Human Capital Formation in Childhood and Adolescence

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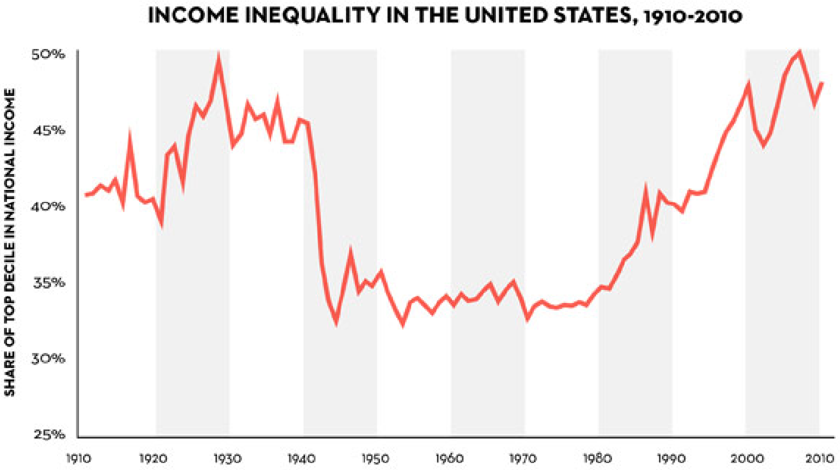
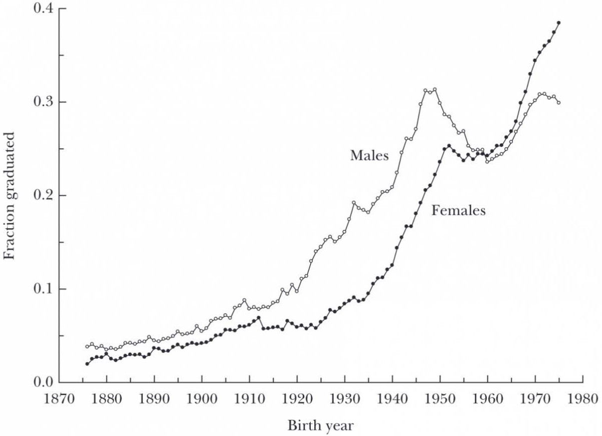
Evolution of Inequality in USA



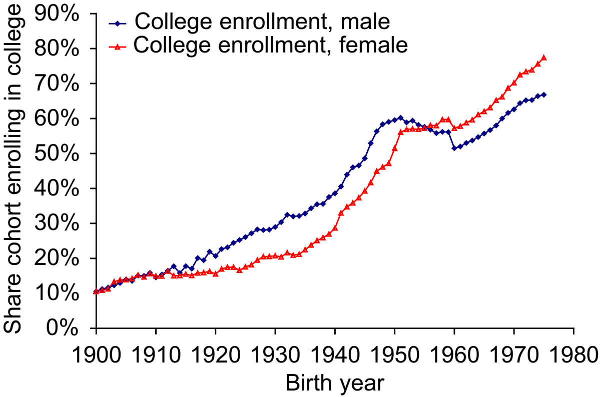
Katz and Goldin (2007): College Graduation in USA

Figure 1
College Graduation Rates (by 35 years) for Men and Women: Cohorts Born from 1876 to 1975



Sources: 1940 to 2000 Census of Population Integrated Public Use Micro-data Samples (IPUMS).

Katz and Goldin (2007): College Enrollment in USA



Katz and Goldin (2007): College Graduation Conditional on Enrollment in USA

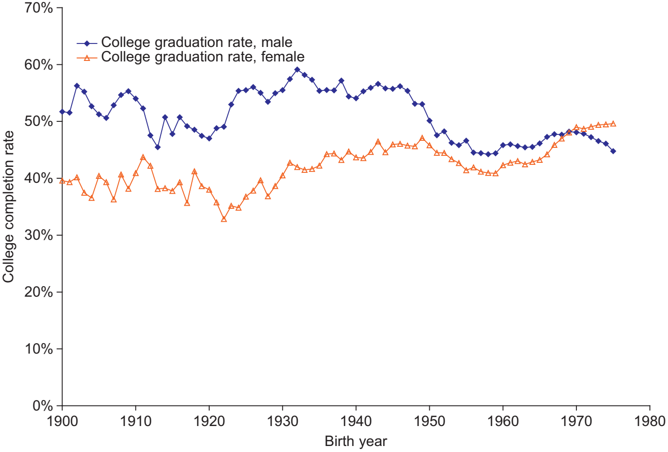
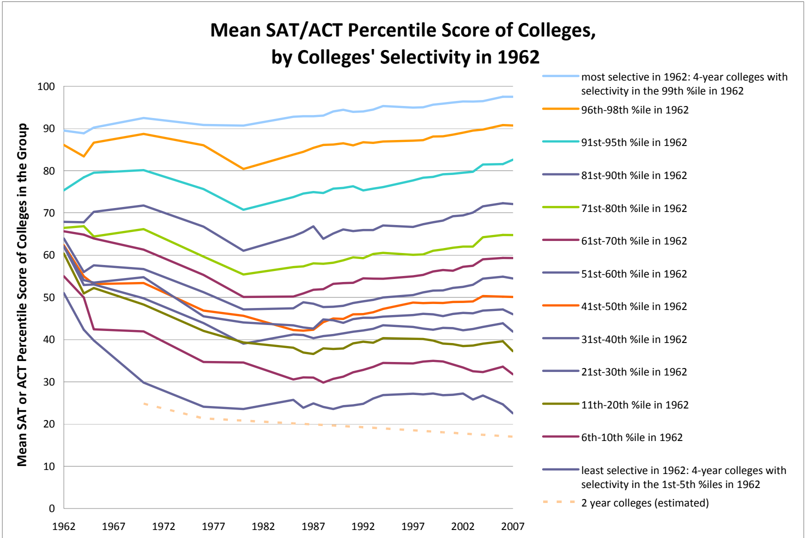


Figure 8.4 Share of College Entrants Receiving BA Degree.

Hoxby (2009): Segmented Markets in Higher Education



Returns and Stocks of Skilled/Unskilled Labor

- Let L_S and L_U denote, respectively, skilled and unskilled labor.
- Let w_S and w_U denote, respectively, skilled and unskilled wage rates.
- Consider the following problem:

$$\min w_S L_S + w_U L_U$$

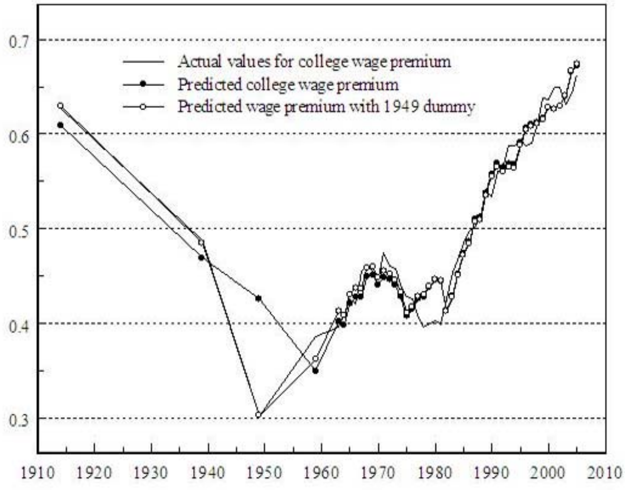
subject to the aggregate production function (where $\gamma \in [0, 1]$ and $\phi \leq 1$):

$$Y = \left[\gamma L_S^{\phi} + (1 - \gamma) L_U^{\phi}\right]^{\frac{1}{\phi}}$$

- Solution satisfies:

$$\ln \frac{w_S}{w_U} = \ln \frac{\gamma}{1 - \gamma} + (\phi - 1) \ln \frac{L_S}{L_U}$$

Katz and Goldin (2007): Model vs Data



Plan: Data on skill formation

- Inequality in skills and inequality in adult socio-economic outcomes.
- Inequality in investments and inequality in skills.
- Increasing inequality in skills.
- Increasing inequality in investments.
- Evidence from RCTs.

Figure 1: The Probability of Educational Decisions, by Endowment Levels, Dropping from Secondary School vs. Graduating

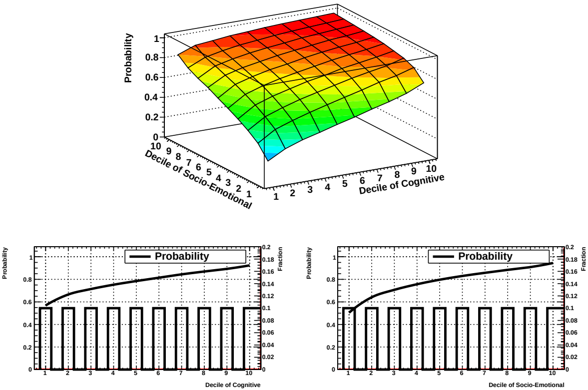


Figure 2: The Probability of Educational Decisions, by Endowment Levels, **HS Graduate** vs. College Enrollment

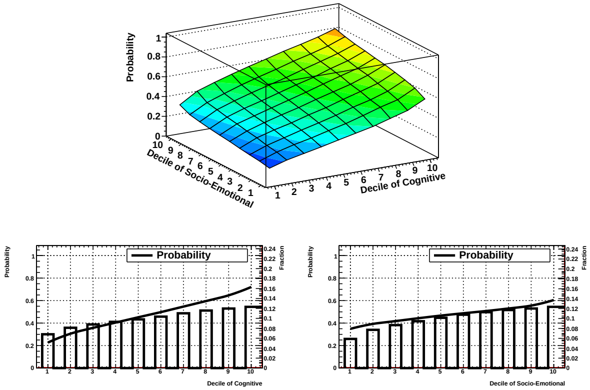


Figure 3: The Probability of Educational Decisions, by Endowment Levels, **Some College** vs. **4-year college degree**

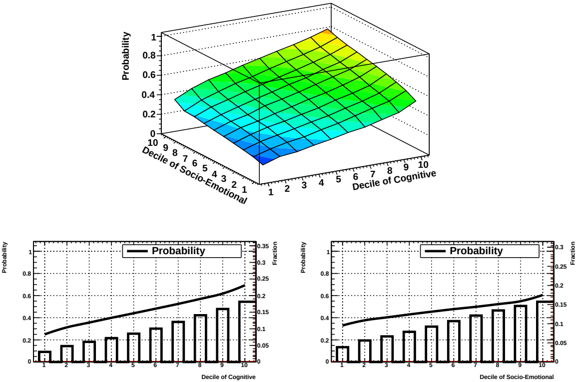
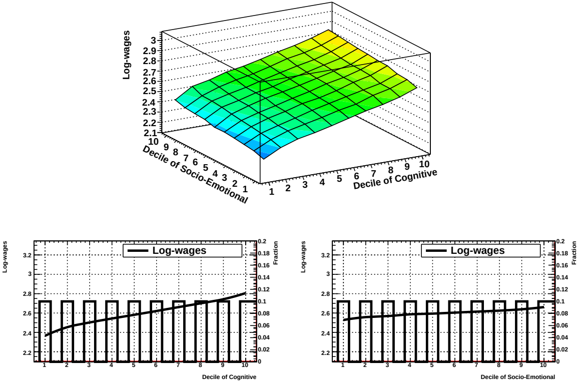
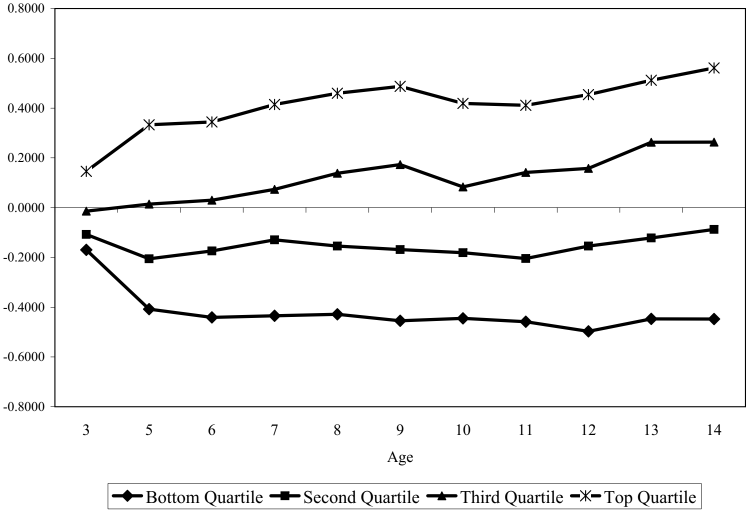


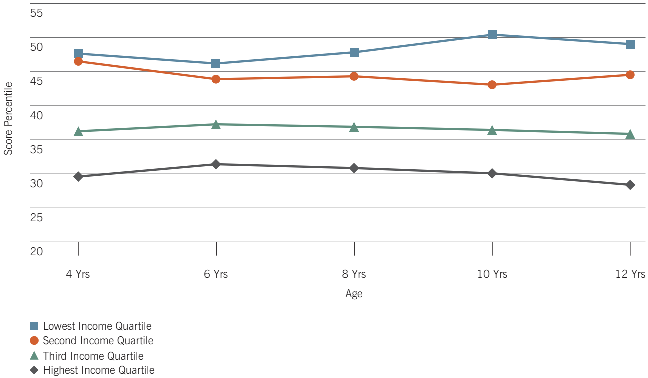
Figure 4: The Effect of Cognitive and Socio-emotional endowments, (log) Wages



Inequality in Cognitive Skills as Children Age

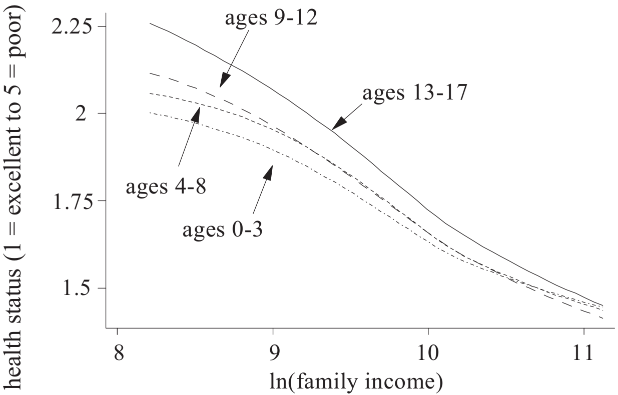


Average percentile rank on anti-social behavior score, by income quartile



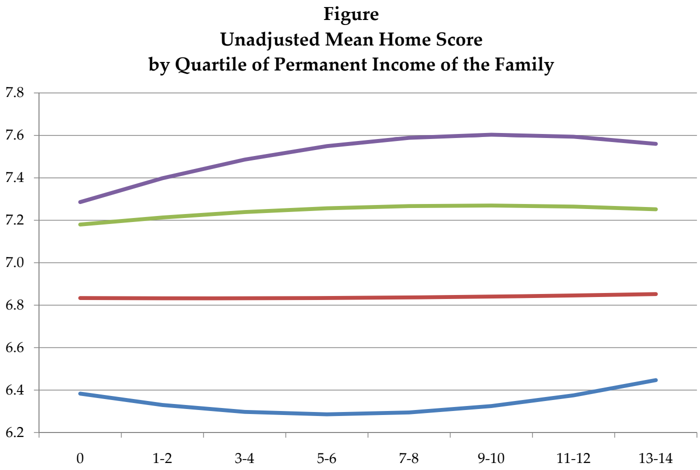
Inequality in Health as Children Age

Health and income for children and adults, U.S. National Health Interview Survey 1986-1995.*

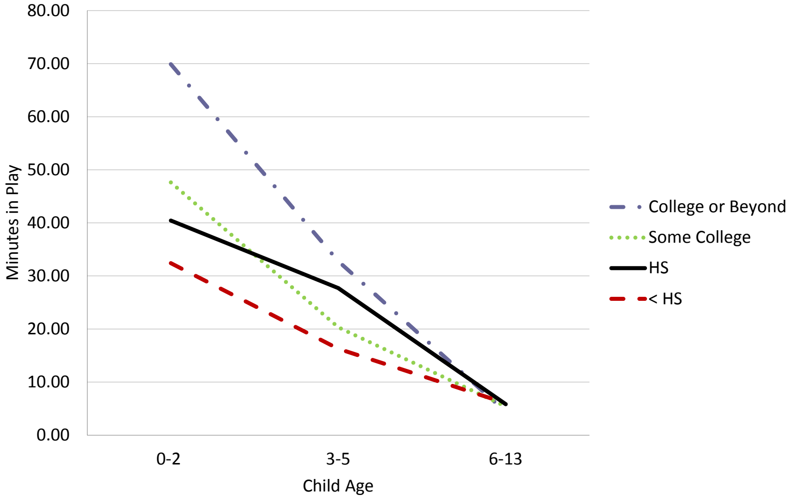


* From Case, A., Lubotsky, D. & Paxson, C. (2002), American Economic Review, Vol. 92, 1308-1334.

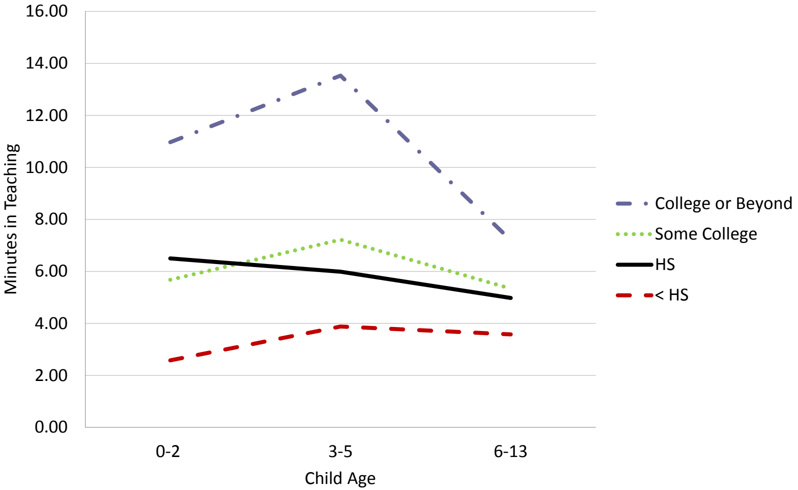
Inequality in Investments as Children Age



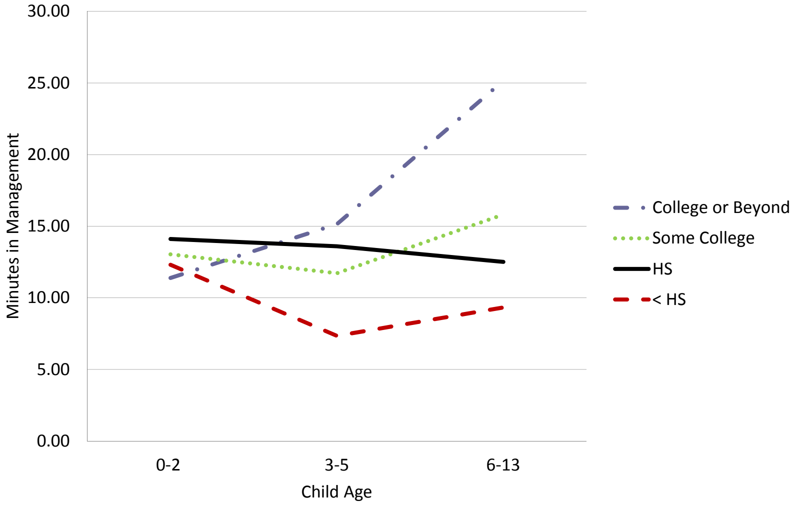
Inequality in Investments as Children Age



Inequality in Investments as Children Age

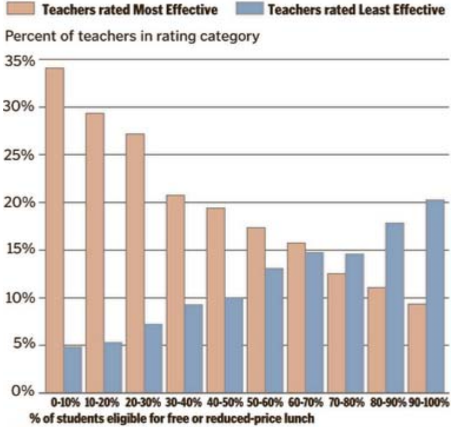


Inequality in Investments as Children Age



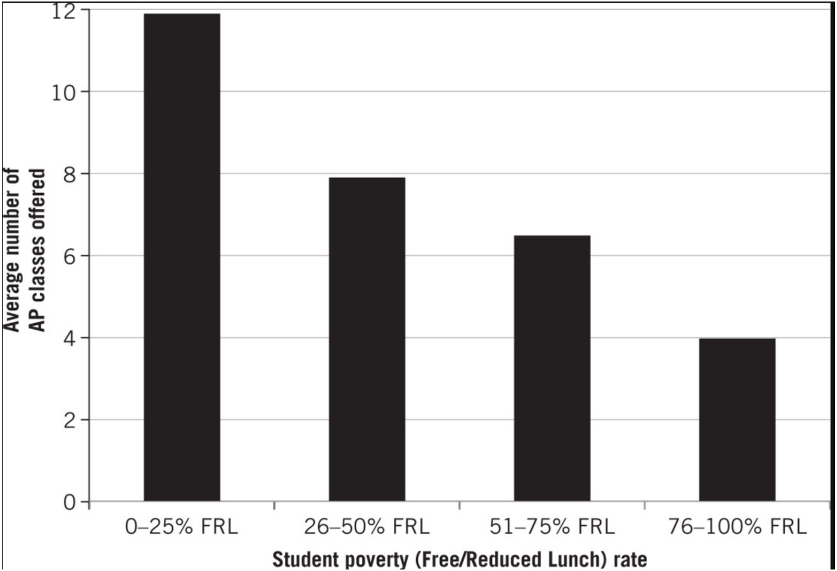
How teacher ratings relate to a school's poverty level

Teachers who receive the state's top value-added rating — "Most Effective" — are likely to be in schools with fewer poor students, based on value-added ratings for teachers at 1,720 public schools. Of 1,035 teachers at the wealthiest schools, 34 percent got the top rating. In contrast, of 2,411 teachers at the poorest schools, just over 9 percent were rated "Most Effective."

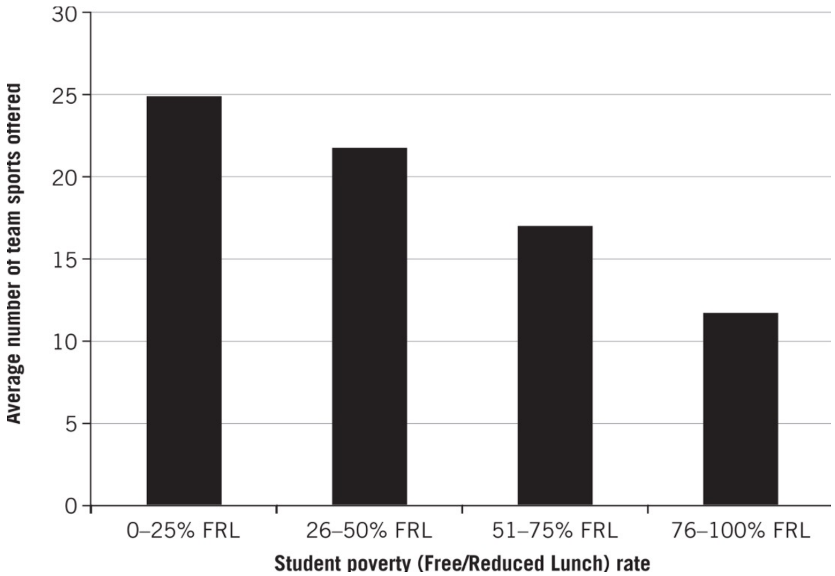


SOURCE: Ohio Department of Education

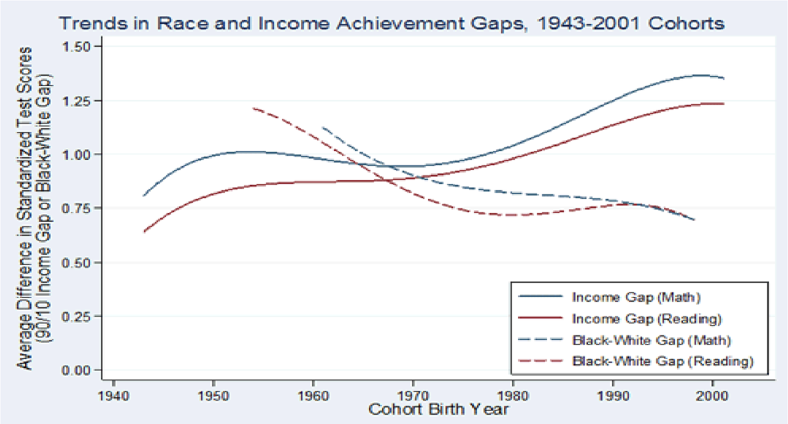
RICH EXNER, JAMES OWENS | THE PLAIN DEALER



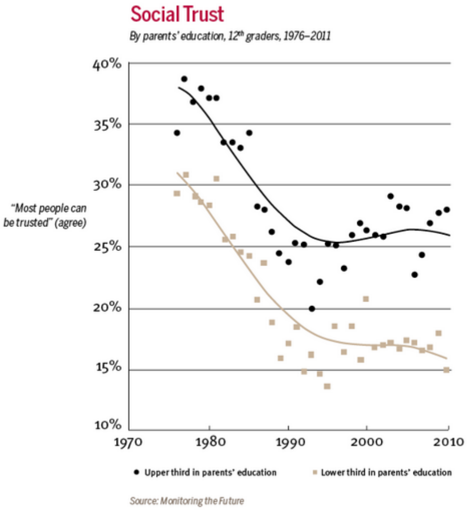
12
10
8
6
4
2
0
Average number of AP classes offered
0–25% FRL
26–50% FRL
51–75% FRL
76–100% FRL
Student poverty (Free/Reduced Lunch) rate


Average number of team sports offered
30
25
20
15
10
5
0
0–25% FRL
26–50% FRL
51–75% FRL
76–100% FRL
Student poverty (Free/Reduced Lunch) rate

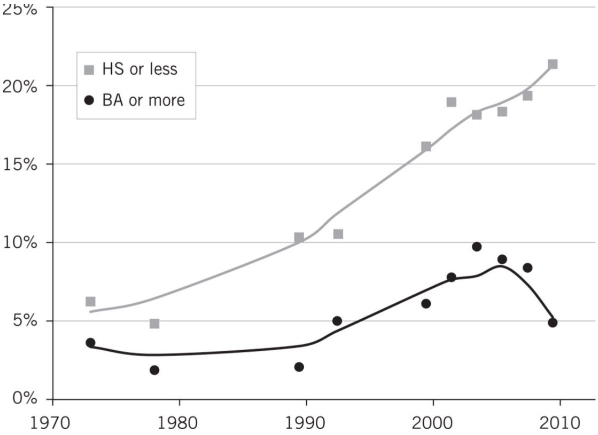
Inequality in Cognitive Skills Over Time



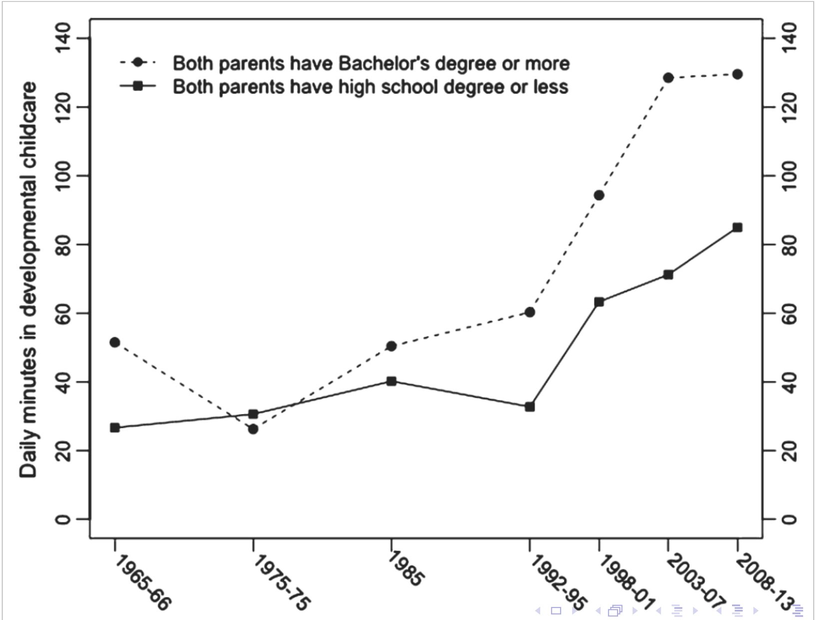
Inequality in Noncognitive Skills Over Time



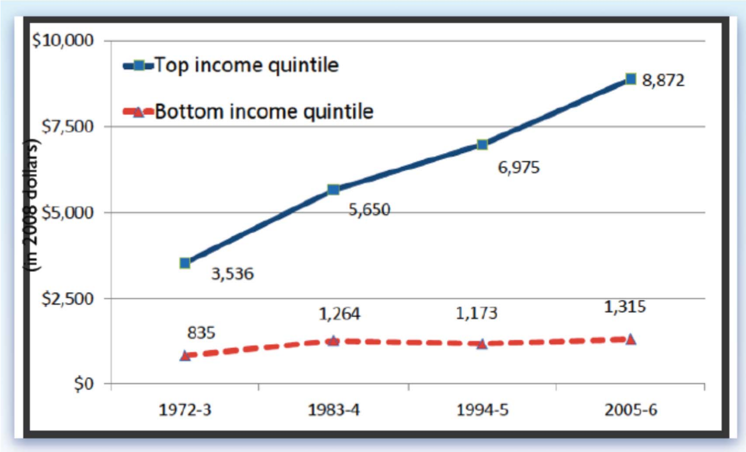
Inequality in Health Over Time



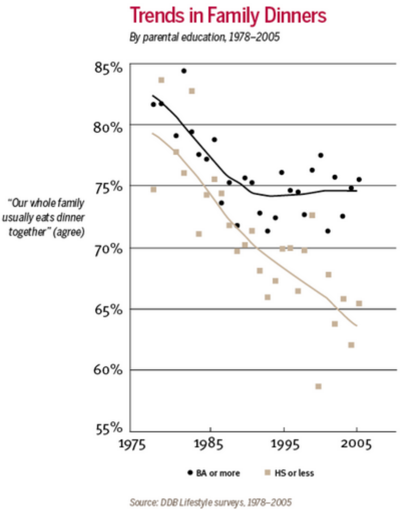
Inequality in Investments Over Time



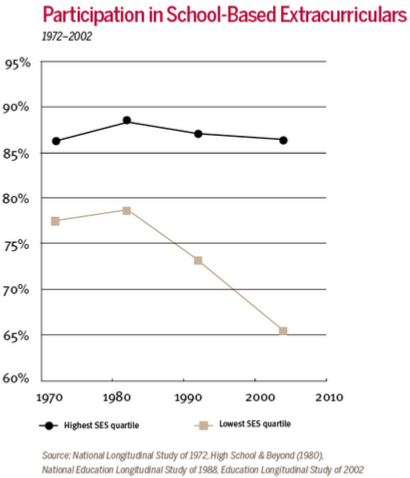
Inequality in Investments Over Time



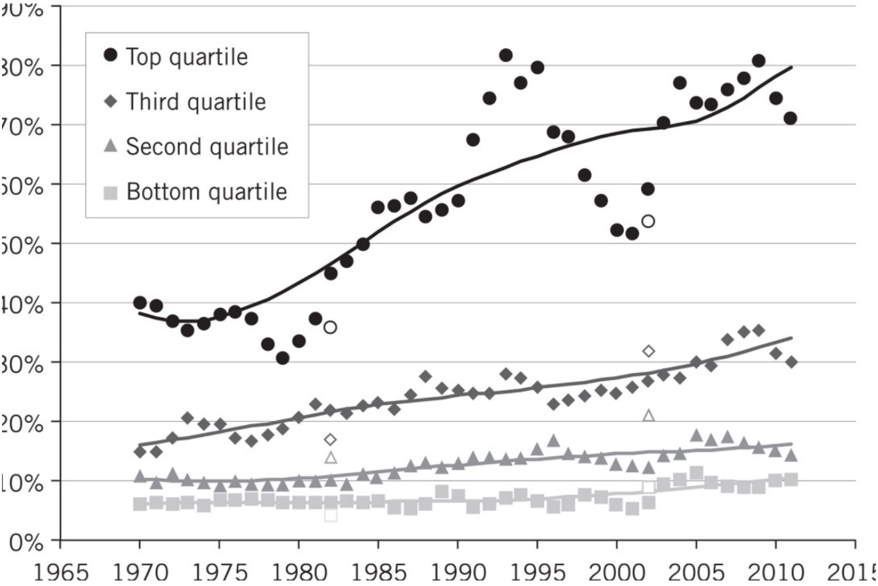
Inequality in Investments Over Time



Inequality in Investments Over Time



Increasing Inequality in College Attendance



Evidence from RCTs in Early Childhood and Adolescence

- Early interventions:
  - Perry Preschool Program
  - Abecedarian
  - Infant Health and Development Program (IHDP)

Early Childhood Education

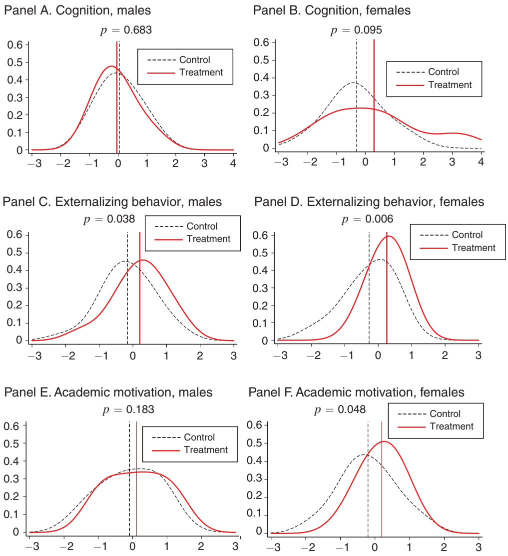


FIGURE 5. KERNEL DENSITIES OF FACTOR SCORES

Early Childhood Education

Table 7: Life-Cycle Outcomes, PPP and ABC

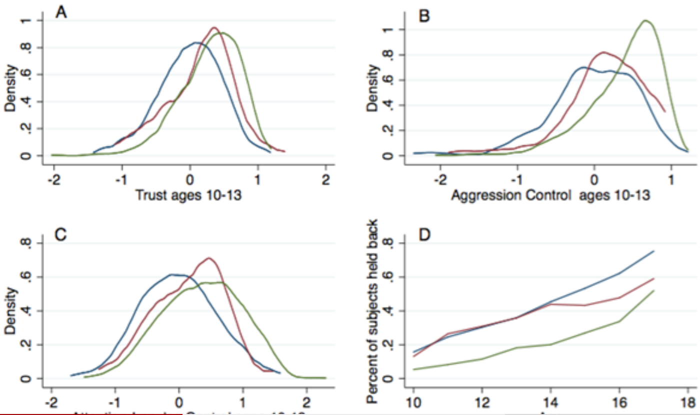
	PPP			ABC		
	Age	Female	Male	Age	Female	Male
Cognition and Education						
Adult IQ	-	-	-	21[c]	10.275	2.588
	-	-	-		**(0.005)**	(0.130)
High School Graduation	19[a]	0.56	0.02	21[c]	0.238	0.176
		(0.000)	(0.416)		**(0.090)**	**(0.100)**
Economic						
Employed	40[a]	-0.01	.29	30[c]	0.147	0.302
		(0.615)	**(0.011)**		(0.135)	**(0.005)**
Yearly Labor Income, 2014 USD	40[a]	\$6,166	\$8,213	30[c]	\$3,578	\$17,214
		(0.224)	(0.150)		**(0.000)**	(0.110)
HI by Employer	40[a]	0.129	0.206	31[b]	0.043	0.296
		(0.055)	(0.103)		(0.512)	**(0.035)**
Ever on Welfare	18–27[a]	-0.27	0.03	30[c]	0.006	-0.062
		(0.049)	(0.590)		(0.517)	**(0.000)**
Crime						
No. of Arrests[d]	≤40[a]	-2.77	-4.88	≤34[c]	-5.061	-6.834
		(0.041)	**(0.036)**		**(0.051)**	(0.187)
No. of Non-Juv. Arrests	≤40[a]	-2.45	-4.85	≤34[c]	-4.531	-6.031
One-sided permutation		**(0.051)**	**(0.025)**		**(0.061)**	(0.181)
Lifestyle						
Self-reported Drug User	-	-	-	30[c]	0.031	-0.438
-	-	-	-		(0.590)	**(0.030)**
Not a Daily Smoker	27[a]	0.111	0.119	-	-	-
		(0.110)	**(0.089)**	-	-	-
Not a Daily Smoker	40[a]	0.067	0.194	-	-	-
		(0.206)	**(0.010)**	-	-	-
Physical Activity	40[a]	0.330	0.090	21[b]	0.249	0.084
		(0.002)	(0.545)		**(0.004)**	(0.866)
Health						
Obesity (BMI >30)	-	-	-	30–34[c]	0.221	-0.292
	-	-	-		(0.920)	**(0.060)**
Hypertension I	-	-	-	30–34[c]	0.096	0.339
	-	-	-		(0.380)	**(0.010)**

Evidence from RCTs in Adolescence

- Becoming-A-Man (B.A.M.) Study:
  - Student training: Learning how to "read" the context to employ the "appropriate" reaction.
- Montreal Longitudinal Study
  - Parent training: Improve monitoring and positive reinforcement; implement non-punitive discipline; and how to better cope with crisis.
  - Child training: Teaching social skills to reduce aggressive behavior (including how to manage anger-inducing situations).

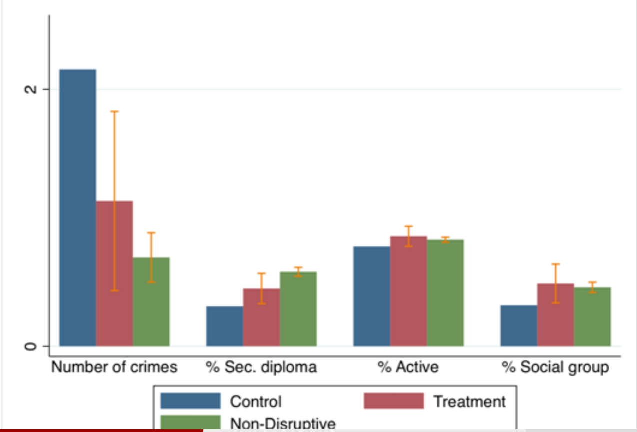
MELS: Algan et al (2014)

Figure 1. Non-cognitive skills and school performance during adolescence. A, B and C show distributions for non-cognitive skills measured in early adolescence for the control, treatment and non-disruptive groups (the non-disruptive boys being those who were not disruptive in kindergarten and did not participate in the experiment as treatment or control: they serve as a normative population baseline). Kolmorgorov-Smirnov test for equality of Treatment and Control distributions gives p-value of 0.003 for Trust, 0.036 for Aggression Control, and 0.023 for Attention-Impulse Control. D shows the increasing gap in the percent of subjects held back at each age. P-value from χ^2 test between Treatment and Control groups is 0.60 at age 10 and 0.01 at age 17.



MELS: Algan et al (2014)

Figure 2. Young Adult Outcomes. As young adults, treatment subjects commit fewer crimes, are more likely to graduate from secondary school, are more likely to be active fulltime in school or work, and are more likely to belong to a social or civic group. The intervention closed part or all of the gap between boys ranked as disruptive in kindergarten but not treated (the control group) and the non-disruptive boys (who represent the normative population). Raw differences are significant for secondary diploma (p-value=0.04) and group membership (p-value=0.05), conditional differences (controlling for group imbalances) are significant for number of crimes (p-value=0.09) and percent active fulltime (p-value=0.03).



Rest of Presentation

- Equation that describes skill formation process.
- Identification and estimation of key parameters of the equation.
- Constraints: Decision maker preference and information set
- Identification and estimation of subjective information set.

Skills Developed in Early Childhood

- **Early development**:
  - Development of language and cognitive skills
  - Development of **executive functions**:
    - Inhibitory control;
    - Working memory;
    - Cognitive flexibility (flexible thinking and set shifting).

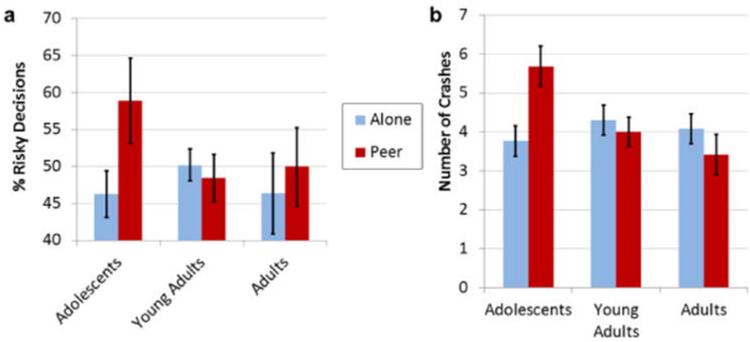
Skills Developed in Adolescence

- **Adolescent development**:

  It seems like people accept you more if you're, like, a dangerous driver or something. If there is a line of cars going down the road and the other lane is clear and you pass eight cars at once, everybody likes that. . . . If my friends are with me in the car, or if there are a lot of people in the line, I would do it, but if I'm by myself and I didn't know anybody, then I wouldn't do it. That's no fun. — Anonymous teenager, as quoted in The Culture of Adolescent Risk-Taking (Lightfoot, 1997, p. 10)

Adolescence and Risk Taking

Figure 2



Differential susceptibility of adolescents to peer influences on Stoplight task performance

Mean (a) percentage of risky decisions and (b) number of crashes for adolescent, young adult, and adult participants when playing the Stoplight driving game either alone or with a peer audience. Error bars indicate the standard error of the mean.

Skills Developed in Adolescence

- **Adolescent development**:
  - Fast development of the reward system potentialized by the influence of peers.
  - Slow development of "rational" decision making system.

Technology of Skill Formation

- We formalize the notion that human capital accumulation is one in which we produce different types of skills at different stages of the lifecycle.
- This notion leads to a technology of skill formation that is described by two parameters:
  - Self-productivity of skills: I learn how inhibit control early on, that helps me learn how to “read” the context before choosing an action when adolescent.
  - Dynamic complementarity: The returns to the development of “reading” context are higher for the children that have learned how to inhibit control early on (and vice-versa).

Optimal Investments in Early Childhood and Adolescence

- Let $h_{i,0}$ and $x_{i,e}$ denote, respectively, human capital at birth and investment during early childhood.
- Let $h_{i,a}$ denote the human capital at beginning of adolescence. Assume that:

$$h_{i,a} = \left[\gamma_e x_{i,e}^{\phi_e} + (1 - \gamma_e) \, h_{i,0}^{\phi_e} \right]^{\frac{1}{\phi_e}}$$

Optimal Investments in Early Childhood and Adolescence

- Let $x_{i,a}$ denote investment during adolescence.
- Let $\overline{h}_i$ denote the human capital at beginning of adulthood. Assume that:
$$\overline{h}_i = \left[\gamma_a x_{i,a}^{\phi_a} + \left(1 - \gamma_a\right) h_{i,a}^{\phi_a}\right]^{\frac{1}{\phi_a}}$$

Optimal Investments in Early Childhood and Adolescence

- Apply recursion and assume $\phi_e = \phi_a = \phi$:

$$\overline{h} = \left\{ \gamma_a x_{i,a}^{\phi} + (1 - \gamma_a)\, \gamma_e x_{i,e}^{\phi} + (1 - \gamma_a)\,(1 - \gamma_e)\, h_{i,0}^{\phi} \right\}^{\frac{1}{\phi}}$$

- Note that:
  - The parameter $1 - \gamma_a$ captures self-productivity.
  - The parameter ϕ captures dynamic complementarity or substitutability.

Optimal Investments in Early Childhood and Adolescence

- The problem of the parent:

$$\min \; x_{i,e} + \frac{1}{1+r} x_{i,a}$$

subject to the technology of skill formation:

$$\overline{h} = \left\{ \gamma_a x_{i,a}^{\phi} + (1-\gamma_a)\,\gamma_e x_{i,e}^{\phi} + (1-\gamma_a)\,(1-\gamma_e)\, h_{i,0}^{\phi} \right\}^{\frac{1}{\phi}}$$

where $\gamma_a \in [0,1]$, $\gamma_e \in [0,1]$, and $\phi \leq 1$.

Boundary Solution when $\phi = 1$

- In this case:

$$\overline{h} = \gamma_a x_{i,a} + (1 - \gamma_a)\,\gamma_e x_{i,e} + (1 - \gamma_a)\,(1 - \gamma_e)\,h_{i,0}$$

- Two investment strategies: Invest early and produce $(1 - \gamma_a)\,\gamma_e$ units of human capital per unit of investment.
- Save in physical assets early and invest $1 + r$ late and produce $(1 + r)\,\gamma_a$ units of human capital.
- Should invest all early if, and only if:

$$\frac{(1 - \gamma_a)\,\gamma_e}{\gamma_a} > 1 + r$$

Boundary Solution when $\phi \to -\infty$

- In this case:
$$\overline{h}_i = \min \{x_{i,a}, x_{i,e}, h_{i,0}\}$$
- The solution to this problem is $x_{i,a} = x_{i,e} = h_{i,0}$ regardless of r.

Interior Solution when $-\infty < \phi < 1$

- The solution to this problem is characterized by the following ratio:

$$\ln \frac{x_{i,e}}{x_{i,a}} = \frac{1}{1-\phi} \ln \left[\frac{(1-\gamma_a)\,\gamma_e}{\gamma_a} \right] + \frac{1}{1-\phi} \ln \left(\frac{1}{1+r} \right)$$

Dual Side of Dynamic Complementarity

- Returns to late investments are higher for the individuals that have high early investments.
- BUT: Returns to early investments are higher for the individuals who will also have high late investments.
- In other words, if the child will not receive high late investments, then the impacts of early investments will be diminished.

Estimating the Technology of Skill Formation

- Return to the recursive formulation of the technology of skill formation:

$$h_{i,t+1} = \left[\gamma_t x_{i,t}^{\phi_t} + (1-\gamma_t)\, h_{i,t}^{\phi_t}\right]^{\frac{\rho_t}{\phi_t}} e^{\eta_{i,t+1}}$$

- Consider (simplified version of) the Kmenta (1967) approximation:

$$\ln h_{i,t+1} = \psi_{t,1} \ln x_{i,t} + \psi_{t,2} \ln h_{i,t} + \psi_{t,3} \ln x_{i,t} \ln h_{i,t} + \eta_{i,t+1}$$

- Where: $\psi_{t,1} = \gamma_t \phi_t$, $\psi_{t,2} = (1-\gamma_t)\phi_t$, and $\psi_{t,3} = \frac{1}{2}\rho_t \phi_t \gamma_t \,(1-\gamma_t)$.
- Possible to decompose $\eta_{i,t+1}$ into permanent and temporary shocks, but not going to do it today.

Estimating the Technology of Skill Formation

- To simplify the math, I will use a simpler version of the Kmenta approximation:

$$\ln h_{i,t+1} = \psi_{t,1} \ln x_{i,t} + \psi_{t,2} \ln h_{i,t} + \psi_{t,3} \ln x_{i,t} \ln h_{i,t} + \eta_{i,t+1}$$

  for $i = 1, ..., I$ and $t = 1, ..., T$.
- I will illustrate three problems in the estimation of the technology of skill formation:
  - Problem 1: data on measures of human capital have no cardinality: anchoring.
  - Problem 2: data on measures of human capital and investment have measurement error: latent factors.
  - Problem 3: data on investment is endogenous: instruments.

Estimating the Technology of Skill Formation

- To simplify the math, I will use a simpler version of the Kmenta approximation:

$$\ln h_{i,t+1} = \psi_{t,1} \ln x_{i,t} + \psi_{t,2} \ln h_{i,t} + \psi_{t,3} \ln x_{i,t} \ln h_{i,t} + \eta_{i,t+1}$$

  for $i = 1, ..., I$ and $t = 1, ..., T$.
- I will illustrate three problems in the estimation of the technology of skill formation:
  - **Problem 1: data on measures of human capital have no cardinality: anchoring.**
  - Problem 2: data on measures of human capital and investment have measurement error: latent factors.
  - Problem 3: data on investment is endogenous: instruments.

Problem 1: Cardinality of Human Capital

- The notion of a production function implies that inputs and output have a well-defined metric.
- You put a units of investments and b units of current-period human capital and you produce x units of next-period human capital.
- Usually units of investments are time (e.g., hours per day) or money (e.g., dollars per month).
- What is the unit of human capital?

Type of scale	Description	Possible statements	Allowed operators	Example
Nominal	Describes qualitative attributes	Identity, countable	=, ≠	Binary variable denoting gender
Ordinal	Describes objects that can be ordered in terms of "greater", "less", or "equal"	Identity, countable, less than/greater than relations	=, ≠, ≤, ≥	Utility levels, test scores, percentile scores
Interval (cardinal)	Describes objects that can be placed in equally spaced units without a true zero point.	Identity, countable, less than/greater than relations, equality of differences	=, ≠, ≤, ≥, +, -	Educational attainment, dates
Ratio (cardinal)	Describes objects that can be placed in equally spaced units that have a true zero point.	Identity, countable, less than/greater than relations, equality of differences, equality of ratios, true zero	=, ≠, ≤, ≥, +, -, ×, ÷	Earnings, length, age

Table

Problem 1: Cardinality of Human Capital

- Let's approach this problem in the following way. Suppose that we have data on labor income, Y_i, at some point in adulthood (e.g., when the individual is 45 years old).
- We can "anchor" human capital at age t before adulthood, $t = 1, ..., T$, through the equation:

$$\ln Y_i = \ln h_{i,t} + \nu_{i,t}$$

- Now $\ln h_{i,t}$ is cardinal. Assume that $\ln h_{i,t} \sim N\left(\mu_h, \sigma^2_{h,t}\right)$, $\nu_{i,t} \sim N\left(0, \sigma^2_{\nu,t}\right)$.
- Note that $\ln Y_i \sim N\left(\mu_h, \sigma^2_{h,t} + \sigma^2_{\nu,t}\right)$

Problem 1: Cardinality of Human Capital

- Now, we have data on scores in standardized tests $M_{i,t,j}$ for $j = 1, ..., J$.
- Assume that the relationship between $M_{i,t,j}$ and $\ln h_{i,t}$ is:

$$M_{i,t,j} = \alpha_{t,j} + \beta_{t,j} \ln h_{i,t} + \varepsilon_{i,t,j}$$

  where $\varepsilon_{i,t,j} \sim N\left(0, \sigma^2_{t,j}\right)$ is measurement error.

- Therefore, we have that $M_{i,t,j} \sim N\left(\alpha_{t,j} + \beta_{t,j}\mu_h, \beta^2_{t,j}\sigma^2_{t,h} + \sigma^2_{t,j}\right)$.
- In particular, note that $M_{i,t,j} | \ln h_{i,t} \sim N\left(\alpha_{t,j} + \beta_{t,j} \ln h_{i,t}, \sigma^2_{t,j}\right)$.

Problem 1: Cardinality of Human Capital

- Solution: We need to transform at least one of the test scores at t so that the transformed measure has cardinality.
- Define $\tilde{m}_{i,t,1} = E\left(\ln Y_i \middle| M_{i,t,1}\right)$ and $s_{t,1} = \frac{\beta_{t,1}^2 \sigma_{t,h}^2}{\beta_{t,1}^2 \sigma_{t,h}^2 + \sigma_{t,j}^2}$
- Use the fact that $\ln Y_i$ and $M_{i,t,1}$ are jointly normal to conclude that:

$$\tilde{m}_{i,t,1} = \left(1 - s_{t,1}\right) \mu_h + s_{t,1} \left(M_{i,t,1} - \alpha_{t,1}\right).$$

Problem 1: Cardinality of Human Capital

- Given that:

$$\tilde{m}_{i,t,1} = (1 - s_{t,1})\,\mu_h + s_{t,1}\,(M_{i,t,1} - \alpha_{t,1})$$

- and that:

$$M_{i,t,j} = \alpha_{t,j} + \beta_{t,j} \ln h_{i,t} + \varepsilon_{i,t,1}$$

- We conclude that:

$$\tilde{m}_{i,t,1} = (1 - s_{t,1})\,\mu_h + s_{t,1} \ln h_{i,t} + \frac{s_{t,1}}{\beta_{t,1}} \varepsilon_{i,t,1}$$

- We need to estimate $s_{t,1}$.

Problem 1: Cardinality of Human Capital

- We need to estimate $s_{t,1}$, but we don't observe $\ln h_{i,t}$. We do observe $\ln Y_i = \ln h_{i,t} + \nu_{i,t}$, so

$$\tilde{m}_{i,t,1} = (1 - s_{t,1})\,\mu_h + s_{t,1} \ln Y_i + \frac{s_{t,1}}{\beta_{t,1}} \varepsilon_{i,t,1} - s_{t,1} \nu_{i,t}$$

- Clearly, we can't use OLS because $\ln Y_i$ is correlated with $\nu_{i,t}$.
- We need an instrument. In particular, we need something that is correlated with $\ln Y_i$ (through $\ln h_{i,t}$), but not correlated with $\varepsilon_{i,t,1}$ or $\nu_{i,t}$.
- We have a few candidates:
    - Investment at period $t-1$.
    - Determinants of investment at period $t-1$ (e.g., random assignment to control or treatment arms of intervention).
    - If nothing else, then $\tilde{m}^*_{i,\tau,1}$ which is leave-one-out estimator of $\tilde{m}_{i,\tau,1}$ where $\tau \neq t$

Problem 1: Cardinality of Human Capital

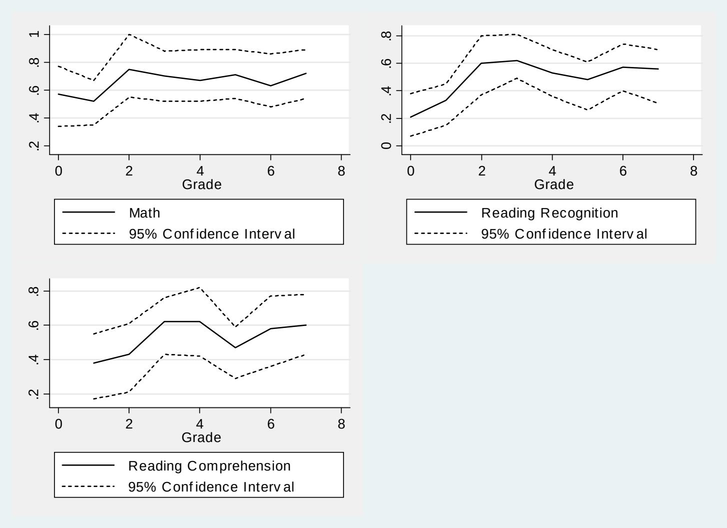
- Use one of these instruments to identify $s_{t,1}$ and define $m_{i,t,1} = \frac{\tilde{m}_{i,t,1}}{s_{t,1}}$

$$m_{i,t,1} = \frac{(1 - s_{t,1})}{s_{t,1}} \mu_h + \ln h_{i,t} + \frac{1}{\beta_{t,1}} \varepsilon_{i,t,1}$$

- Now we have a rescaled score that has a cardinal scale.

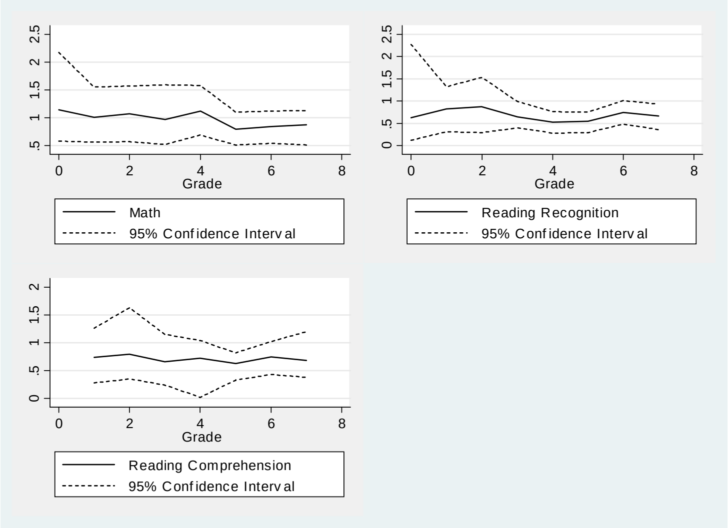
Applications of Anchoring: Bond & Lang (2018)

Figure 1: Raw Difference in Expected White Grade Completion conditional on Test Score



Applications of Anchoring: Bond & Lang (2018)

Figure 2: Measurement Error Adjusted Difference in Achievement in Units of Predicted White Education



Estimating the Technology of Skill Formation

- To simplify the math, I will use a simpler version of the Kmenta approximation:

$$\ln h_{i,t+1} = \psi_{t,1} \ln x_{i,t} + \psi_{t,2} \ln h_{i,t} + \psi_{t,3} \ln x_{i,t} \ln h_{i,t} + \eta_{i,t+1}$$

  for $i = 1, ..., I$ and $t = 1, ..., T$.
- I will illustrate three problems in the estimation of the technology of skill formation:
  - Problem 1: data on measures of human capital have no cardinality: anchoring.
  - **Problem 2: data on measures of human capital and investment have measurement error: latent factors**.
  - Problem 3: data on investment is endogenous: instruments.

Problem 2: Latent Factors

- At every age t we have J test scores and at least one of which (e.g., the first) is anchored:

$$m_{i,t,1} = \frac{(1-s_{t,1})}{s_{t,1}}\mu_h + \ln h_{i,t} + \frac{1}{\beta_{t,1}}\varepsilon_{i,t,1}$$

$$m_{i,t,j} = \alpha_{t,j} + \beta_{t,j} \ln h_{i,t} + \varepsilon_{i,t,j}$$

- At every age t we have J measures of investments:

$$p_{i,t,j} = \delta_{t,j} + \kappa_{t,j} \ln x_{i,t} + \xi_{i,t,j}$$

Problem 2: Latent Factors

- Rewrite in vector form:

$$\boldsymbol{m}_{i,t} = \boldsymbol{\alpha}_t + \boldsymbol{\beta}_t \ln h_{i,t} + \boldsymbol{\varepsilon}_{i,t}$$

- At every age t we have J measures of investments:

$$\boldsymbol{p}_{i,t} = \boldsymbol{\delta}_t + \boldsymbol{\kappa}_t \ln x_{i,t} + \boldsymbol{\xi}_{i,t}$$

Problem 2: Latent Factors

- Estimate $\boldsymbol{\alpha}_t$, $\boldsymbol{\beta}_t$, $\boldsymbol{\delta}_t$, $\boldsymbol{\kappa}_t$, matrix Σ_ϵ and matrix Σ_ξ to predict Bartlett scores:

$$\ln h^B_{i,t} = \left[\boldsymbol{\beta}^{'}_t \Sigma^{-1}_\epsilon \boldsymbol{\beta}_t\right]^{-1} \left[\boldsymbol{\beta}^{'}_t \Sigma^{-1}_\epsilon \left(\boldsymbol{m}_{i,t} - \boldsymbol{\alpha}_t\right)\right]$$

$$\ln x^B_{i,t} = \left[\boldsymbol{\kappa}^{'}_t \Sigma^{-1}_\xi \boldsymbol{\kappa}_t\right]^{-1} \left[\boldsymbol{\kappa}^{'}_t \Sigma^{-1}_\xi \left(\boldsymbol{p}_{i,t} - \boldsymbol{\delta}_t\right)\right]$$

Problem 2: Latent Factors

- Estimate $\boldsymbol{\alpha}_t$, $\boldsymbol{\beta}_t$, $\boldsymbol{\delta}_t$, $\boldsymbol{\kappa}_t$, matrix Σ_ϵ and matrix Σ_ξ to predict Bartlett scores:

$$\ln h_{i,t}^B = \ln h_{i,t} + \left[\boldsymbol{\beta}_t^{'}\Sigma_\epsilon^{-1}\boldsymbol{\beta}_t\right]^{-1}\left[\boldsymbol{\beta}_t^{'}\Sigma_\epsilon^{-1}\boldsymbol{\varepsilon}_{i,t}\right]$$

$$\ln x_{i,t}^B = \ln x_{i,t} + \left[\boldsymbol{\kappa}_t^{'}\Sigma_\xi^{-1}\boldsymbol{\kappa}_t\right]^{-1}\left[\boldsymbol{\kappa}_t^{'}\Sigma_\xi^{-1}\boldsymbol{\xi}_{i,t}\right]$$

Problem 2: Latent Factors

- Note that:

$$\ln h_{i,t}^{B} = \ln h_{i,t} + \tilde{\varepsilon}_{i,t}$$

$$\ln x_{i,t}^{B} = \ln x_{i,t} + \tilde{\xi}_{i,t}$$

- Note that $\tilde{\varepsilon}_{i,t} \sim N\left(0, \left[\boldsymbol{\beta}_t^{'}\Sigma_{\epsilon}^{-1}\boldsymbol{\beta}_t\right]^{-1}\right)$ and $\tilde{\xi}_{i,t} \sim N\left(0, \left[\boldsymbol{\kappa}_t^{'}\Sigma_{\xi}^{-1}\boldsymbol{\kappa}_t\right]^{-1}\right)$ and the variances are known.
- Using factor scores directly will not work because factor scores inherit measurement error (attenuation bias).
- However, bias is a function of $\left[\boldsymbol{\beta}_t^{'}\Sigma_{\epsilon}^{-1}\boldsymbol{\beta}_t\right]^{-1}$ and $\left[\boldsymbol{\kappa}_t^{'}\Sigma_{\xi}^{-1}\boldsymbol{\kappa}_t\right]^{-1}$ which are known. Therefore, we can account for the bias.

Problem 2: Latent Factors

- Define

$$h_t = \{\ln h_{i,t}\}_{i=1}^{I}$$
$$w_t = \{(\ln h_{i,t}, \ln x_{i,t}, \ln h_{i,t} \times \ln x_{i,t})\}_{i=1}^{I}$$
$$\boldsymbol{\gamma_t} = (\gamma_{t,1}, \gamma_{t,2}, \gamma_{t,3})$$

- Rewrite:

$$h_{t+1} = w_t \boldsymbol{\gamma_t} + \eta_{t+1}$$

- Let $\hat{\boldsymbol{\gamma}}_{\boldsymbol{t}}$ denote the infeasible OLS estimator that uses h and w (assumed to be exogenous).

$$\hat{\boldsymbol{\gamma}}_{\boldsymbol{t}} = \left(w_t^T w_t\right)^{-1} \left(w_t^T h_{t+1}\right)$$

- Easy to show that $\hat{\boldsymbol{\gamma}}_{\boldsymbol{t}}$ is consistent.

Problem 2: Latent Factors

- Let $\hat{\gamma}^{\boldsymbol{B}}$ denote the OLS estimator that uses Bartlett scores h^B and w^B (assumed to be exogenous).

$$\hat{\gamma}_{\boldsymbol{t}}^{\boldsymbol{B}} = \left[\left(w_t^B \right)^T w_t^B \right]^{-1} \left[\left(w_t^B \right)^T h_{t+1}^B \right]$$

- Note that w^B is error-ridden measure of w, so standard attenuation bias arises.
- Difference: attenuation bias is a function of variance of measurement error.
- The bias arises because of matrix $\left[\left(w_t^B \right)^T w_t^B \right]$.

Problem 2: Latent Factors

- The matrices $\left[w_t^T w_t\right]$ and $\left[\left(w_t^B\right)^T w_t^B\right]$ are symmetric with the following elements:

Element	plim $\left[w_t^T w_t\right]$	plim $\left[\left(w_t^B\right)^T w_t^B\right]$
$(1,1)$	$E\left(x_t^2\right)$	$E\left(x_t^2\right)+\boldsymbol{Var}\left(\boldsymbol{\xi_t}\right)$
$(1,2)$	$E\left(x_t h_t\right)$	$E\left(x_t h_t\right)$
$(1,3)$	$E\left(x_t^2 h_t\right)$	$E\left(x_t^2 h_t\right)+\boldsymbol{E}\left(\boldsymbol{h_t}\right)\boldsymbol{Var}\left(\boldsymbol{\xi_t}\right)$
$(2,2)$	$E\left(h_t^2\right)$	$E\left(h_t^2\right)+\boldsymbol{Var}\left(\boldsymbol{\varepsilon_t}\right)$
$(2,3)$	$E\left(x_t h_t^2\right)$	$E\left(x_t h_t^2\right)+\boldsymbol{E}\left(\boldsymbol{x_t}\right)\boldsymbol{Var}\left(\boldsymbol{\varepsilon_t}\right)$
$(3,3)$	$E\left(x_t^2 h_t^2\right)$	$E\left(x_t^2 h_t^2\right)+\boldsymbol{\Delta}$

where

$$\boldsymbol{\Delta}=\boldsymbol{E}\left(\boldsymbol{x_t^2}\right)\boldsymbol{Var}\left(\boldsymbol{\varepsilon_t}\right)+\boldsymbol{E}\left(\boldsymbol{h_t^2}\right)\boldsymbol{Var}\left(\boldsymbol{\xi_t}\right)+\boldsymbol{Var}\left(\boldsymbol{\xi_t}\right)+\boldsymbol{Var}\left(\boldsymbol{\varepsilon_t}\right)$$

Problem 2: Latent Factors

- Define matrix $A = \left(w_t^B\right)^T w_t^B - B$ where

$$B = \begin{bmatrix} \boldsymbol{Var}\left(\xi_t\right) & 0 & \boldsymbol{E}\left(h_t\right)\boldsymbol{Var}\left(\xi_t\right) \\ & \boldsymbol{Var}\left(\varepsilon_t\right) & \boldsymbol{E}\left(x_t\right)\boldsymbol{Var}\left(\varepsilon_t\right) \\ & & \boldsymbol{\Delta} \end{bmatrix}$$

- Feasible estimator $\hat{\gamma}^{\boldsymbol{A}}$ is consistent:

$$\hat{\gamma}^{\boldsymbol{A}} = \left[\left(w_t^B\right)^T w_t^B - B\right]^{-1}\left[\left(w_t^B\right)^T h_{t+1}^B\right]$$

Figure 3
Share of Residual Variance in Measurements of Cognitive Skills Due to the Variance of Cognitive Factor (Signal) and Due to the Variance of Measurement Error (Noise)

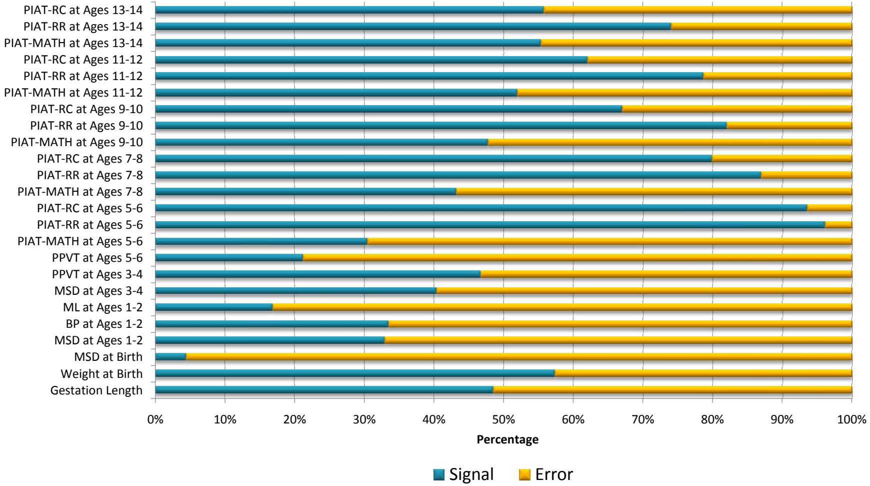


Figure 4A
Share of Residual Variance in Measurements of Noncognitive Skills Due to the Variance of Noncognitive Factor (Signal) and Due to the Variance of Measurement Error (Noise)

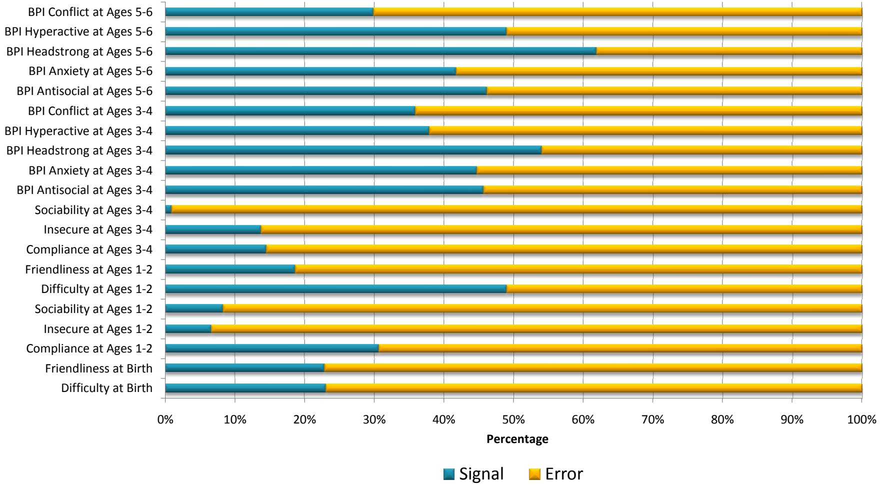


Figure 4B
Share of Residual Variance in Measurements of Noncognitive Skills Due to the Variance of Noncognitive Factor (Signal) and Due to the Variance of Measurement Error (Noise)

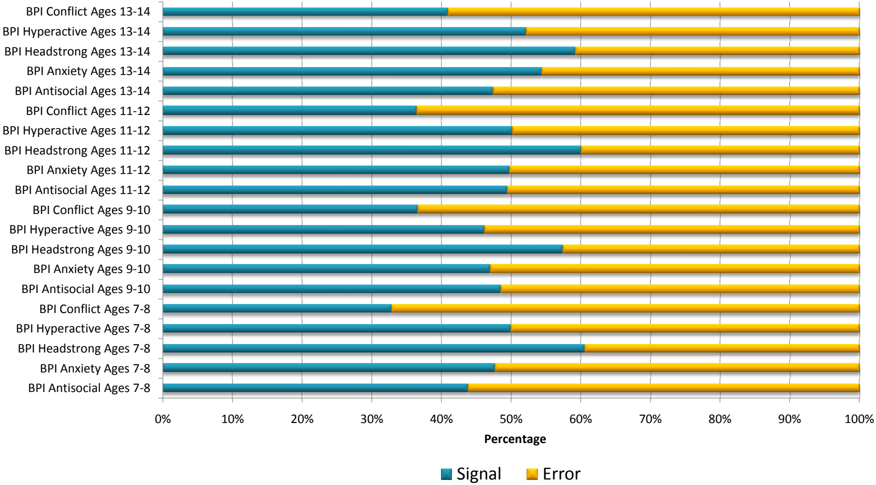


Figure 5A

Share of Residual Variance in Measurements of Investments Due to the Variance of Investment Factor (Signal) and Due to the Variance of Measurement Error (Noise)

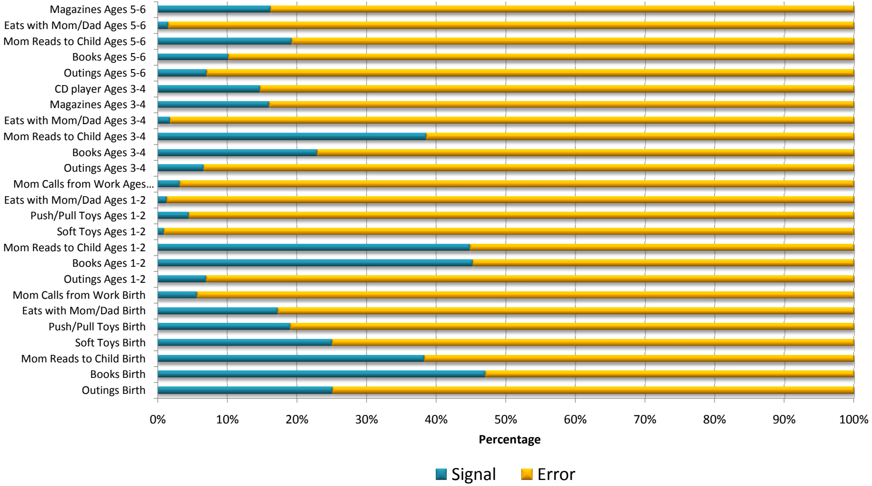


Figure 5B
Share of Residual Variance in Measurements of Investments Due to the Variance of Investment Factor (Signal) and Due to the Variance of Measurement Error (Noise)

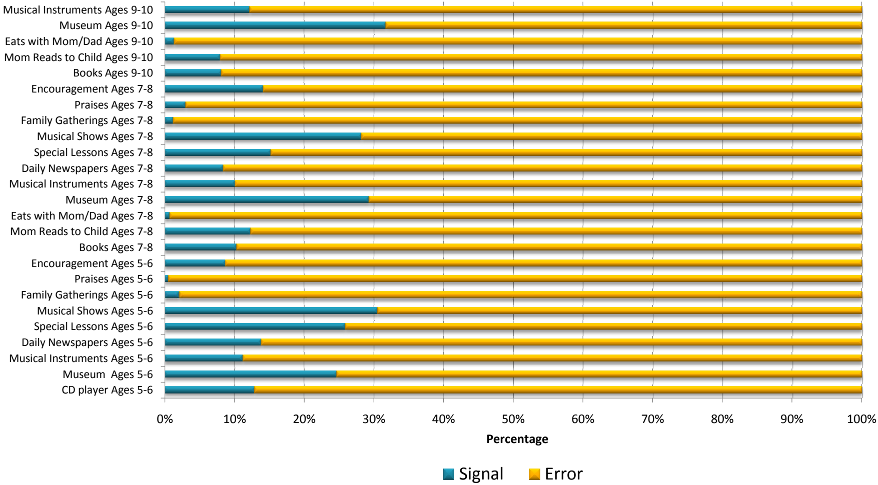
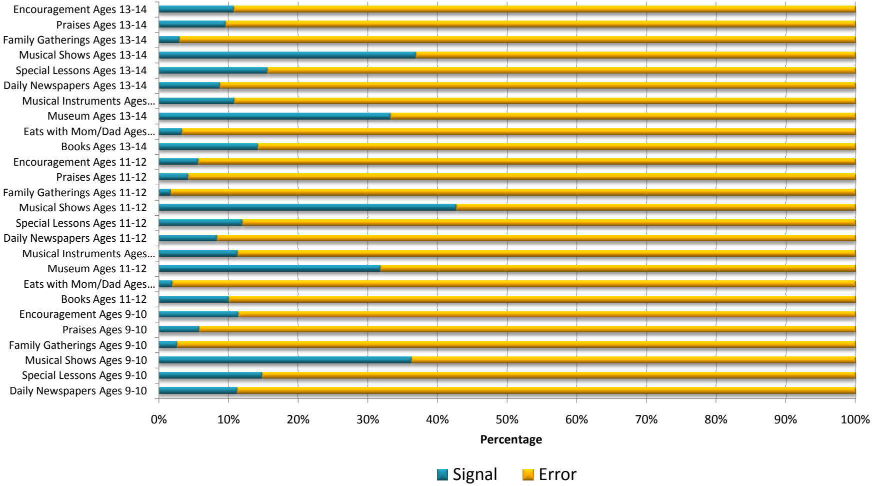


Figure 5C
Share of Residual Variance in Measurements of Investments Due to the Variance of Investment Factor (Signal) and Due to the Variance of Measurement Error (Noise)



Estimating the Technology of Skill Formation

- To simplify the math, I will use a simpler version of the Kmenta approximation:

$$\ln h_{i,t+1} = \psi_{t,1} \ln x_{i,t} + \psi_{t,2} \ln h_{i,t} + \psi_{t,3} \ln x_{i,t} \ln h_{i,t} + \eta_{i,t+1}$$

  for $i = 1, ..., I$ and $t = 1, ..., T$.
- I will illustrate three problems in the estimation of the technology of skill formation:
  - Problem 1: data on measures of human capital have no cardinality: anchoring.
  - Problem 2: data on measures of human capital and investment have measurement error: latent factors.
  - **Problem 3: data on investment is endogenous: instruments.**

Problem 3: Instruments

- Note that:

$$\ln h_{i,t+1} = \psi_{t,1} \ln x_{i,t} + \psi_{t,2} \ln h_{i,t} + \psi_{t,3} \ln x_{i,t} \ln h_{i,t} + \eta_{i,t+1}$$

$$\ln x_{i,t} = z_{i,t} + v_{i,t}$$

- Here $z_{i,t}$ is the instrument.
- Valid instruments address not only endogeneity ($\ln x_{i,t}$ correlated with η_{t+1}) but also problems created by measurement error in $\ln x^B_{i,t}$.
- Instrument does not address bias due to measurement error in $\ln h^B_{i,t}$ unless we have a specific instrument for $\ln h_{i,t}$.

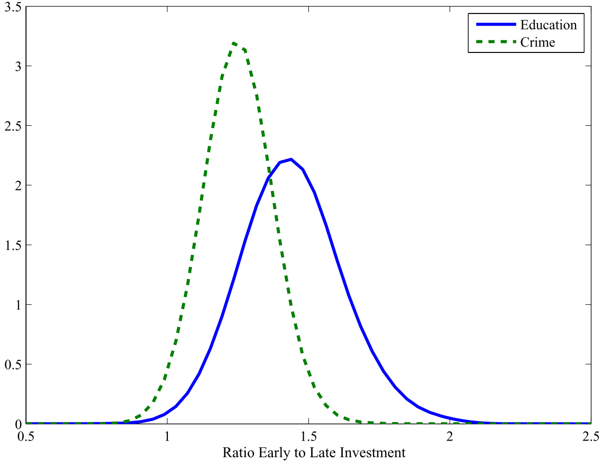


FIGURE 6.—Densities of ratio of early to late investments maximizing aggregate education versus minimizing aggregate crime.

Hart and Risley (1995): Children's Vocabulary Size

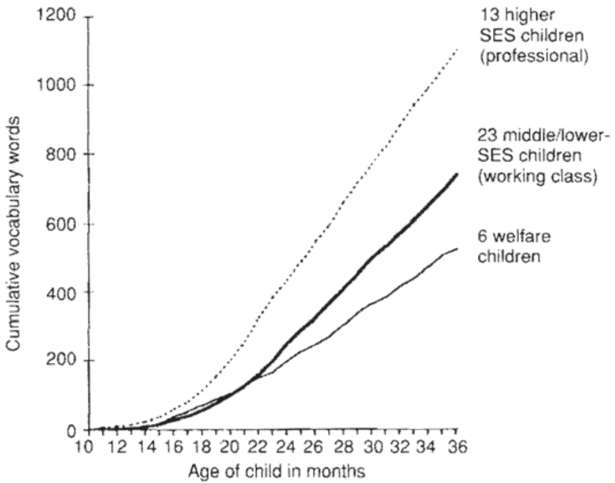
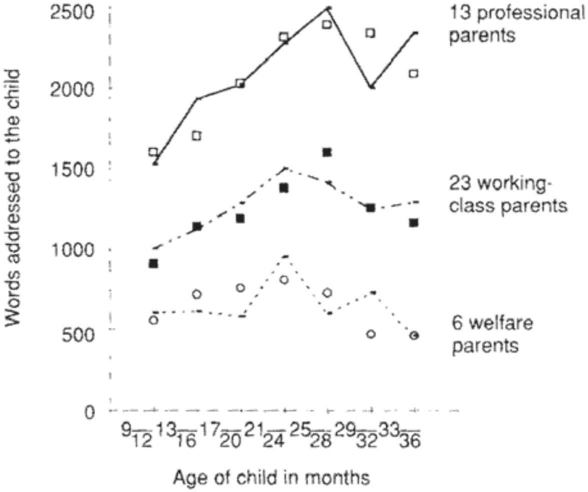


Figure 2. The widening gap we saw in the vocabulary growth of children from professional, working-class, and welfare families across their first 3

Hart and Risley (1995): Adult Words per Hour



Extending the Theory: Preferences

- Preferences are represented by the following utility function:

$$U\left(c, h_1, h_1^R\right) = \ln c + \alpha \ln h_1 + \beta 1\left(\ln h_1 \leq \ln h_R\right)$$

- Where:
  - c is consumption;
  - h_1 is the child's human capital at the end of the early childhood period;
  - h_R is the parent's reference point for the child's human capital level at the end of the early childhood period.
  - From the point of view of the parent, $\ln h_R \sim N\left(\mu_R, \sigma_R^2\right)$.

Theory: Budget Constraint

- I assume that parents cannot borrow or save:

$$c + px = y$$

- Where:
  - p is the relative price of the investment good;
  - x is the investment good;
  - y is the family income during the early childhood period.

Theory: Technology of Skill Formation

- I assume that the child's human capital at the end of the early childhood period is determined according to:

$$\ln h_1 = \gamma_0 + \gamma_1 \ln h_0 + \gamma_2 \ln x + \nu$$

- Where:
  - h_0 is the child's human capital at birth;
  - ν is a shock that is unanticipated by the parent and unobserved by the economist.
  - From the point of view of the parent, $\gamma_k \sim N\left(\mu_k, \sigma_k^2\right)$.

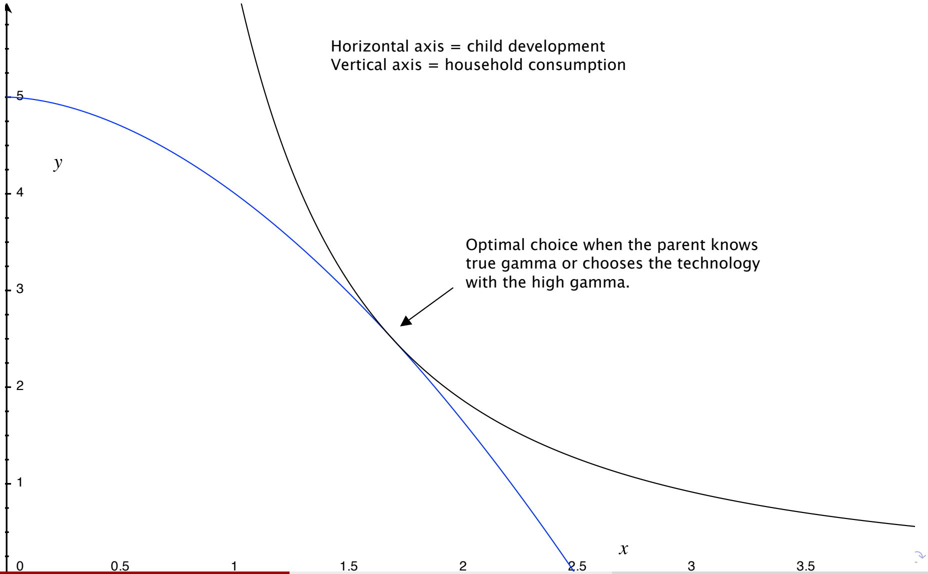
Theory: Parent's Information Set

- The parent's information set:

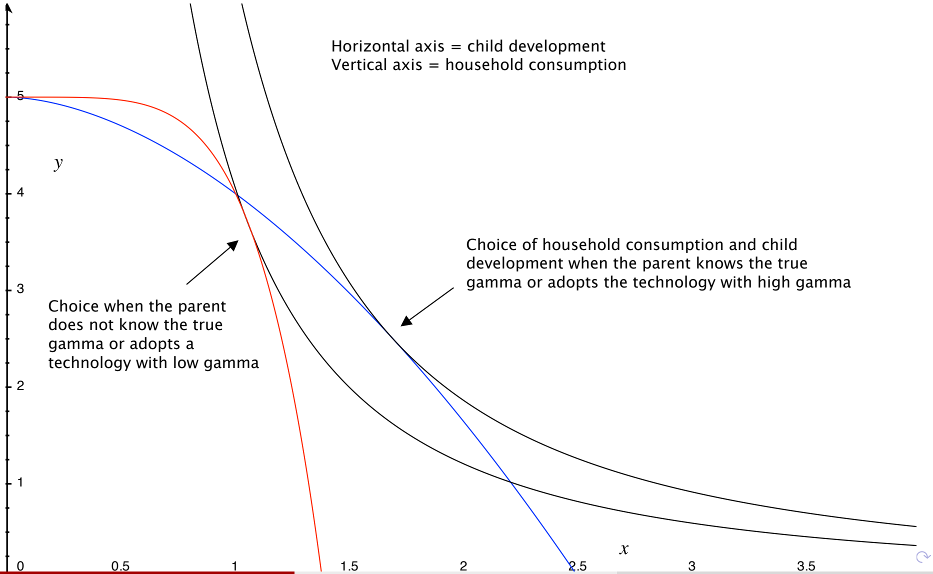
$$\Omega = \left\{ p, y, h_0, \epsilon, \Phi\left(\mu_R, \sigma_R^2\right), \left[\Phi\left(\mu_k, \sigma_k^2\right)\right]_{k=0}^{3} \right\}$$

- Note that from the point of view of the parent:
  - $\Phi\left(\mu_R, \sigma_R^2\right)$ is the parent's perceived distribution of $\ln h_R$.
  - $\Phi\left(\mu_k, \sigma_k^2\right)$ is the parent's perceived distribution of γ_k.
- We do not impose any a priori restrictions on the parameters of these distributions.

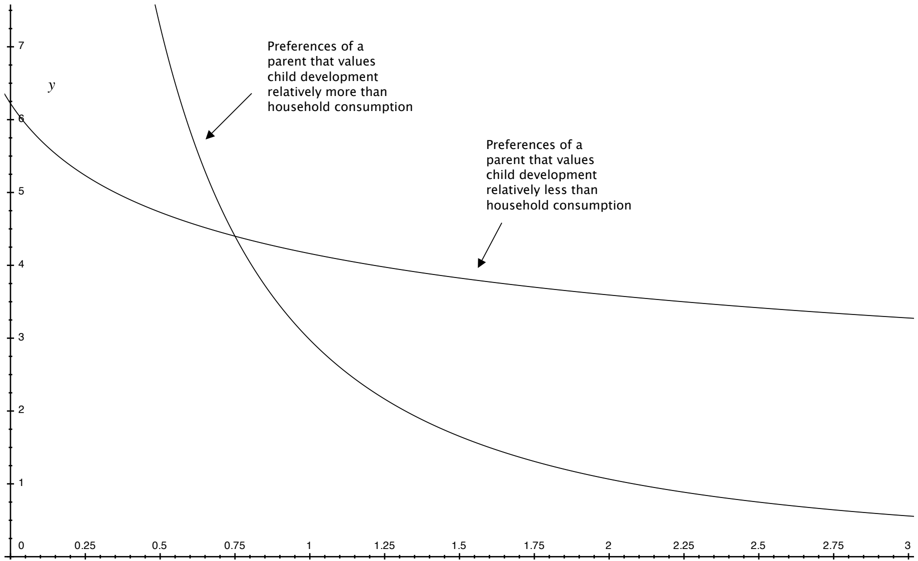
Typical Textbook Model



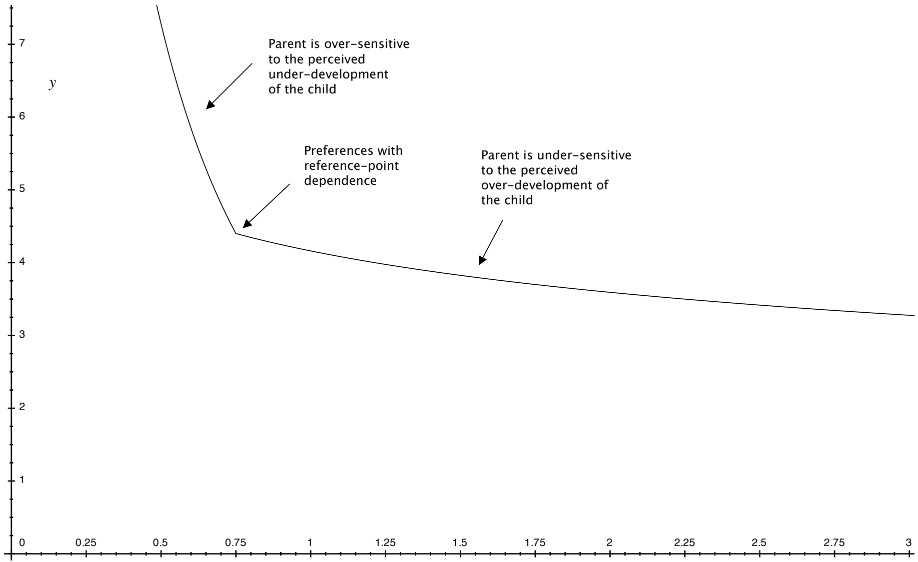
Introducing Heterogeneity in Beliefs



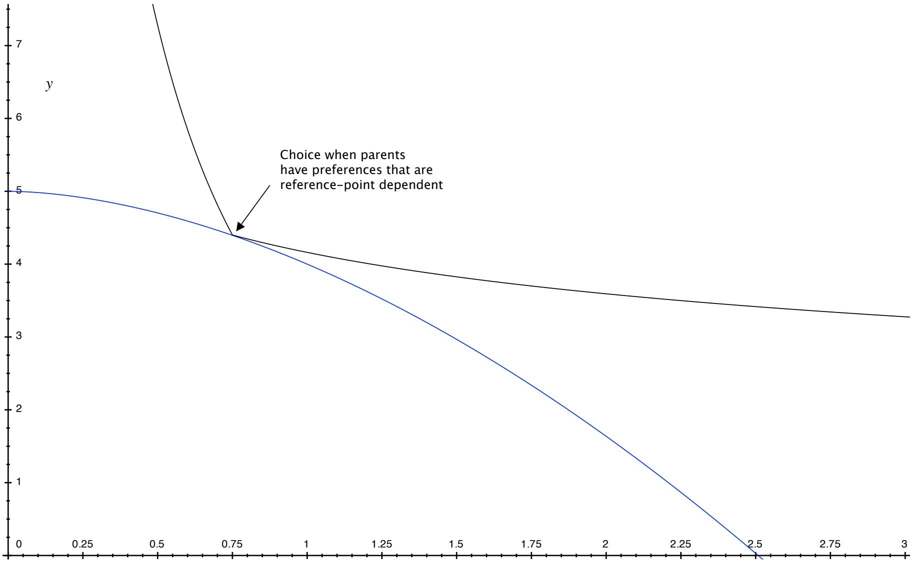
Introducing Heterogeneity in Reference Points



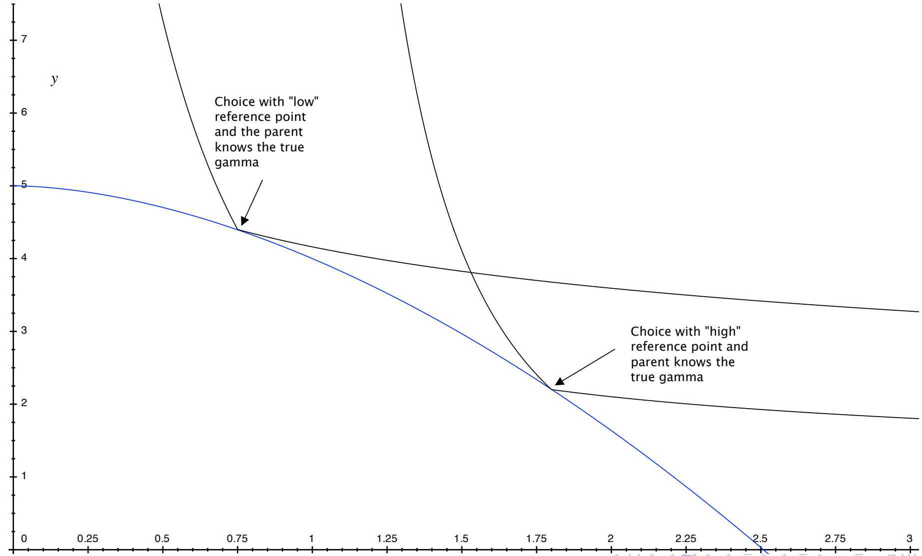
Introducing Heterogeneity in Reference Points



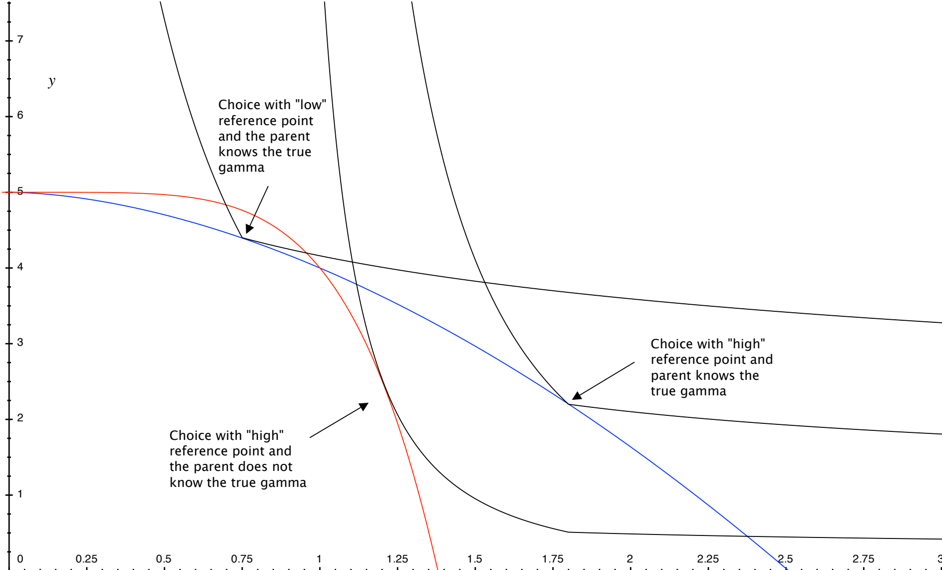
Introducing Heterogeneity in Reference Points



Introducing Heterogeneity in Reference Points



Introducing Heterogeneity in Reference Points



Three Papers

- Can we elicit maternal subjective expectations?
  - **Cunha, Elo, and Culhane (2013, revised 2017)**.
- Does intervention that provide information and objective feedback affect parental beliefs, investments, and development?
  - **Cunha, Gerder, and Nihtiavova (2018).**
- Do reference points affect parental investments in children?
  - **Wang, Puentes, Behrman, and Cunha (2018)**: Use RCT to see if reference points affect children's height by age 2 years.

Cunha, Elo, and Culhane (2013): Project Timeline

- Philadelphia Human Development (PHD) Study.
  - Round 1: Elicit maternal subjective expectations during 2nd trimester of 1st pregnancy.
  - Round 2: Measure maternal investments when child is 9-12 months old.
  - Round 3: Measure child development when child is 22-26 months old.
  - **Round 4: RCT about language development when child is 28-32 months old.**

Defining Subjective Expectation

- The technology of skill formation is:

$$\ln h_{i,1} = \psi_0 + \psi_1 \ln h_{0,i} + \psi_2 \ln x_i + \psi_3 \ln h_{0,i} \ln x_i + \nu_i$$

- Let Ψ_i denote the mother's information set.
- Let $E\left(\psi_j \middle| h_{0,i}, x_i, \Psi_i\right) = \mu_{i,j}$ and assume that $E\left(\nu_i \middle| \Psi_i\right) = 0$.
- From the point of view of the mother:

$$E\left(\ln h_{i,1} \middle| h_{0,i}, x_i, \Psi_i\right) = \mu_{i,0} + \mu_{i,1} \ln h_{0,i} + \mu_{i,2} \ln x_i + \mu_{i,3} \ln h_{0,i} \ln x_i$$

Model: Preferences and budget constraint

- Consider a simple static model. Parent's utility is:

$$u\left(c_i, h_{i,1}; \alpha_{i,1}, \alpha_{i,2}\right) = \ln c_i + \alpha_{i,1} \ln h_{i,1} + \alpha_{i,2} \ln x_i$$

- Budget constraint is:

$$c_i + p x_i = y_i.$$

Model

- The problem of the mother is to maximize expected utility subject to the mother's information set, the budget constraint, and the technology of skill formation.
- The solution is

$$x_i = \left[\frac{\alpha_{i,1} \left(\mu_{i,2} + \mu_{i,3} \ln h_{0,i} \right) + \alpha_{i,2}}{1 + \alpha_{i,1} \left(\mu_{i,2} + \mu_{i,3} \ln h_{0,i} \right) + \alpha_{i,2}} \right] \frac{y_i}{p}$$

- Clearly, we cannot separately identify α_i from $\mu_{i,\gamma}$ if we only observe x_i, y_i, and p.

Eliciting subjective expectations: Steps

- Measure actual child development: MSD and Item Response Theory (IRT).
- Develop the survey instrument to elicit beliefs $E\left[\ln h_{i,1} \middle| h_0, x, \psi_i\right]$:
  - Reword MSD items.
  - Create hypothetical scenarios of h_0 and x.
- Estimate beliefs from answers allowing for error in responses.

SECTION 3: MOTOR AND SOCIAL DEVELOPMENT

PART H: (22 MONTHS - 3 YEARS, 11 MONTHS)

MOTHER/GUARDIAN:

If ______________________ is **at least 22 months** old, but **not yet 4 years** old, please answer these 15 questions.
Child's Name

1.	Has your child ever let someone know, without crying, that wearing wet (soiled) pants or diapers bothered him/her?	YES.... 1 NO..... 0	72/
2.	Has your child ever spoken a partial sentence of 3 words or more?	YES.... 1 NO..... 0	73/
3.	Has your child ever walked upstairs by himself/herself without holding on to a rail?	YES.... 1 NO..... 0	74/
4.	Has your child ever washed and dried his/her hands without any help except for turning the water on and off?	YES.... 1 NO..... 0	75/
5.	Has your child ever counted 3 objects correctly?	YES.... 1 NO..... 0	76/

Eliciting beliefs: Item response theory

- Let $d_{i,j}^{*} = b_{0,j} + b_{1,j}\left(\ln a_i + \frac{b_{2,j}}{b_{1,j}}\theta_i\right) + \eta_{i,j}$
- We observe $d_{i,j} = 1$ if $d_{i,j}^{*} \geq 0$ and $d_{i,j} = 0$, otherwise.
- Measure of (log of) human capital: $\ln h_i = \ln a_i + \frac{b_{2,j}}{b_{1,j}}\theta_i$.
- In this sense, θ_i is deviation from typical development for age.

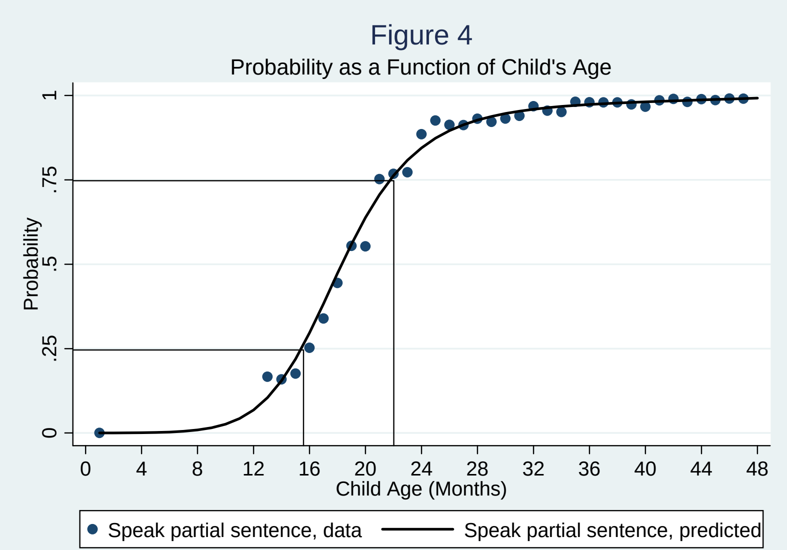


Figure 4
Probability as a Function of Child's Age
Probability
0
.25
.5
.75
1
0
4
8
12
16
20
24
28
32
36
40
44
48
Child Age (Months)
Speak partial sentence, data
Speak partial sentence, predicted

Eliciting beliefs: Changing wording of the MSD Instrument

- In order to measure $E\left[\ln h_{i,1} \middle| h_0, x, \psi_i\right]$, we take the tasks from the MSD Scale, but instead of asking: *"Has your child ever spoken a partial sentence with three words or more?"*, we ask:
- **Method 1**: **How likely is it that a baby will speak a partial sentence with three words or more by age 24 months?**
- **Method 2: What is the youngest and oldest age a baby learns to speak a partial sentence with three words or more?**

Eliciting beliefs: Scenarios of human capital and investments

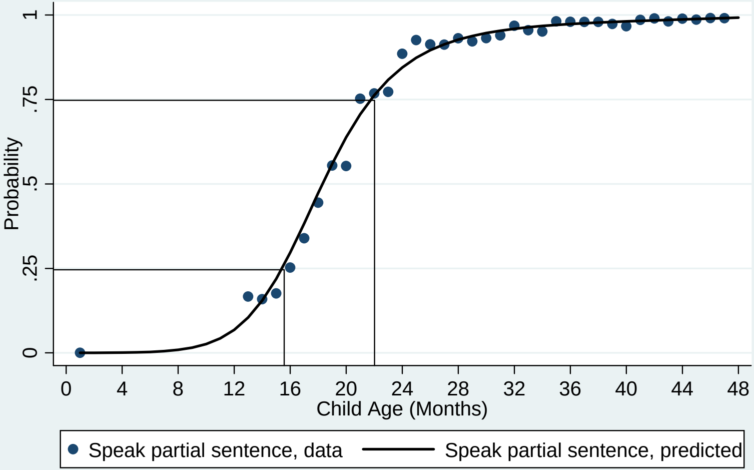
- We consider four scenarios:
  - Scenario 1: Child is healthy at birth (e.g., normal gestation, birth weight, and birth length) and investment is high (e.g., six hours per day).
  - Scenario 2: Child is healthy at birth and investment is low (e.g., two hours per day).
  - Scenario 3: Child is not healthy at birth (e.g., premature, low birth weight, and small at birth) and investment is high.
  - Scenario 4: Child is not healthy at birth and investment is low.
- Scenarios are described to survey respondents through a video.

Method 1: Transforming probabilities into mean beliefs

- **Method 1**: **How likely is it that a baby will speak a partial sentence with three words or more by age 24 months?**
- Let's say that when investment is high – that is, when $x = \overline{x}$ – the mother states that there is a 75% chance that the child will learn how to speak a partial sentence with three words or more.
- And when investment is low– that is, when $x = \underline{x}$ – the mother states that there is a 25% chance that the child will learn how to speak a partial sentence with three words or more.
- We convert this probability statement into an age-equivalent statement using the NHANES data.

Figure 4

Probability as a Function of Child's Age



Method 2: Transforming age ranges into probabilies

- **Method 2: What is the youngest and oldest age a baby learns to speak a partial sentence with three words or more?**
- Let's say that when investment is high, so that $x = \overline{x}$, the mother states that the youngest and oldest ages a baby will learn how to speak a sentence with three words or more are, respectively, 18 and 28 months.
- And when investment is low, so that $x = \underline{x}$, the mother states that the ages are 20 and 30 months.
- We need to transform the age ranges into probabilities. We use the age ranges to estimate a mother-specific IRT model.

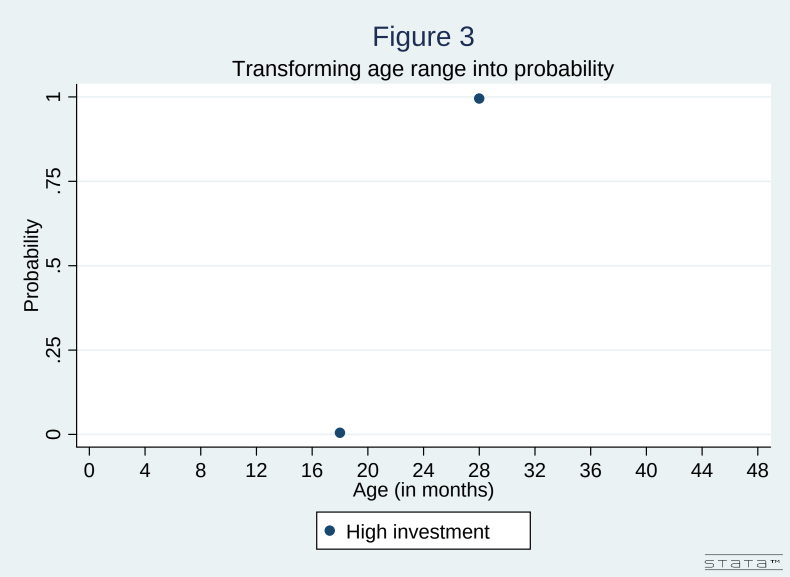


Figure 3
Transforming age range into probability
Probability
0
.25
.5
.75
1
0
4
8
12
16
20
24
28
32
36
40
44
48
Age (in months)
High investment

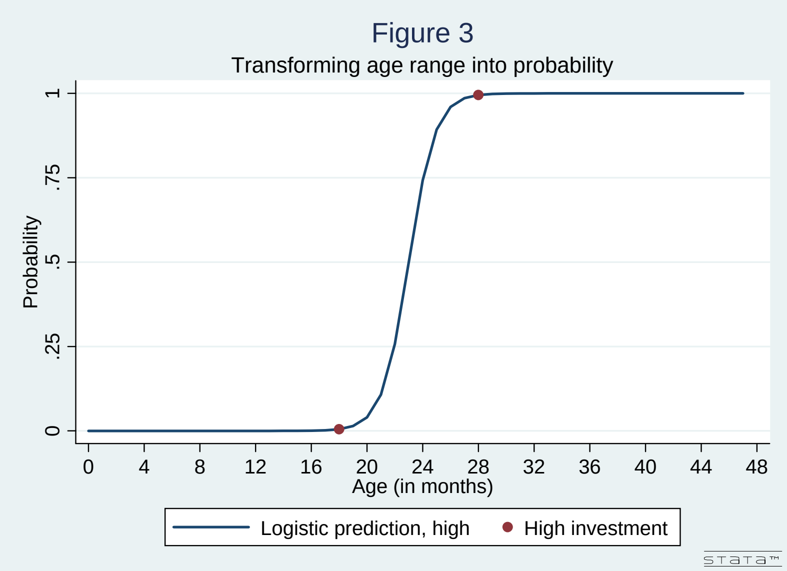
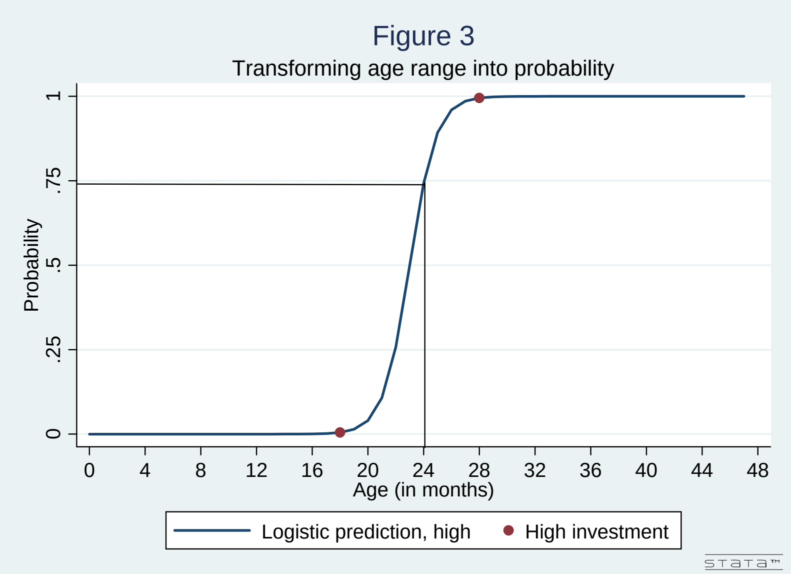


Figure 3
Transforming age range into probability
Probability
0
.25
.5
.75
1
0
4
8
12
16
20
24
28
32
36
40
44
48
Age (in months)
Logistic prediction, high
High investment



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Probability
0
.25
.5
.75
1
0
4
8
12
16
20
24
28
32
36
40
44
48
Age (in months)
Logistic prediction, high
High investment

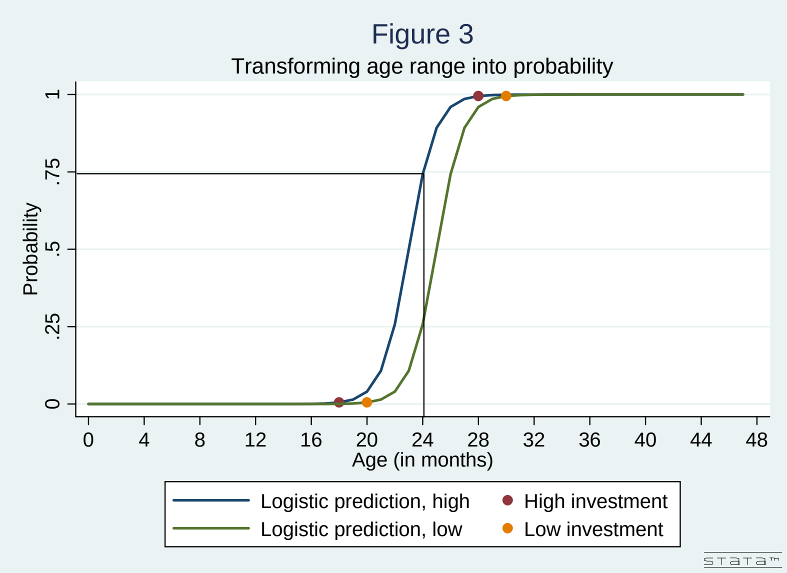


Figure 3
Transforming age range into probability
Probability
0
.25
.5
.75
1
0
4
8
12
16
20
24
28
32
36
40
44
48
Age (in months)
Logistic prediction, high
Logistic prediction, low
High investment
Low investment
stata™

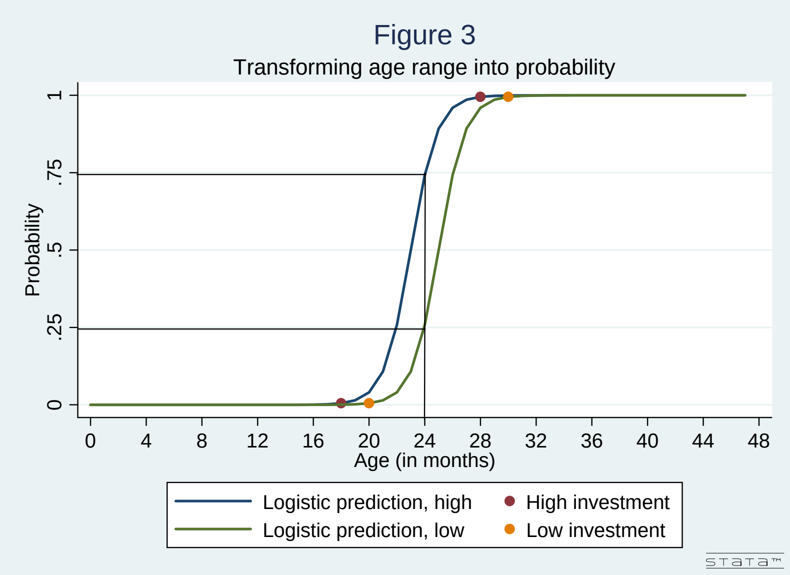


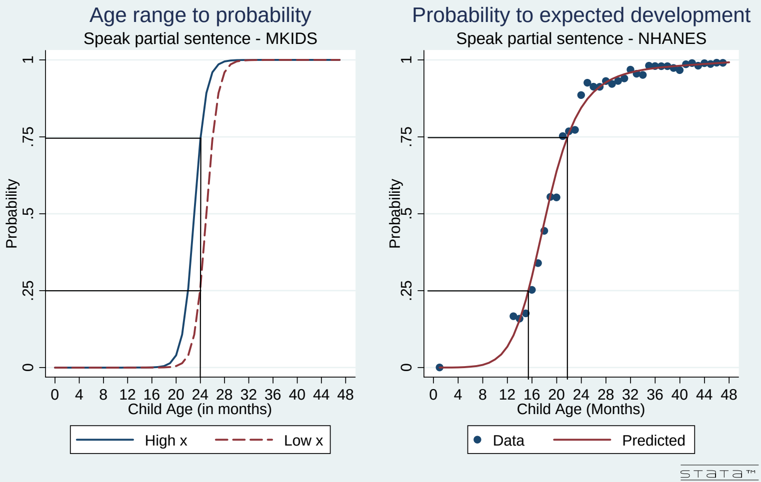
Figure 3
Transforming age range into probability
Probability
0
.25
.5
.75
1
0
4
8
12
16
20
24
28
32
36
40
44
48
Age (in months)
Logistic prediction, high
Logistic prediction, low
High investment
Low investment

Method 2: Transforming probabilities into mean beliefs

- **Method 2: Given scenario for h_0 and x, how likely is it that a baby will speak a partial sentence with three words or more by age 24 months?**
- Given maternal supplied age range and the logistic assumption, we conclude that when $x = \overline{x}$, the mother believes that there is a 75% chance that the child will learn how to speak a partial sentence with three words or more.
- Analogously, when $x = \underline{x}$, the mother believes that there is a 25% chance that the child will learn how to speak a partial sentence with three words or more.
- We convert this probability statement into an age-equivalent statement using the NHANES data.

Figure 3

Expected development for two levels of investments (x)



Recovering mean beliefs: Measurement error model

- Let $\ln q^L_{i,j,k}$ denote an error-ridden measure of $E\left[\ln h_{i,1} \middle| h_{0,k}, x_k, \psi_i\right]$ generated by "how likely" questions:

$$\ln q^L_{i,j,k} = E\left[\ln h_{i,1} \middle| h_{0,k}, x_k, \psi_i\right] + \epsilon^L_{i,j,k}.$$

- Let $\ln q^A_{i,j,k}$ denote an error-ridden measure of $E\left[\ln h_{i,1} \middle| h_{0,k}, x_k, \psi_i\right]$ generated by "age range" questions:

$$\ln q^A_{i,j,k} = E\left[\ln h_{i,1} \middle| h_{0,k}, x_k, \psi_i\right] + \epsilon^A_{i,j,k}.$$

- For each scenario, we have multiple measures of the same underlying latent variable.

Recovering mean beliefs:

- Use technology of skill formation, and the mother's information set, to obtain:

$$\ln q^L_{i,j,k} = \mu_{i,0} + \mu_{i,1} \ln h_{0,k} + \mu_{i,2} \ln x_k + \mu_{i,3} \ln h_{0,k} \ln x_k + \epsilon^L_{i,j,k}$$

.

$$\ln q^A_{i,j,k} = \mu_{i,0} + \mu_{i,1} \ln h_{0,k} + \mu_{i,2} \ln x_k + \mu_{i,3} \ln h_{0,k} \ln x_k + \epsilon^A_{i,j,k}.$$

- We have a factor model where:
  - $\mu_i = (\mu_{i,0}, \mu_{i,1}, \mu_{i,2}, \mu_{i,3})$ are the latent factors;
  - $\lambda_k = (1, h_{0,k}, \ln x_k, \ln h_{0,k} \ln x_k)$ are the factor loadings;
  - $\epsilon_{i,j,k} = \left(\epsilon^L_{i,j,k}, \epsilon^A_{i,j,k}\right)$ are the uniquenesses.

Eliciting beliefs: Intuitive explanation

- Let $E\left[\ln h_{i,1}\middle|\, h_0, h, \Psi_i\right]$ denote maternal expectation of child development at age 24 months conditional on the child's intial level of human capital, investments, and the mother's information set.
- Assume, for now, technology is Cobb-Douglas.
- Suppose we measure $E\left[\ln h_{i,1}\middle|\, h_0, x, \Psi_i\right]$ at two different levels of investments:

$$E\left[\ln h_{i,1}\middle|\, h_0, \overline{x}, \Psi_i\right] = \mu_{i,0} + \mu_{i,1}\ln h_0 + \mu_{i,2}\ln \overline{x}$$

$$E\left[\ln h_{i,1}\middle|\, h_0, \underline{x}, \Psi_i\right] = \mu_{i,0} + \mu_{i,1}\ln h_0 + \mu_{i,2}\ln \underline{x}$$

- Subtracting and re-organizing terms:

$$\mu_{i,2} = \frac{E\left[\ln h_{i,1}\middle|\, h_0, \overline{x}, \Psi_i\right] - E\left[\ln h_{i,1}\middle|\, h_0, \underline{x}, \Psi_i\right]}{\ln \overline{x} - \ln \underline{x}}$$

Important issue

- We could use only one MSD item to elicit beliefs.
- But, if we use more items, we can relax assumptions about measurement error.
- And, we can check for consistency in answers.

Figure 5

Comparing answers across scenarios

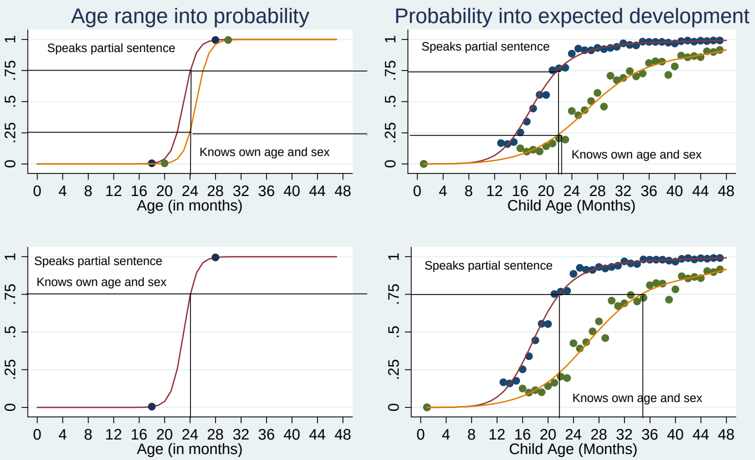


Table 5

Correlation between MSE and demographic characteristics of PHD Study Participants

VARIABLES	Standardized $\mu_{i,\psi,1}$	Standardized $\mu_{i,\psi,2}$	Standardized $\mu_{i,\psi,3}$
Dummies for household income (y)			
$\$25{,}000\ per\ year \leq y < \$55{,}000\ per\ year$	0.2243	0.3452	0.1908
	(0.1003)	(0.0928)	(0.1027)
$\$55{,}000\ per\ year \leq y < \$105{,}000\ per\ year$	-0.1701	0.3662	-0.2460
	(0.1265)	(0.1209)	(0.1135)
$y \geq \$105{,}000\ per\ year$	-0.5060	0.4694	-0.5276
	(0.1278)	(0.1405)	(0.1203)
Constant	-0.2746	-0.5133	0.0514
	(0.1581)	(0.1758)	(0.1664)
Observations	822	822	822
R-Squared	0.0709	0.0641	0.0900

Robust standard errors in parentheses.

Table 6

Correlation between the HOME Score and MSE

VARIABLES	Dependent variable: Standardized HOME Score					
	Both	Both	How Likely Only	How Likely Only	Age Range Only	Age Range Only
Standardized μ_1	-0.0237	-0.0015	-0.0946	-0.0577	-0.0136	0.0306
	(0.0813)	(0.0740)	(0.0799)	(0.0742)	(0.0585)	(0.0530)
Standardized μ_2	0.1667	0.1141	0.1185	0.0980	0.1699	0.0834
	(0.0449)	(0.0385)	(0.0435)	(0.0395)	(0.0446)	(0.0383)
Standardized μ_3	-0.0856	0.0096	-0.0401	0.0344	-0.0581	-0.0137
	(0.0673)	(0.0611)	(0.0662)	(0.0618)	(0.0479)	(0.0422)
Demographic characteristics included?*	No	Yes	No	Yes	No	Yes
Observations	687	687	687	687	687	687
R-squared	0.0369	0.2695	0.0343	0.2706	0.0314	0.2655

Robust standard errors in parentheses.

*Note: The following variables describe demographic characteristics: A dummy variable that takes the value one if the mother's year of birth is between 1978 and 1987 and zero otherwise; a dummy variable that takes the value one if the mother's year of birth is between 1988 and 1997 and zero otherwise; a dummy variable that takes the value one if the mother is Hispanic and zero otherwise; a dummy variable that takes the value one if the mother is non-Hispanic black and zero otherwise;

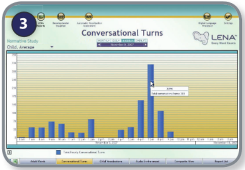
Cunha, Gerdes, and Nihtianova (2018): Language Environment

- Group sessions of approximately 12-15 parents.
- Lasts 13 weeks.
- Each week there is a one-hour session:
  - Importance of language environment for language development.
  - Tips on how to improve language enviroment.
  - Objective feedback of the language environment based on recording provided by the parent.

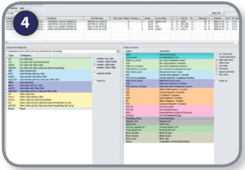
Recording of the Parental Environment

Turn on the DLP and place it in the pocket of the child's LENA clothing.

After completing recording, plug the DLP into a PC running LENA Pro. The sophisticated language environment analysis software automatically uploads and processes the audio file.

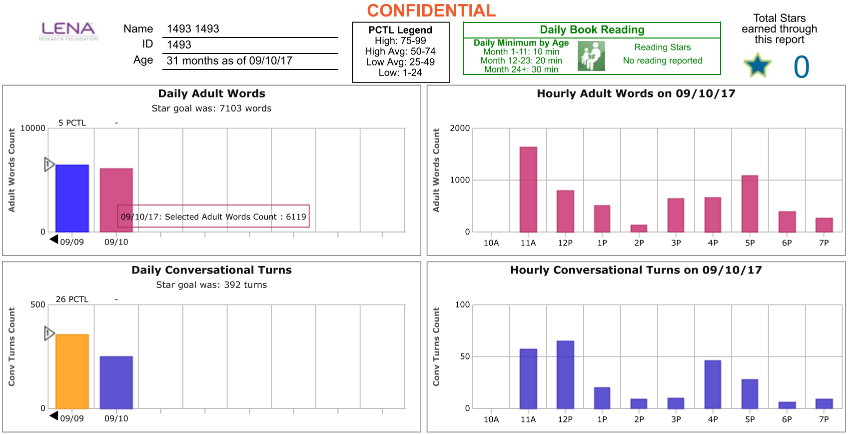


The software generates the LENA reports and other analyses.

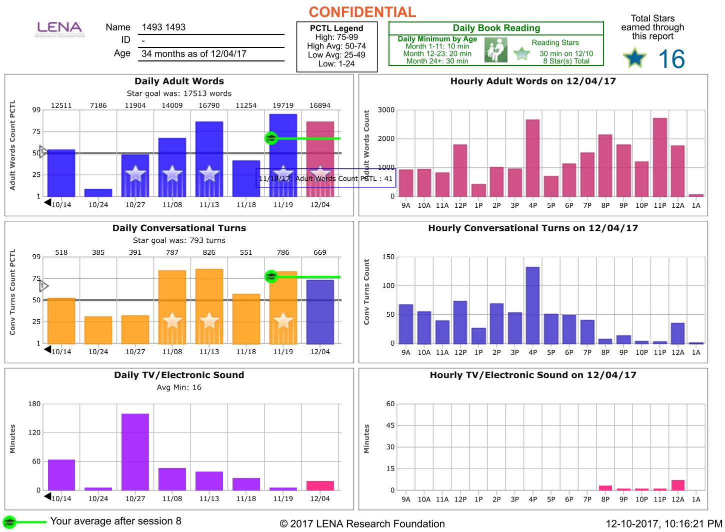


Export data from LENA Pro to mine your LENA data and perform custom in-depth analyses.

Baseline: AWC and CTC



During Intervention: AWC and CTC



Impact of the Intervention on the Language Environment

VARIABLES	(1) Random assignment OLS	(2) Random Assignment Selection	(6) Dummy for Attendance IV
Adult Word Counts	1,216.687 (1,549.894)	1,210.724 (1,346.493)	2,361.115 (2,615.559)
Conversation Turns	116.021* (64.473)	117.170** (57.078)	225.151** (113.421)
Child Vocalizations	444.771 (281.505)	465.820* (259.943)	863.127* (496.136)
AVA (Standardized Score	0.255 (0.155)	0.269* (0.141)	0.466* (0.256)
Observations	91	128	91

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Impact of the Intervention on Parental Beliefs

Table 7

LEME Study: Maternal Beliefs

	(1)	(2)	(3)
VARIABLES	Baseline	Endline	
Random assignment	0.2581	0.3310**	
	(0.1453)	(0.1143)	
Dummy for LENA Start attendance			0.6301***
			(0.2086)
Observations	134	128	128
R-squared	0.0840	0.2156	0.1971

Robust standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

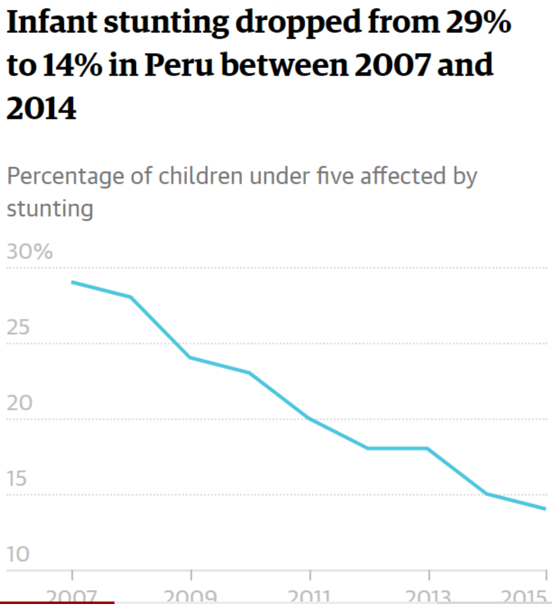
Impact of Beliefs on Language Environment

Table 8
LEME Study
Impact of Maternal Beliefs on Conversation Turn Counts

VARIABLES	(1) First Stage Dependent variable: Maternal Beliefs	(2) Second Stage Dependent Variable: Conversation Turns
Random assignment	0.4331***	
	(0.1234)	
Maternal Beliefs		289.5987**
		(118.4140)
Observations	91	91
R-squared	0.6804	0.4874

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Wang, Puentes, Behrman, and Cunha (2018)



Data

- Nutritional supplementation trial from 1969 until 1977:
  - A high-protein nutritional supplement was delivered in the two treatment villages (Atole)
  - A non-protein supplement was delivered in two control villages (Fresco).
  - Initial Height, Height at Month 24 Protein (and Calorie) intakes every 3 months in first 2 years (24-hour and 72-hour recall)
  - Prices of eggs, chicken, pork, beef, dry beans, corn, and rice.

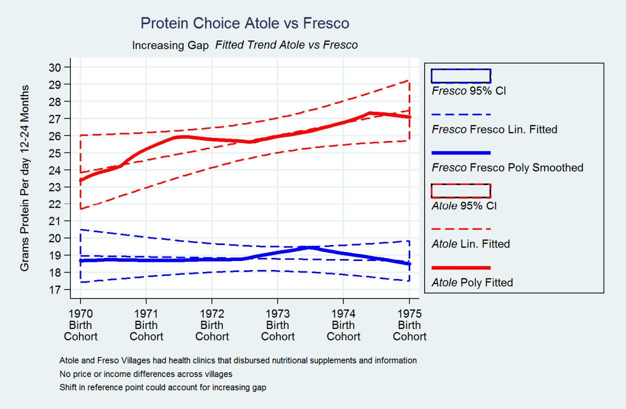
Identification Argument

- Adaptive expectations: Reference points for age two height at year t were determined by the height of children born in year $t - 2$.
- We show this implies two exclusion restrictions:
  - Random assignment to treatment or control: Identifies coefficients on investment in production function.
  - Interaction between random assignment and calendar time: Identifies preference parameter on reference point.

Consumption of Protein in Treatment vs Control Villages

Estimation: Identification III

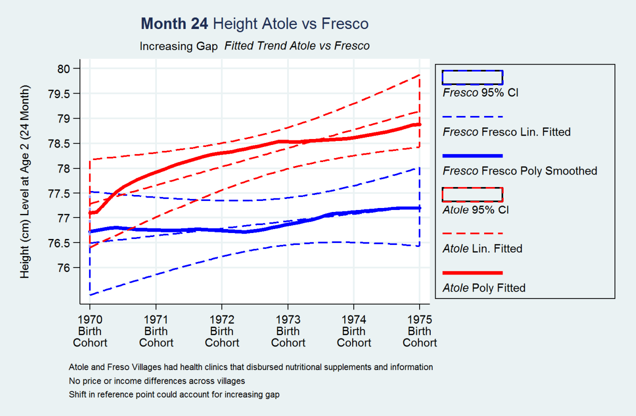
- We find that there is a gap in protein choice up to month 24 between Atole and Fresco villages.
- Income is constant over time, and price is the same across locations.
- Only the increasing reference point gap, through λ, can explain the choice gap's widening.



Age Two Height in Treatment vs. Control Villages

Estimation: Identification II

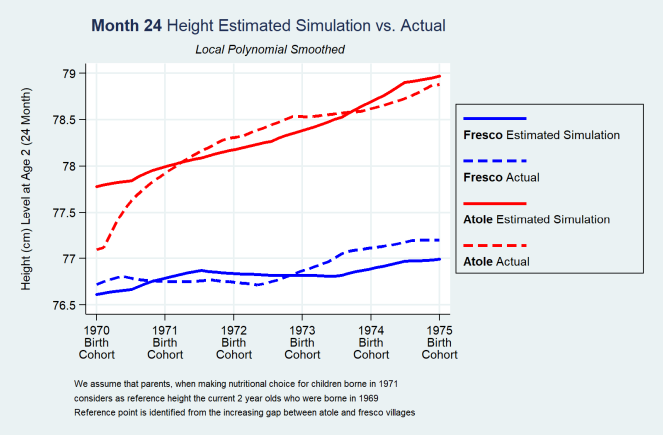
- We find that there is a gap in height at month 24 between Atole and Fresco villages.
- The gap is increasing over time.
- These curves are our $\mu_{R_{yv}}$



Model Fit: Height

Estimation Results: Fit of the Model III

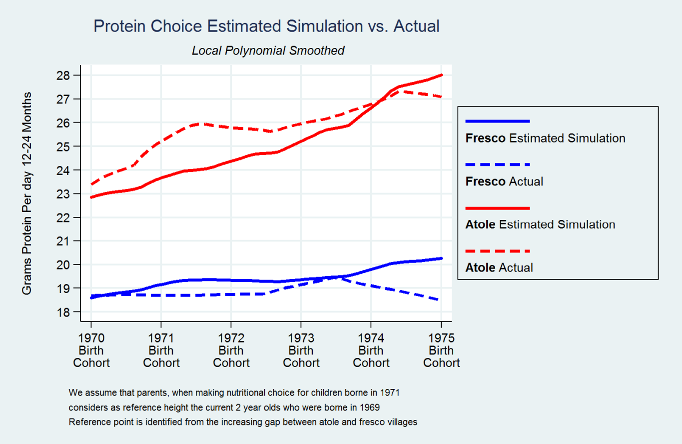
Fit general Height Change pattern.



Model Fit: Protein Consumption

Estimation Results: Fit of the Model IV

Fit Protein Trend over time

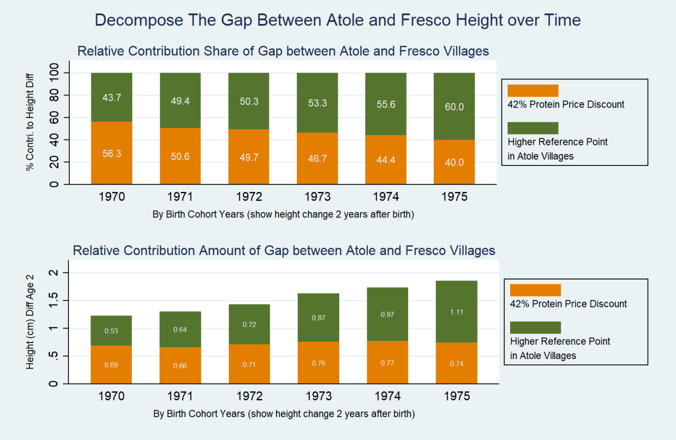


Decomposition: Price Discount vs Reference Point

Counterfactual 1a

For children in fresco villages:

- **Yellow**: Increase in height with just 42% price discount, fixing reference point.
- **Green**: Remaining contribution from reference point change



Conclusion: Skill Formation

- Inequality in socio-economic outcomes is partly caused by inequality in human capital.
- Inequality in human capital is partly caused by inequality in investments in human capital during early childhood, adolescence, and early adulthood.
- Inequality in stocks of human capital has been increasing.
- Inequality in investments in human capital has also been increasing.
- Correlational studies. It is not determinate if we can make causal links from this data.

Conclusion: Skill Formation

- At different stages of the lifecycle, investments produce different dimensions of human capital.
- The skills acquired in one stage of the lifecycle promote the emergence of other skills in later stages (self-productivity).
- The skills acquired in different stages of the lifecycle complement each other (dynamic complementarity, “success begets success”).
- The evidence is built on estimation of technology of skill formation.
- To do so, we showed how we can
  - Address lack of cardinality of measures of human capital.
  - Address measurement error in measures of human capital.
  - Address endogeneity of investments.

Conclusion: Skill Formation

- Inequality in early investments in human capital is partially determined by:
  - Parental beliefs about the technology of skill formation.
  - Parental beliefs about what constitutes "normal" development.
- Intervention that provides information with feedback based on objective information positively affects parental beliefs, investments, and development.
- This is common in the intervention in Philadelphia, but also the intervention in Peru regarding stunting.

Conclusion: Skill Formation

- Lots of work for young researchers:
  - Theory: How to model within family decision making processes? How to model these processes when parents are not
  - Theory: How to model parent-child interaction (child is a "player").
  - Data: How to measure investments? How to measure human capital in cardinal ways?
  - Data: Implement and evaluate pilot programs that can foster human capital formation.
  - Data and Theory: Identify mechanisms to validate or reject theories and to identify new opportunities for interventions.