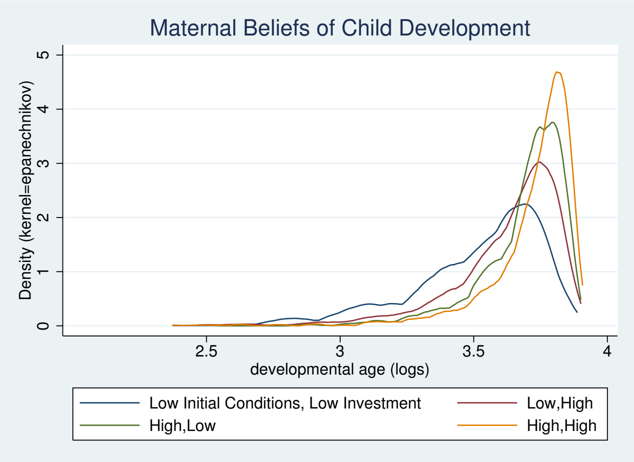
- Now:

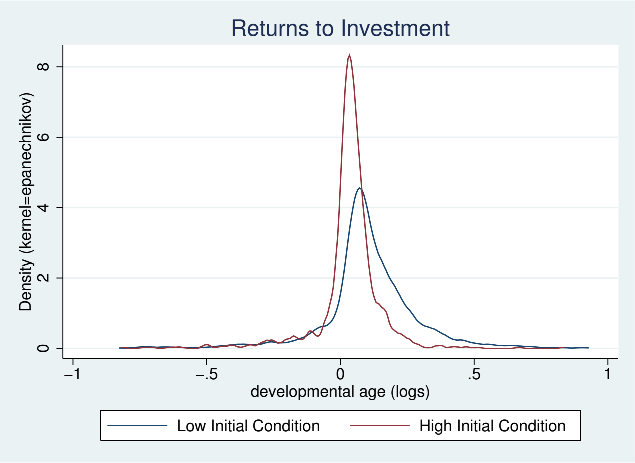
$$h_{i,d,s} = \mu_0 + \mu_1 \ln h_0^s + \mu_2 \ln x_1^s + \eta_{i,d,s}$$

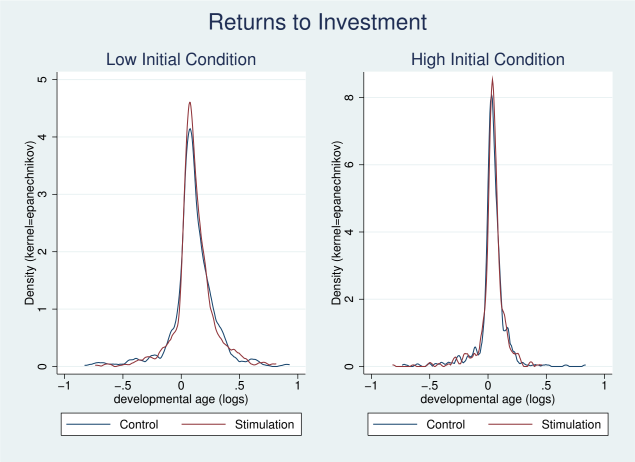
- Beliefs are latent factors with fixed factor loadings. We can relax assumptions on $\eta_{i,d,s}$.

Table: Answers about the Outcomes of Maternal Investment and Initial Conditios

		Mean	St. Dv.	Min	Max
Low Initial Conditions	Low Investment	18.3	6.2	9	48
		23.5	7.3	10	48
		29.5	8.8	11	48
	High Investment	15.8	5.7	9	48
		20.1	6.8	9	48
		25.0	8.2	9	48
High Initial Conditions	Low Investment	14.4	4.8	9	48
		18.0	5.6	9	48
		22.3	7.2	10	48
	High Investment	13.5	5.3	9	48
		16.7	5.9	9	48
		20.3	7.2	9	48


Maternal Beliefs of Child Development
Density (kernel=epanechnikov)
0
1
2
3
4
5
2.5
3
3.5
4
developmental age (logs)
Low Initial Conditions, Low Investment
Low,High
High,Low
High,High


Returns to Investment
Density (kernel=epanechnikov)
0
2
4
6
8
−1
−.5
0
.5
1
developmental age (logs)
Low Initial Condition
High Initial Condition


Returns to Investment
Low Initial Condition
High Initial Condition
Density (kernel=epanechnikov)
developmental age (logs)
0
1
2
3
4
5
6
8
−1
−.5
.5
Control
Stimulation

nvestment.

$$ElnH^i_t = \quad ElnH^i_t = \delta_0 + \delta_1 lnH^i_{t-1} + \delta_2 lnx^i_t + \delta^1_3 lnH^i_{t-1} lnx^i_t, \quad t = 1,$$

Table: Objective Estimation of the Production Function

	First Stage		Second Stage	
	Scaled	Standardized	Scaled	Standardized
	Log of Investment		Log of Human Capital at Follow Up	
Intercept	0.0877 (0.5422)	0.0000 (0.1541)	1.7260 (0.1142)	0.0000 (0.5967)
Log of Human Capital at Baseline	1.1626 (0.1879)	0.1896 (0.0306)	0.4630 (0.0910)	0.3776 (0.0742)
Log of Investments at Follow Up			0.1461 (0.0707)	0.7309 (0.1533)
Treatment dummy	0.1740 (0.0587)	0.0909 (0.0307)		

$$E_i ln(h_1) = \mu_{0,i} + \mu_{1,i} ln(h_0) + \mu_{2,i} ln(X) + \mu_{3,i}[ln(h_0) ln(X)]$$

Our procedure yields estimates of the coefficients for each mother:

Table: Estimation of Perceived Production Function

Dependent Variable: Expected Human Capital at Follow Up					
	Cobb Douglas		Translog		
	Scaled	Standardized	Scaled	Standardized	Fraction $\|t\| > 2$
Intercept	1.7410 (0.0401)	0.0000 (0.0058)	-0.0450 (0.1313)	0.0000 (0.0053)	6.67%
Human Capital at Baseline	0.5703 (0.0132)	0.4651 (0.0108)	1.1922 (0.0450)	0.4630 (0.0107)	70.50%
Investment	0.0542 (0.0023)	0.2712 (0.0115)	0.5638 (0.0357)	0.2677 (0.0114)	24.00%
Investment x Human Capital at B.			-0.1775 (0.0122)	-0.1385 (0.0096)	2.50%

Table: Beliefs and Demographic Characteristics

VARIABLES	Standardized μ_0	Standardized μ_1	Standardized μ_2
Treatment dummy	0.0018	-0.0118	0.0410
	(0.0607)	(0.0611)	(0.0611)
Child is male	0.0132	-0.0042	-0.0448
	(0.0613)	(0.0616)	(0.0613)
Standardized Human Capital at Baseline	-0.0350	0.0283	0.0464
	(0.0296)	(0.0298)	(0.0298)
Standardized Household Wealth	-0.0456	0.0387	0.0518*
	(0.0311)	(0.0312)	(0.0313)
Mother's Standardized Raven Score	-0.1960***	0.1661***	0.2230***
	(0.0309)	(0.0311)	(0.0326)
Constant	-0.0179	0.0150	0.0212
	(0.0524)	(0.0522)	(0.0552)
Observations	1,017	1,017	1,017
R-squared	0.0487	0.0350	0.0634

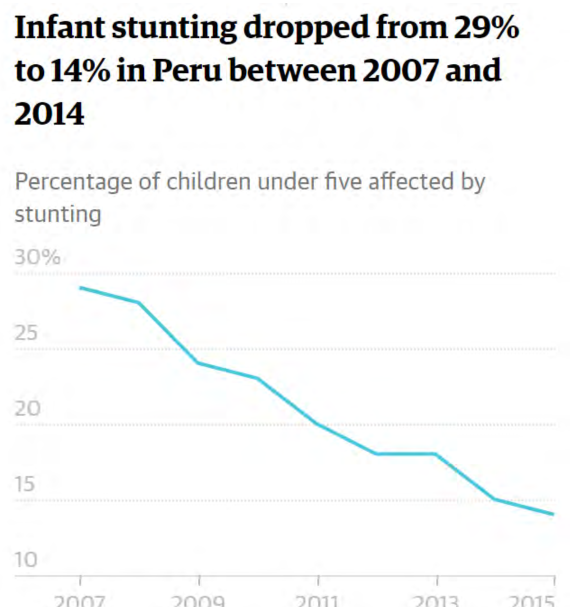
Standard errors (in parentheses) are clustered at municipality level, $^{*}\ p < 0.10$, $^{**}\ p < 0.05$, $^{***}\ p < 0.01$

Table: Beliefs and Investment

VARIABLES	Standardized Log Investment		Standardized Time		Standardized Activities		Standardized Materials	
Standardized μ_2	0.1084***	0.0389	0.0912***	0.0617*	0.0652**	0.0106	0.0755**	0.0150
	(0.0294)	(0.0292)	(0.0309)	(0.0317)	(0.0286)	(0.0289)	(0.0300)	(0.0303)
Dummy for Treatment	0.1762***	0.1801***	0.0266	0.0299	0.2745***	0.2789***	0.0152	0.0150
	(0.0622)	(0.0593)	(0.0625)	(0.0620)	(0.0620)	(0.0606)	(0.0628)	(0.0603)
Dummy for Male		-0.0081		-0.0031		-0.0034		-0.0115
		(0.0597)		(0.0620)		(0.0607)		(0.0609)
Standardized Human Capital at Birth		0.1469***		0.0830***		0.1042***		0.1220***
		(0.0303)		(0.0302)		(0.0340)		(0.0315)
Standardized Household Wealth		0.1196***		0.0287		0.0791***		0.1473***
		(0.0315)		(0.0328)		(0.0303)		(0.0336)
Mother's Standardized Raven Score		0.1862***		0.0819**		0.1567***		0.1446***
		(0.0325)		(0.0330)		(0.0319)		(0.0347)
Constant	-0.0911**	-0.0877	-0.0152	-0.0147	-0.1400***	-0.1394**	-0.0091	-0.0020
	(0.0458)	(0.0544)	(0.0436)	(0.0543)	(0.0454)	(0.0546)	(0.0459)	(0.0563)
Observations	1,017	1,017	1,017	1,017	1,017	1,017	1,017	1,017
R-squared	0.0199	0.1072	0.0086	0.0256	0.0234	0.0741	0.0058	0.0783

Standard errors (in parentheses) are clustered at municipality level, $^{*}\ p < 0.10$, $^{**}\ p < 0.05$, $^{***}\ p < 0.01$

Wang, Puentes, Behrman, and Cunha (2018)



Data

- Nutritional supplementation trial from 1969 until 1977:
  - A high-protein nutritional supplement was delivered in the two treatment villages (Atole)
  - A non-protein supplement was delivered in two control villages (Fresco).
  - Initial Height, Height at Month 24 Protein (and Calorie) intakes every 3 months in first 2 years (24-hour and 72-hour recall)
  - Prices of eggs, chicken, pork, beef, dry beans, corn, and rice.

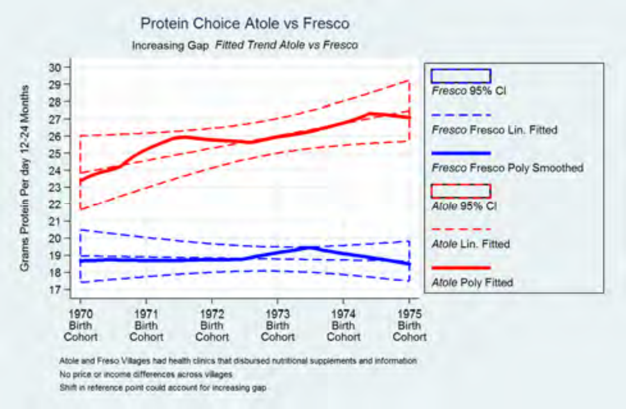
Identification Argument

- Adaptive expectations: Reference points for age two height at year t were determined by the height of children born in year $t-2$.
- We show this implies two exclusion restrictions:
  - Random assignment to treatment or control: Identifies coefficients on investment in production function.
  - Interaction between random assignment and calendar time: Identifies preference parameter on reference point.

Consumption of Protein in Treatment vs Control Villages

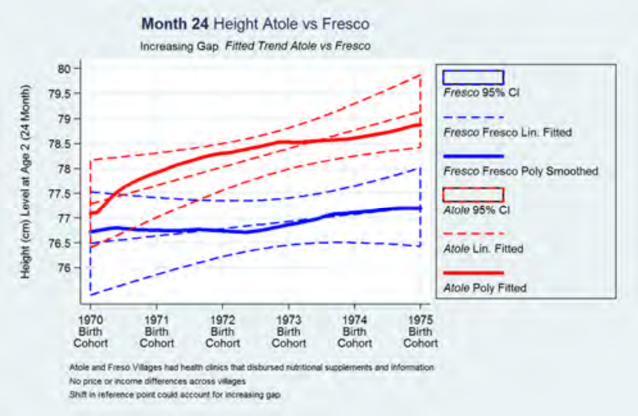
Estimation: Identification III

- We find that there is a gap in protein choice up to month 24 between Atole and Fresco villages.
- Income is constant over time, and price is the same across locations.
- Only the increasing reference point gap, through λ, can explain the choice gap's widening.



Estimation: Identification II

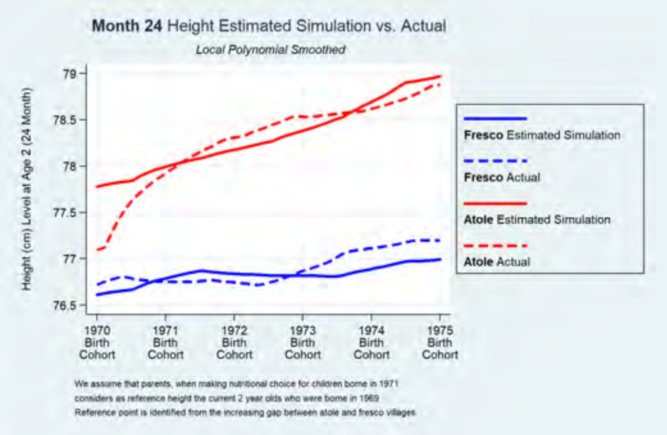
- We find that there is a gap in height at month 24 between Atole and Fresco villages.
- The gap is increasing over time.
- These curves are our $\mu_{R_{yv}}$



Model Fit: Height

Estimation Results: Fit of the Model III

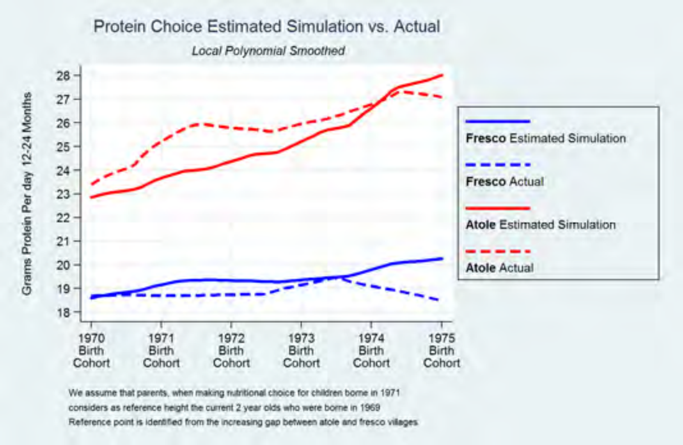
Fit general Height Change pattern.



Model Fit: Protein Consumption

Estimation Results: Fit of the Model IV

Fit Protein Trend over time



Decomposition: Price Discount vs Reference Point

Counterfactual 1a

For children in fresco villages:

- **Yellow**: Increase in height with just 42% price discount, fixing reference point.
- **Green**: Remaining contribution from reference point change

Cunha, Gerdes, and Nihtianova: LENA Start

- Group sessions of approximately 12-15 parents.
- Importance of language environment for language development.
- Lasts 13 weeks.
- Each week parent provides one 16-hour recording of the child's language environment.
- Team analyzes data and provides feedback to parents.

LENA: How it works

Measuring Quality and Quantity of Time: LENA Pro

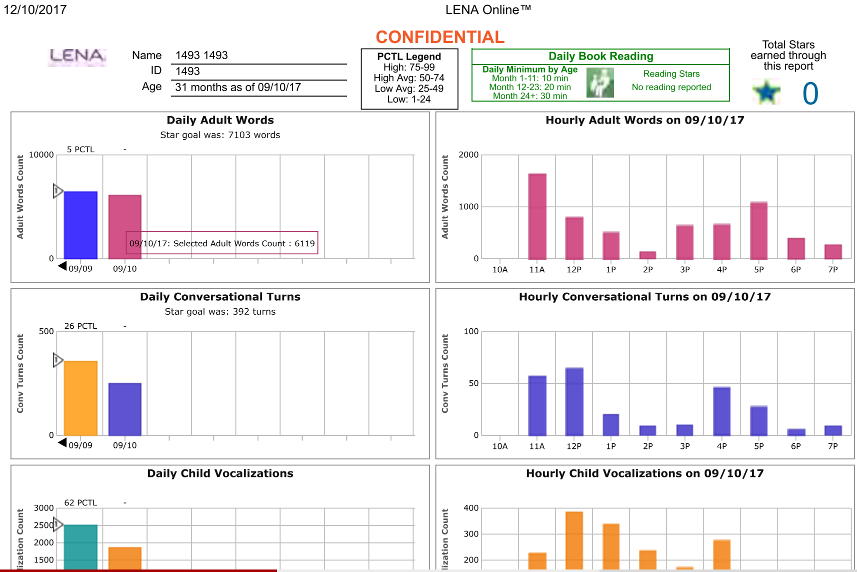
Turn on the DLP and place it in the pocket of the child's LENA clothing.

After completing recording, plug the DLP into a PC running LENA Pro. The sophisticated language environment analysis software automatically uploads and processes the audio file.

The software generates the LENA reports and other analyses.

Export data from LENA Pro to mine your LENA data and perform custom in-depth analyses.

Baseline: AWC and CTC



During Intervention: AWC and CTC

Conclusion: Skill Formation

- Inequality in socio-economic outcomes is partly caused by inequality in human capital.
- Inequality in human capital is partly caused by inequality in investments in human capital during early childhood, adolescence, and adulthood.
- Inequality in stocks of human capital is increasing in the last 20 years.
- Inequality in investments in human capital is also increasing in the last 20 years.

Conclusion: Skill Formation

- At different stages of the lifecycle, investments produce different dimensions of human capital.
- To estimate production functions of human capital:
  - Address lack of cardinality of measures of human capital.
  - Address measurement error in measures of human capital.
  - Address endogeneity of investments.
- Previous work shows that some of the inequality in investments is due to inequality in family resources: family income, parents' education, etc.
- We don't know much about parental preferences, parental information sets, and other constraints that families face when choosing how much to invest in their children.
- This lack of knowledge limits our ability to think of new public policies that can foster human capital formation.

Conclusion: Skill Formation

- Previous work shows that some of the inequality in investments is due to inequality in family resources: family income, parents' education, etc.
- We don't know much about the nature and importance of heterogeneity in parental preferences (that can be manipulated by policy); how this heterogeneity predicts investments; and whether public policy can affect these preferences; and if so, the quantitative importance of this mechanism.
- Same is true for parental beliefs.
- Even more problematic is that parents may face many oher constraints that are so far unidentified by theoretical or empirical work.
- Lots of work for young, talented researchers with interest in theory, in empirical work, or in any convex combination of the two.

Conclusion: Skill Formation

- Lots of work for young researchers:
  - Theory: How to model within family decision making processes? How to model these processes when parents are not
  - Theory: How to model parent-child interaction (child is a "player").
  - Data: How to measure investments? How to measure human capital in cardinal ways?
  - Data: Implement and evaluate pilot programs that can foster human capital formation.
  - Data and Theory: Identify mechanisms to validate or reject theories and to identify new opportunities for interventions.