The "Effect" of Children on Female Labor Supply

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Empirical studies have found that fertility, especially the number of young children in the household, is an important "determinant" of female labor supply.

From early on, however, researchers have expressed concern that, because fertility is subject to control, and is thus a choice, it should not be treated as exogenous with respect to labor supply decisions.

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This concern led to a large literature on estimating labor supply functions in the presence of endogenous fertility choice.

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This led researchers to search for variables that affect fertility, but do not affect labor supply. The original aim of consistently estimating the labor supply function was augmented to include identifying the "effect" of fertility on labor supply.

Is the analogy to a simultaneous equations framework valid?

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No!

To understand the issues involved, it is useful, **as always**, to begin with a behavioral model.

Consider a lifetime model in which the household decides on the supply of labor of the wife (say, total hours of work over the lifetime, h) and on the number of children to bear, n.

Specifically the household is assumed to maximize the utility function

$$U = U(C, T - h, n; \epsilon)$$

subject to

$$C = y + wh - p_n n,$$

where *y* is the husband's lifetime earnings, *w* is the wife's lifetime wage, p_n is the price of bearing and rearing a child and ϵ is a vector of unobserved preference shifters that affect the marginal utilities of leisure and children, (ϵ_l, ϵ_n), assumed to satisfy $E(\epsilon | y, w, p_n) = 0)$.

The wife is assumed to work positive hours over the lifetime and to bear a positive number of children (also treated as continuous), so that the solution is interior.

The maximization problem yields the Marshallian demand functions. Notice that the determinants of *h* and *n* are exactly the same.

$$h = h(y, w, p_n; \epsilon_l, \epsilon_n),$$

$$n = n(y, w, p_n; \epsilon_l, \epsilon_n).$$

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$$h = h(y, w, p_n; \epsilon_l, \epsilon_n),$$

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There is therefore no variable within the model that can affect fertility that does not also affect labor supply.

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$$h = h(y, w, p_n; \epsilon_l, \epsilon_n),$$

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To formulate such a regression, consider solving the fertility demand function for one of its arguments, say p_n,

$$p_n = n_3^{-1}(n, y, w; \epsilon_l, \epsilon_n)$$

and substituting this function into the labor supply function, giving

$$h = h(y, w, n_3^{-1}(n, y, w; \epsilon_l, \epsilon_n); \epsilon_l, \epsilon_n)$$
$$= h^*(y, w, n; \epsilon_l, \epsilon_n).$$

Fertility, n, takes the place of p_n in the hours function. An increase in fertility will induce a couple with given y and w to alter the wife's labor supply according to

$$\partial h^*/\partial n = (\partial h/\partial p_n)(\partial n/\partial p_n)^{-1},$$

the ratio of the uncompensated (p_n) price effect on h to its effect on n.

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One could alternatively solve the fertility demand equation for either y or w. In those cases:

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In the context of this procedure, then, there is no single interpretation of the "effect" of fertility on labor supply. The interpretation of the relationship between fertility and hours worked depends on what else is included in the hours function.

It is convenient to work with linearized versions of the demand functions. Specifically,

$$h = \alpha_0 + \alpha_1 y + \alpha_2 w + \alpha_3 p_n + \gamma_1 \epsilon_l + \gamma_2 \epsilon_n,$$

$$n = \beta_0 + \beta_1 y + \beta_2 w + \beta_3 p_n + \theta_1 \epsilon_l + \theta_2 \epsilon_n.$$

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Upon solving for p_n and substituting the resulting equation is

$$h = (\alpha_0 - \frac{\alpha_3}{\beta_3}\beta_0) + (\alpha_1 - \frac{\alpha_3}{\beta_3}\beta_1)y + (\alpha_2 - \frac{\alpha_3}{\beta_3}\beta_2)w + \frac{\alpha_3}{\beta_3}n +$$
$$(\gamma_1 - \frac{\alpha_3}{\beta_3}\theta_1)\epsilon_l + (\gamma_2 - \frac{\alpha_3}{\beta_3}\theta_2)\epsilon_n$$

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$$= \pi_0 + \pi_1 y + \pi_2 w + \pi_3 n + \pi_4 \epsilon_l + \pi_5 \epsilon_n.$$

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The wage coefficient, π_2, is not the uncompensated wage effect on hours, α_2, nor is the income coefficient, π_1, the Marshallian income effect, α_1. And, the fertility coefficient, π_3 is the ratio of uncompensated (p_n) price effects.

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The reason is that fertility, *n*, depends on the unobserved preference shifters ϵ_l and ϵ_n.

To see what the bias depends on, consider π_3. To simplify the expression, suppose that $\pi_1 = \pi_2 = 0$. In that case, the ols estimator is

$$\widehat{\pi_3} = \pi_3 + \pi_4\theta_1 \frac{\sigma^2_{\epsilon_l}}{\sigma^2_n} + \pi_5\theta_2 \frac{\sigma^2_{\epsilon_n}}{\sigma^2_n}.$$

The bias can be of either sign, depending on the signs of $\pi_4\theta_1$ and $\pi_5\theta_2$, and their respective weights, the extent to which the variance in fertility is accounted for by the variance in the leisure and fertility unobserved preference shifters.

Are the other π 's biased as well?

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In general, all of the estimates will be biased because of the endogeneity of fertility.

$$h = (\alpha_0 - \frac{\alpha_3}{\beta_3}\beta_0) + (\alpha_1 - \frac{\alpha_3}{\beta_3}\beta_1)y + (\alpha_2 - \frac{\alpha_3}{\beta_3}\beta_2)w + \frac{\alpha_3}{\beta_3}n +$$
$$(\gamma_1 - \frac{\alpha_3}{\beta_3}\theta_1)\epsilon_l + (\gamma_2 - \frac{\alpha_3}{\beta_3}\theta_2)\epsilon_n$$
$$= \pi_0 + \pi_1 y + \pi_2 w + \pi_3 n + \pi_4 \epsilon_l + \pi_5 \epsilon_n.$$

To see that, consider the thought experiment of varying *y* holding *n and w* constant. As long as β_1 is not zero, *n* would be constant in the sample of couples with different *y* both because those couples vary in p_n and because they vary in ϵ_l and ϵ_n. Thus, conditional on *n*, *y* would be correlated with the unobservables. The same is true of *w*.

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These biases might lead a researcher to think about applying two-stage least squares.

In that case, the price, p_n, would serve as the identifying variation (IV). Such a two-stage least squares procedure would provide consistent estimates of the π 's . Alternatively, if the researcher had substituted *n* for *y* or for *w*, a different set of π 's would be consistently estimated based on linearized versions.

Why would a researcher, with data on y, w, and p_n, not simply estimate the Marshallian labor supply function?

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Estimating it recovers both the income effect, the uncompensated wage effect (and thus the compensated wage effect) as well as the response of labor supply to a change in the price of bearing and rearing children (all the alphas).

Why would a researcher, with data on y, w, and p_n, not simply estimate the Marshallian labor supply function?

THERE IS NO REASON.

Suppose, contrary to the above assumption, that the researcher has data on y and w, but not on p_n.

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The estimates of the income and wage effects will be biased if y and/or w are correlated with p_n . The extent of the bias depends on these intercorrelations and on the importance of p_n in determining hours of work.

It is not possible, a priori, to tell which of the biases, from omitting p_n or from simply including n, is larger.

Of course, two-stage least squares is unavailable because p_n, being unobserved, cannot be used as an identifying exclusion restriction.

What is to be made of the notion of identifying the "effect" of fertility on labor supply?

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One way to give that question content, in the context of the lifetime decision-making we are considering, is to imagine that one could experimentally randomly assign couples different numbers of children at the beginning of their decision-making lives.

Random assignment would ensure that the number of children is independent of all of the determinants of labor supply, that is, of y, w, p_n, ϵ_l and ϵ_n.

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We could (nonparametrically or parametrically) estimate the function

$$h_i = h(\overline{n_i}) + u_i,$$

where $\overline{n_i}$ represents the fixed number of children assigned to couple *i*.

Randomization ensures that $\overline{n_i}$ is independent of u_i .

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$$U = U(C, l, \overline{n}; \epsilon)$$

subject to

$$C = y + wh - p_n \overline{n}.$$

$y, w, p_n,$ and ϵ are independent of *n* by the randomization.

This maximization problem has been considered in another context, that of rationioning, for example, the rationing of commodities in war time.

To interpret the effect of changing (the rationed) number of children on labor supply, $dh_i/d\overline{n_i}$, it is useful to define the restricted expenditure function:

$$e^r(U, w, p_n, \overline{n}) = p_n\overline{n} + \min_l [C + wl; U(C, l, \overline{n}) \geq U].$$

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The unrestricted expenditure function cannot be greater than the restricted expenditure function and will be equal to it when the rationed amount of children is equal to the amount the couple would have chosen anyway.

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Demand functions that contain rationed goods are also called *conditional demand functions*.

$$\frac{\partial l}{\partial \overline{n}}\Big|_U = \frac{\partial l}{\partial p_n}\Big|_U \div \frac{\partial n}{\partial p_n}\Big|_U,$$

The effect of increasing the (rationed) number of children on leisure hours (utility constant) is equal to the ratio of the cross compensated substitution effect on leisure to the own compensated substitution effect on children.

Empirical Application:

What variation in observed fertility can replicate rationing?

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Because women with more children are also more likely to experience a twin birth, simply because they had more draws, RW compare the labor supply behavior of women who experienced a twin birth at their first birth to women who did not have a twin birth at their first birth.

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Given its random occurrence, having a twin on the first birth is thus like the social experiment we would like to conduct.

Because the experiment is given by nature, the use of twins exemplifies what has been called the *natural experiment approach*.

Given the available data, RW measure labor supply by whether a woman is currently participating in the labor force.

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A model that is consistent with that measure being an interior solution to the labor supply problem is one in which the couple's utility depends on the fraction of periods over the lifetime, weeks in the case of the data used by RW, that the woman works. One can consider that fraction to measure the continuous variable h/T in the lifetime model.

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As long as all women in the sample work at least one week over their lifetimes, the effect of having a twin at the first birth can be interpreted as the ratio of compensated price effects.

RW report estimates of the effect of having a twin at the first birth on the number of children ever born and on participation in ten year age intervals (15-24, 25-34 and 35-44) for two groups of women, those having their first birth under the age of 25 and those under the age of 35.

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Women who had their first birth under the age of 25 have *almost exactly one additional child* in the 15-24 age interval relative to women who did not have twins

The corresponding labor force participation rate of those women with twins is .371 percentage points *less*.

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And, perhaps surprisingly, ten years beyond that, their labor force participation is .142 percentage points *more*.

What possible explanations might there be for this pattern?

1. Leisure is highly substitutable over the life cycle.

    Having taken more leisure at earlier ages because of the birth of twins, less leisure is taken later.

2. Children and leisure are complements when children are young.

At ages 25-34, the differential in fertility between those women having twins at the first birth and those not having twins falls from one child to .654 children and by the time the women reach ages 35-44, the differential is only .150 children.

2. Children and leisure are complements when children are young.

At ages 25-34, the differential in fertility between those women having twins at the first birth and those not having twins falls from one child to .654 children and by the time the women reach ages 35-44, the differential is only .150 children.

Thus, the ultimately greater labor force participation of twins mothers at ages 35-44 may, in part, be due to their exercise of fertility control at later ages rather than to the substitutability of leisure across time.

Impact of Twins-First Birth on Life-Cycle Fertility: Synthetic Cohorts from the Pooled Sample

Age at First Birth	Age Group 15–24	25–34	35–44
CEB:			
Under 25	1.055	.654	.150
	(.224)	(.299)	(.428)
Under 35	. . .	.631	.312
		(.259)	(.312)

TABLE 3

IMPACT OF TWINS-FIRST BIRTH ON LIFE-CYCLE LABOR FORCE PARTICIPATION, SYNTHETIC COHORTS FROM THE POOLED SAMPLE: MAXIMUM LIKELIHOOD TRANSFORMED LOGIT COEFFICIENTS

	Age Group		
Age at First Birth	15–24	25–34	35–44
Under 25	−.371	−.102	.142
	(.212)	(.105)	(.102)
Under 35	...	−.096	.060
		(.096)	(.078)