Static Labor Force Participation

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Static Model of Married Women's Labor Force Participation

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C_a = household consumption

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$$U(0,0) = 0,\ U_C > 0,\ U_{CC} < 0, U(C,1) < U(C,0), \frac{\partial U}{\partial C}\bigg|_{P=0} \lesseqgtr \frac{\partial U}{\partial C}\bigg|_{P=1}$$

Marginal utility from consumption may increase/decrease with labor participation.

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Budget Constraint:

$$C_a = y_a + w_a P_a,$$

Alternative-Specific Utilities:

$$U^1 = U(y + w, 1)$$

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Participation Function:

$$P = P(y, w)$$

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Assume a cross section of married couples with different ages of the wife for whom we have the following information:

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(i) If $y_{aj} = y_{ak}$ and $w_{aj} = w_{ak}$ then $P_{aj} = P_{ak}$

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$$(ii) \text{ If } y_{aj} = y_{ak},\ w_{aj} > w_{ak} \text{ then } P_{aj} \geqslant P_{ak}$$

These may not be true in the data.

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Why might the theory be rejected?

(i) Theory is correct, but w, y, and/or P are incorrectly measured.

(ii) P depends on more than just w and y, e.g., preferences for the wife's leisure differ among households with the same y and w.

(iii) The theory is wrong in a fundamental way – the assumption of static optimization is wrong, households do not optimize.

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Need external validation of the true values of *y*, *w* and *P*.

(ii) Allow for preferences to depend on other household characteristics that might affect the value of the wife’s leisure, e.g., the number of young children in the household. The rejection criterion would condition on these characteristics.

But, there are likely be *unmeasured* characteristics that are related to household preferences.

How can we explicitly account for them?

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Denoting the unobserved preference for leisure by ϵ, write

$$U = U(C, P; \epsilon(1-P))$$

where

$$U_{\epsilon|P=0} > 0$$

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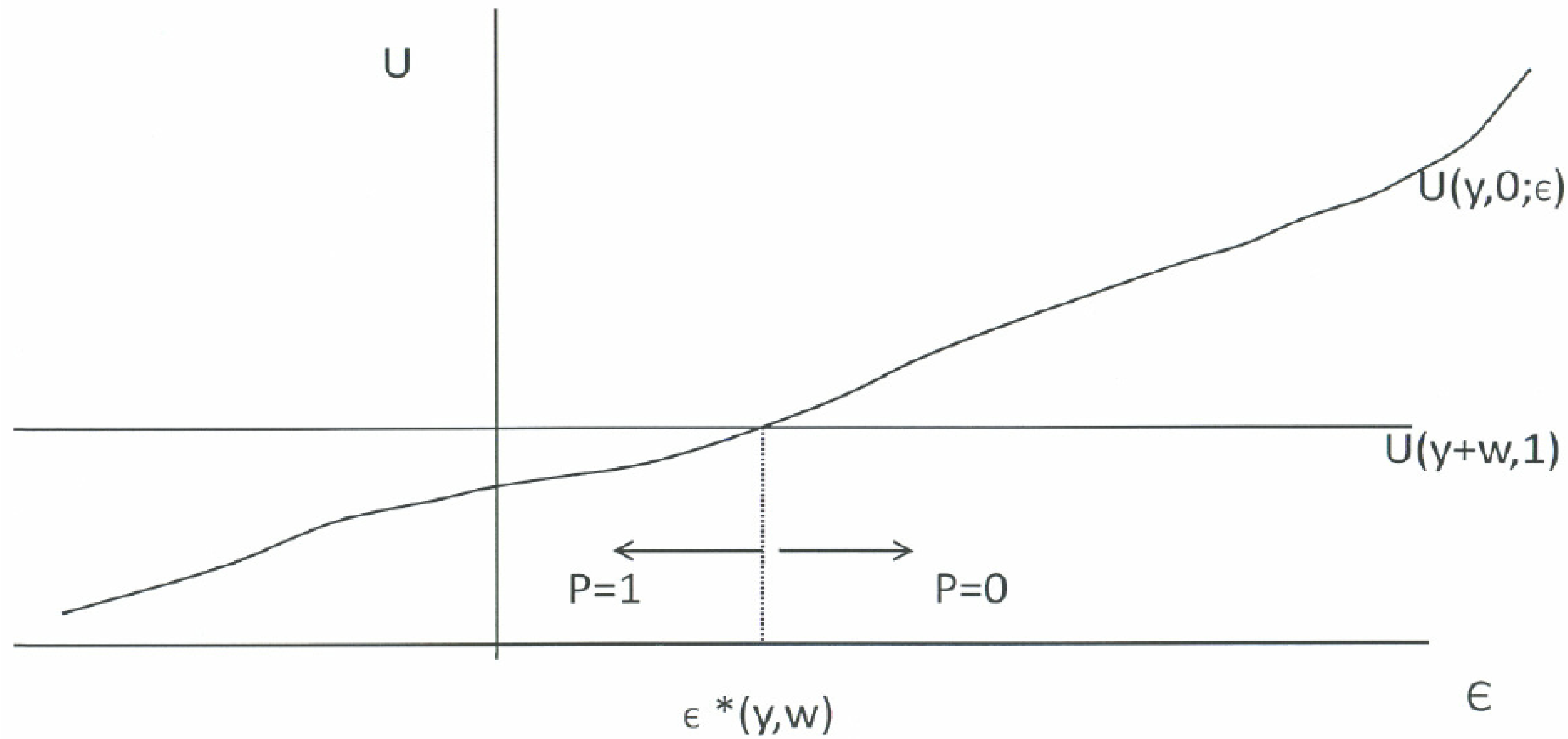
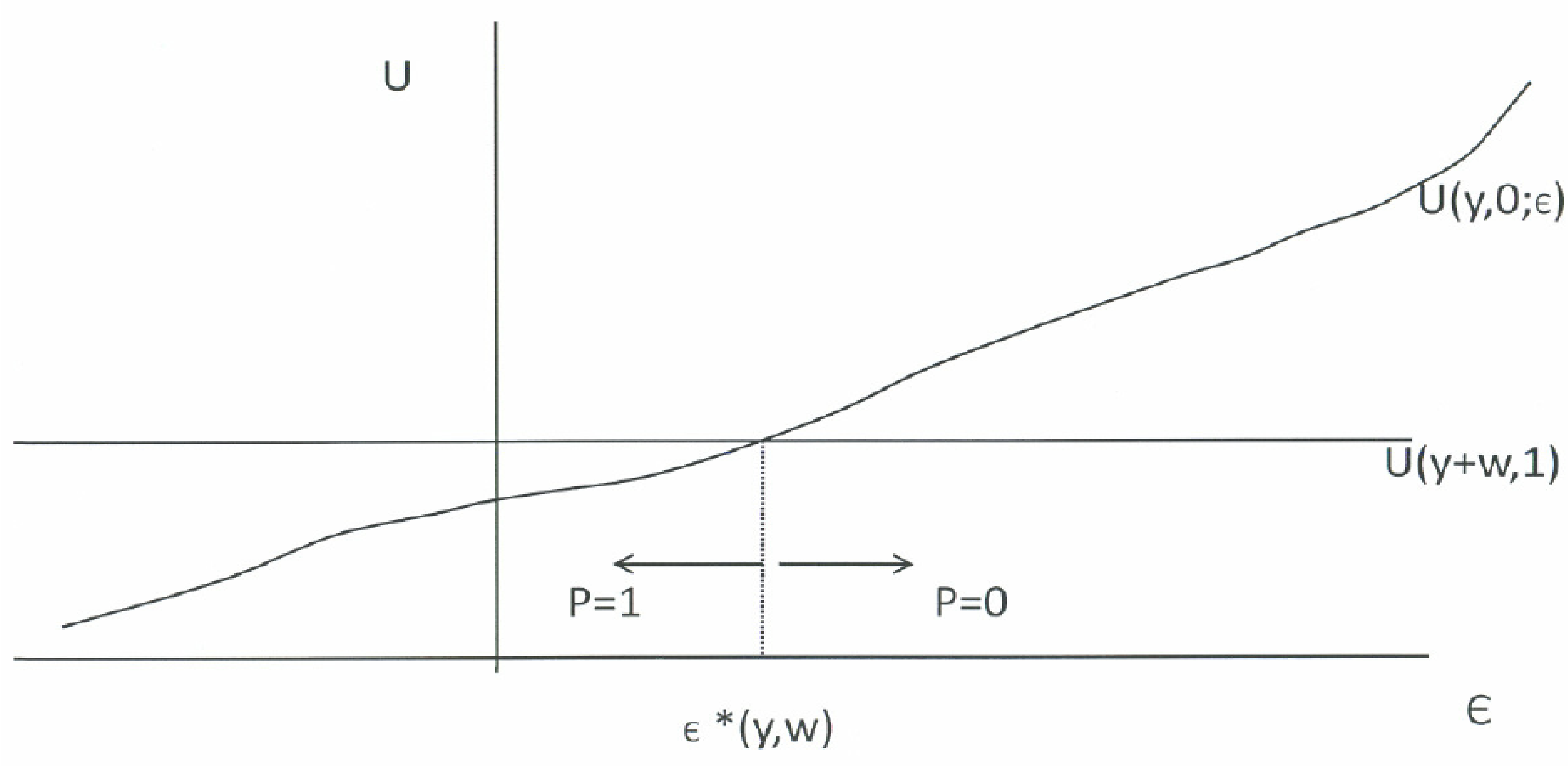
or,

$P = 1$ iff $\epsilon \leq \epsilon^*(y,w)$

$P = 0$ otherwise

where $\epsilon \leq \epsilon^*(y,w)$ is the threshold value of ϵ at which the couple is just indifferent between *P=1* and *P=0*.


U
U(y,0;ϵ)
U(y+w,1)
P=1
P=0
ϵ *(y,w)
ϵ



Properties of the cut-off value, $\epsilon^*(y,w)$:

$$\frac{\partial \epsilon^*(y,w)}{\partial w} > 0 \qquad \frac{\partial \epsilon^*(y,w)}{\partial y} \lesseqgtr 0$$

As y goes up, the value of wife staying at home may go up/down.

Let $\epsilon \sim F_{\epsilon|y,w}$. Then, participation is probabilistic from our perspective, though not from the household's.

$$\Pr(P = 1|y, w) = \int_{-\infty}^{\epsilon^*(y,w)} dF_{\epsilon|y,w}$$

$$= F_{\epsilon|y,w}(\epsilon^*(y, w)) = G(y, w)$$

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Note that *G(y, w)* is a composite of *F* and *U*, both of which depend on *y* and *w*. Thus, comparative static effects of *w* and *y* on the participation probability confound changes in the utility of participation and non-participation and changes in the distribution of unmeasured preferences.

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If we assume that ϵ is distributed independently of w and y, that is, $F_{\epsilon|y,w} = F_\epsilon$

$$\frac{\partial \Pr(P = 1|y,w)}{\partial w} = G_w(y,w) = f_\epsilon(\epsilon^*(y,w))\frac{\partial \epsilon^*}{\partial w} > 0$$

$$\frac{\partial \Pr(P = 1|y,w)}{\partial y} = G_y(y,w) = f_\epsilon(\epsilon^*(y,w))\frac{\partial \epsilon^*}{\partial y} \lesseqgtr 0$$

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The model thus has the testable implications that the *probability* of working is increasing in the wage level. Testing that proposition requires estimation.

Estimation:

The participation function is thus $P(y, w; \epsilon)$. It is determined by the primitives *U* and *F* and the assumption of maximization. The participation probability function, *G(y, w)*, is itself not a primitive.

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Questions:

1. What can we learn about the features of *U*, *F* and *G* given cross sectional data on *P*, *y* and *w*?

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Questions:

1. What can we learn about the features of *U*, *F* and *G* given cross sectional data on *P*, *y* and *w*?

2. How does what we learn depend on *a priori* assumptions?

Approaches to Estimation – A General Taxonomy

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Parametric vs. Non-Parametric:

Parametric approaches impose either a functional form assumption on *U* or *G*, and/or a distributional assumption on *F*.

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Parametric vs. Non-Parametric: *Parametric* approaches impose either a functional form assumption on *U* or *G*, and/or a distributional assumption on *F*. *Non-Parametric* approaches usually restrict the functions to be a member of a broad class, such as the class of continuous and differentiable functions.

Structural vs. Non-Structural:

Structural approaches recover the primitive functions, *U* and *F.*

Non-structural approaches recover *G*.

The following table describes the approaches that we will consider for the labor force participation model.

	nonparametric (NP)	parametric (P)
nonstructural (NS)	√	√
structural (S)	X	√

Note: See Matzkin (1992, Ecma) for the NP-S case.

NP-NS Approach:

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Note that

$$E(P|y,w) = \Pr(P = 1|y,w) \cdot 1 + [1 - \Pr(P = 1|y,w)] \cdot 0$$
$$= \Pr(P = 1|y,w)$$

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$$= \Pr(P = 1|y,w)$$

Thus, $P = E(P|y,w) + u = G(y,w) + u$

where $E(u|y,w) = 0$ by construction (u=P-E(u|y,w)). The G function can be estimated by non-parametric regression.

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The model can be tested by checking whether

$$G_w(w,y) > 0 \text{ for all } w,y$$

P-NS Approach:

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Choose a functional form for G. Examples are:

1. Linear

$$G(w, y) = \alpha_0 + \alpha_1 y + \alpha_2 w$$

which yields the linear probability model

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A test of the model is whether $\alpha_2 > 0$.

P-NS Approach

2. a. Probit

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where Λ is the logistic function $\exp(x)/(1+\exp(x))$.

Denoting the parameters of the G function by Θ^G, the likelihood function is

$$L(\Theta^G; \text{data}) = \Pi_{i=1}^{I} G(y_i, w_i)^{P_i}[1 - G(y_i, w_i)]^{1-P_i}$$

P-S Approach:

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Denoting the parameters of the utility function by Θ^U and those of the distribution function by Θ^F,

$$L(\Theta^U, \Theta^F; \text{ data})$$

$$= \Pi_{i=1}^{I} F_\epsilon(\epsilon^*(y, w))^{P_i}[1 - F_\epsilon(\epsilon^*(y, w))]^{1-P_i}$$

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We will consider three alternatives.

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2. Assume a distribution for *F* and leave *U* free.

3. Assume a functional form for both *U* and *F*.

P-S Approach

Cases 1 and 3: As an illustration, assume the utility function to be

$$U(C,P;\epsilon) = C + \alpha_1(1-P) + \alpha_2(1-P)C + \epsilon(1-P)$$

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Threshold value for $\epsilon,\ \epsilon^*(y,w)$:

$$\epsilon^*(y,w) = w - \alpha_2 y - \alpha_1$$

P-S Approach

Case 1:

L(;data) $= \Pi_{i=1}^{I} G(y_i, w_i)^{P_i}[1 - G(y_i, w_i)]^{1-P_i}$

where

$G(y_i, w_i)$= $F_{\epsilon}(w - \alpha_2 y - \alpha_1)$

P-S Approach

Case 3: Assume further that the preference for leisure is normally distributed $(\epsilon \sim N(0, \sigma_\epsilon^2))$. Then,

$$G(y, w) = \Phi\left(\frac{w - \alpha_2 y - \alpha_1}{\sigma_\epsilon}\right)$$

where Φ is the standard cumulative normal.

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A test of the model comes down to the condition that

$$\frac{\partial \Pr(P = 1 | w, y)}{\partial w} = \sigma_\epsilon^{-1} \phi\left(\frac{w - \alpha_2 y - \alpha_1}{\sigma_\epsilon}\right) > 0 \text{ for all } y, w.$$

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This condition is equivalent to a test that the coefficient on the wage, σ_ϵ^{-1}, is positive.

P-S Approach

Case 2: Specify a distribution for ϵ and non-parametrically estimate $\epsilon^*(y, w)$.

For example, if $\epsilon \sim N(0, \sigma_\epsilon^2)$, it is possible to identify $\epsilon^*(y, w)$ up to scale, that is, up to a normalization of σ_ϵ.

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For example, if $\epsilon \sim N(0, \sigma_\epsilon^2)$, it is possible to identify $\epsilon^*(y,w)$ up to scale, that is, up to a normalization of σ_ϵ.

But, non-parametric estimation $\epsilon^*(y,w)$ does not non-parametrically identify *U.*

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2. perform policy evaluation

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Conditional on achieving the goal of estimation, the fewer extra-theoretic assumptions the better. Are there tradeoffs between achieving a goal and reliance on extra-theoretic assumptions?

Ex-Ante Policy Evaluation

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Is it possible within a non-parametric framework?

Consider, for example, the imposition of a wage tax, where none had existed before and suppose we non-parametrically estimate $G(y, w)$

NP-NS: Ex-Ante Policy Evaluation

A proportional wage tax changes the budget constraint:

$$C = y + (1 - \tau)wP$$
$$= y + w'P$$

What is the participation probability under the new tax regime?

NP-NS: Ex-Ante Policy Evaluation

A proportional wage tax changes the budget constraint:

$$
\begin{aligned}
C &= y + (1-\tau)wP \\
&= y + w'P
\end{aligned}
$$

The participation probability with the tax is $G(y, w')$.

NP-NS: Ex-Ante Policy Evaluation

A proportional wage tax changes the budget constraint:

$$C = y + (1-\tau)wP$$
$$= y + w'P$$

The participation probability with the tax is $G(y, w')$.

Letting $h(w,y)$ be the sample density of w, y, the effect of the tax on the participation rate in the sample is

$$\int_{S_w}\int_{S_y}[G(w',y) - G(w,y)]h(w,y)dwdy$$

where S_w and S_y denote the support of w and y.

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If, for example, the supports are not the same, then it will be necessary to extrapolate outside the range of support observed in the sample using a P-NS or P-S approach.

NP-NS: Ex-Ante Policy Evaluation

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If, for example, the supports are not the same, then it will be necessary to extrapolate outside the range of support observed in the sample using a P-NS or P-S approach.

Even if the supports are the same, there will always be values of *w* for which there are no observation with a wage of w'. For example, the minimum wage in the population has no analog in w' (The new minimum wage is lower than the minimum wage observable in the old regime).

P-S: Ex-Ante Policy Evaluation

P-S: The effect of the wage tax in the P-S approach is

$$\Phi\left(\frac{(1-\tau)w - \alpha_2 y - \alpha_1}{\sigma_\epsilon}\right) - \Phi\left(\frac{w - \alpha_2 y - \alpha_1}{\sigma_\epsilon}\right)$$

Identification of σ_ϵ is critical for performing the policy evaluation.

Ex-Ante Policy Evaluation

If one has sufficient data, non-parametric estimation of the policy effect is preferred.

The P-S approach provides a more precise estimate, but may not fit the observations well.

Model vs Data: An Example

To fix ideas, we begin with the static labor force participation model.

$$U_a(C_a, P_a; x_a) = C_a + \alpha_1(1 - P_a) + \alpha_2 C_a(1 - P_a) + \alpha_3(1 - P_a)n_a$$

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C: consumption,

P: participation

n: number of young children

Let c be the cost of childcare if the woman works.

Let y be husband's income.

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s: years of schooling,

a: age

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Household's state space: $\Omega_a = \{ y_{a,} n_{a,} s, a, \xi_a \}$

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Household's state space:

$$\Omega_a = \{y_a, n_a, s, a, \xi_a\}$$

State space observable to the researcher:

$$\Omega_a^- = \{y_a, n_a, s, a\}$$

Alternative-Specific Utilities:

$$U^1 = y_a - cn_a + \exp(\gamma_0 + \gamma_1 s + \gamma_2 a - \gamma_3 a^2 + \xi_a),$$

$$U^0 = y_a + \alpha_1 + \alpha_2 y_a + \alpha_3 n_a.$$

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Decision Rule:

$$P_a = 1 \text{ if } \xi_a \geq \log(\alpha_1 + \alpha_2 y_a + (\alpha_3 + c)n_a) - \gamma_0 - \gamma_1 s - \gamma_2 a + \gamma_3 a^2 = \xi_a^*(\Omega_a^-)$$

$$= 0, \text{ otherwise.}$$

Alternative-Specific Utilities:

$$U^1 = y_a - cn_a + \exp(\gamma_0 + \gamma_1 s + \gamma_2 a - \gamma_3 a^2 + \xi_a),$$

$$U^0 = y_a + \alpha_1 + \alpha_2 y_a + \alpha_3 n_a.$$

Decision Rule:

$$P_a = 1 \text{ if } \xi_a \geq \log(\alpha_1 + \alpha_2 y_a + (\alpha_3 + c)n_a) - \gamma_0 - \gamma_1 s - \gamma_2 a + \gamma_3 a^2 = \xi_a^*(\Omega_a^-)$$

$$= 0, \text{ otherwise.}$$

The woman works if, given her age, education, husband's income and number of children, her ξ_a exceeds the critical value $\xi_a^*(\Omega_a^-)$.

Data: Cross-section of $i = 1, \ldots, I$ married women

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Likelihood function:

$$L(\Theta; data) = \Pi_{i=1}^{I} [\Pr(P_{ia} = 1, w_{ia} | \Omega_{ia}^{-})]^{P_{ia}} [\Pr(P_{ia} = 0) | \Omega_{ia}^{-})]^{1-P_{ia}}$$

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$$\Pr(P_{ia} = 1, w_{ia} | \Omega_{ia}^{-}) = \Pr(P_{ia} = 1 | w_{ia}, \Omega_{ia}^{-}) \Pr(w_{ia} | \Omega_{ia}^{-}).$$

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$$\Pr(w_{ia} > \alpha_1 + \alpha_2 y_{ia} + (\alpha_3 + c)n_{ia}|w_{ia}, \Omega_{ia}^{-})$$

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What values can this probability take?

Only 0 or 1. Why?

Either the condition inside the probability statement holds or it does not hold.

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Thus, the parameters must satisfy the set of restrictions that

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In particular, it must be satisfied for the person with the lowest observed wage. Clearly that observation will have an extreme effect on the parameter estimates.

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   For example, we can assume that α_1 differs across people, i.e., replace α_1 with $\alpha_{1i} = \alpha_1 + \epsilon_i$

   What complication does this raise?

   We lose the analytical solution for the cutoff value.

$$\xi_a - \log(\alpha_1 + \epsilon_i + \alpha_2 y_a + (\alpha_3 + c) n_a) - \gamma_0 - \gamma_1 s - \gamma_2 a + \gamma_3 a^2 \lesseqgtr 0$$

2. Assume that the wage data are measured with error, for example, that the observed (reported) wage measures the true wage with a proportionate error,

$$\log w_{ia}^{o} = \log w_{ia} + \eta_{ia},$$

where

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What advantage does this have over the previous method?

The decision rule is unchanged, so that the cutoff value has an analytical form. The parameter restriction that we had before only holds with respect to the true wage, not with respect to reported wages.

How does the likelihood function change?

$$\Pr(P_{ia} = 1, w_{ia}^{o} | \Omega_{ia}^{-})$$

$$= (w_{ia}^{o})^{-1} \Pr(\xi_{ia} \geq \xi_{ia}^{*}(\Omega_{ia}^{-}), u_{ia} = \log w_{ia}^{o} - (\gamma_0 + \gamma_1 s_i + \gamma_2 a_i - \gamma_3 a_i^2))$$

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Given normality, this becomes

$$\Pr(P_{ia} = 1, w^o_{ia} | \Omega^-_{ia}) = (1 - \Phi(\frac{\xi^*_{ia}(\Omega^-_{ia}) - \rho \frac{\sigma_\xi}{\sigma_u} u_{ia}}{\sigma_\xi \sqrt{1 - \rho^2}}) \frac{1}{\sigma_u} \phi(\frac{u_{ia}}{\sigma_{u_{ia}}}),$$

4

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The component of the likelihood function for non-workers is

$$\Pr(P_{a} = 0) = \Phi(\xi_{ia}^{*}/\sigma_{\xi})$$