Optimal Parenting Styles: Evidence from a Dynamic Game with Multiple Equilibria

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- It is controversial if leaving discretion to children is a better approach to parenting than setting strict limits
- It has been recently been suggested that the "Tiger" parenting model, as opposed to "Western" parenting, is the main source of academic success of Asian children with respect to their peers
- Addressing this debate has potentially important implications for public policies that ease parents' monitoring cost by restricting children's recreational activities

- The strategic interaction between parents and children has received limited attention in economics
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- The purpose of this paper is to address the "parenting-debate" by analyzing the development of adolescent skills as an equilibrium outcome of the *repeated* interaction between parents and children
- I extended my previous work by estimating a "fully" dynamic model whose equilibria hold under standard conditions adopted in the principal-agent model literature and abstracting from *permanent* asymmetric information
- Results indicate that the dynamic aspect is important to understand the quantitative impact of alternative parenting strategies

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Parenting in the Data

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Table: Curfew Limit by Age

	12-13	13-14	14-15
Parents	67.04	54.79	46.31
Jointly/Child	32.96	45.21	53.69
N	1341	1305	1274

1984 cohort

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Parents	22.09	11.67	9.08
Jointly/Child	77.91	88.33	90.92
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Table: TV Limit by Age

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Parents	35.94	19.23	14.01
Jointly/Child	64.06	80.77	85.99
N	1341	1305	1278

1984 cohort

Table: Curfew by Race

	Black	Hispanic	White
Parents	75.76	67.45	64.82
Jointly/Child	24.24	32.55	35.18
N	435	381	901

1984 cohort, age:12-13

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Statistics on Parental Limits

Table: Curfew by Race

	Black	Hispanic	White
Parents	75.76	67.45	64.82
Jointly/Child	24.24	32.55	35.18
N	435	381	901

1984 cohort, age:12-13

Table: Friends Limit by Race

	Black	Hispanic	White
Parents	34.59	28.16	17.44
Jointly/Child	65.41	71.84	82.56
N	425	380	900

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Statistics on Parental Limits

Table: Average PIAT and Limits

	Limits on TV	Limits on Curfew	Limits on Friends
Jointly/Child	73.66	75.66	74.48
Parents	70.49	70.87	66
T-test	0.02	0	0

all cohorts, pooled data

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all cohorts, pooled data

Table: Average GPA and Limits

	Limits on TV	Limits on Curfew	Limits on Friends
Jointly/Child	280.1	287.79	282.23
Parents	293.6	271.38	271.14
T-test	0.35	0.04	0.42

all cohorts, pooled data

Table: Average CAT-ASVAB and Limits

	Limits on TV	Limits on Curfew	Limits on Friends
Jointly/Child	49059	52659	51592
Parents	47115	44831	32687
T-test	0.4	0	0

all cohorts, pooled data

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Statistics on Limits

Table: Limits and Race

	Age 12-13				Age 13-14				Age 14-15			
	Curfew											
	Parents	Child	Jointly	N	Parents	Child	Jointly	N	Parents	Child	Jointly	N
Hispanic	67.45	3.94	28.61	381	60.06	5.38	34.56	353	52.19	5.83	41.98	343
Black	75.52	3	21.48	433	63.06	7.65	29.29	379	56.87	5.49	37.64	364
White	64.55	2.37	33.08	928	51.76	4.42	43.38	883	39.51	9.5	50.98	863
	Friends' Limits											
Hispanic	28.16	43.36	28.16	380	13.31	55.81	30.88	353	10.17	62.5	27.33	344
Black	34.87	36.49	28.64	433	17.32	52.23	30.45	381	14.25	56.99	28.77	365
White	17.35	51.08	31.57	928	10.24	62.8	26.96	879	7.86	68.21	23.93	865
	TV Limits											
Hispanic	35.96	35.17	28.87	381	18.64	48.59	32.77	354	14.53	55.81	29.65	344
Black	35.33	35.33	29.33	433	20.63	52.65	26.72	378	10.68	62.74	26.58	365
White	37.5	27.8	34.7	928	20.39	50.4	29.22	883	15.95	60.12	23.93	865

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Information structure and order of moves

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- The conflict b/w players stems from a mismatch in preferences, i.e. there are no behavioral differences which can advocated to justify parental intervention

The order of moves on the stage game is as follows:

- Conditional on the stock of human capital, K_t and the beliefs about parents choose a parenting style $\rho_t \in \mathbb{R}^n$

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- Child's new human capital K_{t+1}, becomes public

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- Child's new human capital K_{t+1}, becomes public
- The stage game is repeated

Primitives

Preferences

- Child cares about leisure and his adult human capital

$$u_t = \begin{cases} u(l_t) & \text{if } t = 1, 2, \ldots, T \\ \Xi(K_{T+1}) & \text{when the game is over} \end{cases}$$

  with u and Ξ increasing
- Parent cares suffers from the monitoring cost and cares about the child's adult human capital

$$w_t = \begin{cases} -c(\rho_t) & \text{if } t = 1, 2, \ldots, T \\ \Pi(K_{T+1}) & \text{when the game is over} \end{cases}$$

  with c and Π increasing

Primitives

The Evolution of Skills

- Cognitive skills evolve stochastically according to a distribution $F(K_{t+1}|a_t, K_t)$ such that
  - $K_{t+1} \in [\underline{k}, \overline{k}]$ for any K_t and a_t
  - $F(K_{t+1}|a'', K_t)$ FOSD $F(K_{t+1}|a', K_t)$ for any $a'' > a'$, K_t
  - $F(K_{t+1}|a_t, K'')$ FOSD $F(K_{t+1}|a_t, K')$ for any $K'' > K'$, a_t
- I capture evolution in noncognitive skills through the changes in the discount factor $\delta(K_t)$. Endogenous formation of time preferences. A possible parametrization is:

$$\delta(K_t) = \frac{\exp(K_t)}{1 + \exp(K_t)}$$

- The model captures:
  - The cross and self-productivity of skills: $\uparrow \delta(K_t) \Rightarrow \uparrow a_t \Rightarrow \uparrow K_{t+1} \Rightarrow \delta(K_{t+1})$

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- $\tau \in [0, 1]$ is a random variable with conditional distribution $G(\tau|\rho)$
- Let $R = (\rho_1, \ldots, \rho_N)$ be ordered according to FOSD order, i.e.

$$\rho'' > \rho' \Leftrightarrow G(\tau|\rho'') \geq G(\tau|\rho')$$

  e.g. ρ'' is stricter than ρ' if and only if $\rho'' > \rho'$
- A parenting style can be interpreted as a set of rules the parent imposes on the child's recreational activities

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- A parenting style can be interpreted as a set of rules the parent imposes on the child's recreational activities
- The stochastic nature of the map between ρ and τ reflects the existence of an imperfect monitoring problem
- Higher the variance of τ conditional on ρ more pervasive the moral hazard problem

Strategies and Equilibria

- The parent's history is $H^{t,p} = (K^t, a^{t-1}, \rho^{t-1}, \tau^t)$
- The child's history is $H^{t,c} = (K^t, a^{t-1}, \rho^t, \tau^t)$ where $x^t = (x_1, \ldots, x_t)$
- The parent's strategy is a map $\phi^p : H^{t,p} \to R$
- The child's strategy is a map $\phi^c : H^{t,c} \to [0, 1]$
- An optimal strategy profile for the child, $\vec{\phi}^c = \langle \phi_t^c(H^t) \rangle_{t=1}^T$, is given by a sequence of strategies such that:

$$\vec{\phi}^c \in argmax \;\; E\Big[\sum_{t=1}^{T} v(\phi_t^c(H^{t,c}))|\vec{\phi}^p\Big] \tag{1}$$

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$$\vec{\phi}^p \in argmax \;\; E\Big[\sum_{t=1}^{T} w(\phi_t^p(H^{t,c}))|\vec{\phi}^c\Big] \tag{2}$$

Recursive Representation

- I focus on Markov Strategies in which $H^{t,c} = (K_t, \tau_t)$, $H^{t,p} = K_t$
- The problem solved by the child is

$$V(K) = \left\langle \max_{a \geq \tau} \left[u(1-a) + \delta(K) \int_{\underline{k}}^{\overline{k}} V'(K') dF(K'|K, a) \right] \right\rangle$$

- $V' = \Xi$ if $t = T$
- V' incorporates the parent's strategy at $t+1$ if $t \leq T-1$

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- Let $a(\tau, K)$ denote the child's best response function
- The parent takes it as given and solves

$$W(k) = \left\langle \max_{\rho} \left[-c(\rho) + \beta \int_0^1 \int_k W'(K') dF(K'|K, a(\tau, K)) dG(\tau|\rho) \right] \right\rangle$$

- Let $\rho(K)$ denote the best response correspondence of the parent

Monotone Comparative Statics

- Let $f : X \times \Theta \to \mathbb{R}$ with $X \subseteq \mathbb{R}^n$, and $\Theta \subseteq \mathbb{R}^m$.
  - We say that f has the single crossing property in (x, θ), if and only if for any $x'' > x'$, $\theta'' > \theta'$

$$f(x'', \theta'') - f(x', \theta'') \geq (>) 0 \Rightarrow f(x'', \theta') - f(x', \theta') \geq (>) 0$$

Equivalently we say that $f(x, \theta'')$ dominates $f(x, \theta')$ according to the single crossing order $f(x, \theta') \succeq_I f(x, \theta')$

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  - If f has ID in (x, θ) then f has the SCP in (x, θ)
- (Milgrom & Shannon): $argmax_x f(x, \theta)$ is increasing in θ if and only if f has the SCP

Characterizing Equilibria: Punishing Strategy Equilibria

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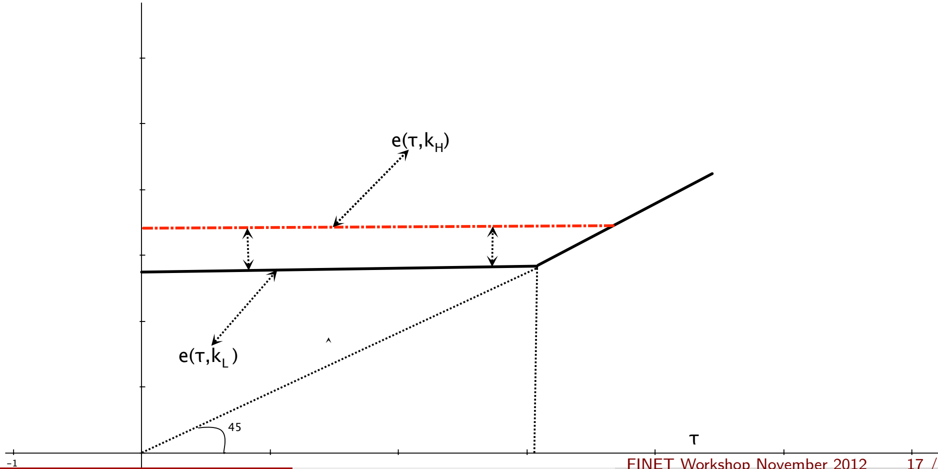
Characterizing Equilibria: Punishing Strategy Equilibria

- Under which conditions is $\rho(K)$ decreasing in K for any t?
- In order to apply the standard Milgrom-Shannon theorem I need to show that the objective function of the child has the single-crossing property in (K, ρ)
- This is complicated by the fact that the parent's payoff function involve non-primitive objects - $a(\tau, K)$ and the value functions

Best Responses: Graphical Analysis

Figure: Properties of Best Responses



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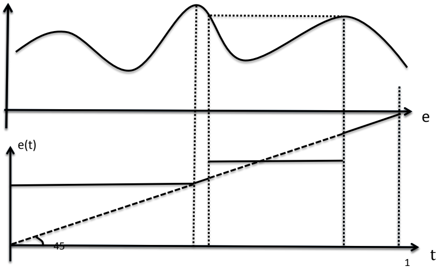

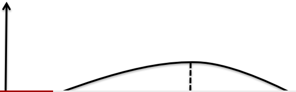

Properties of the Best Response Function

Lemma

- *If $a(\tau, K)$ is increasing in K (I), then $a(\tau, K') \succeq_{IDO} a(\tau, K'')$ for any $K'' > K'$.*
- *If the objective function of the child is single-peaked (U) and (I) holds, $\Delta(K) = a(\tau'', K) - a(\tau', K)$ is decreasing in K*

An intuitive approach:

1. If for any t (U) and (I) hold

Properties of the Best Response Function

Lemma

- *If* $a(\tau, K)$ *is increasing in* K *(I), then* $a(\tau, K') \succeq_{IDO} a(\tau, K'')$ *for any* $K'' > K'$.
- *If the objective function of the child is single-peaked (U) and (I) holds,* $\Delta(K) = a(\tau'', K) - a(\tau', K)$ *is decreasing in* K

An intuitive approach:

1. If for any t (U) and (I) hold
2. The parent's payoff function preserves the DD property of $\rho_t(K_t)$ then

MS is applicable $\Rightarrow \rho_t(K_t)$ is decreasing in K_t for any t

Sufficient Conditions

Lemma

If

1. *If* $F(K|k,a) = F_1(K|k) + F_2(K|a)$
2. F_2 *has the CDFC property, i.e. is convex in* a
3. u *is concave*

then $\rho_t(K_t)$ *is decreasing in* K_t.

Main ingredients

- 1) exploits the fact that the sum of supermodular functions is supermodular by approximating an increasing function as a sum of steps function
- An increasing and concave (convex) transformation of a function with DD(ID) has the DD(ID) ((2) and (3))
- A theorem by Vives&Van Zandt preserves the DD property under integration

Data and Sample

The NLSY97 contains information on:

- the person setting the limit on the 3 activities (first three survey rounds for children born in 1983-1984)
- time spent watching TV and doing homework (first survey round for all cohorts)
- GPA achieved at the end of each academic (only for high school) year and PIAT test scores (first rounds all cohorts and all other rounds only 1984 cohorts)

Patterns

Homework

Table: Proportion of Children Spending Some Time Doing Homework in a Typical Week

Age	Black Children	Hispanic Children	White Children
12	88.51 (296)	85.42 (240)	92.81 (612)
13	86.47 (414)	85.17 (344)	91.5 (871)
14	82.25 (462)	83.14 (344)	90.22 (941)
15	82.71 (133)	82.68 (127)	83.71 (264)

Number of observations in parenthesis

Table: Time Spent Doing Homework in a Typical Week

	Black Children				Hispanic Children				White Children			
Age	Mean	Median	S.D	N	Mean	Median	S.D	N	Mean	Median	S.D	N
12-16	4.34	3.75	3.71	1309	4.75	4	3.8	1058	4.66	4	3.6	2694
12	4.53	4	3.23	262	5.81	5.5	3.59	205	5.12	4.5	3.3	568
13	5.10	4.5	3.53	358	5.55	5	3.57	293	4.98	4.16	3.62	797
14	5.29	4.5	2.29	380	5.48	5	3.33	286	5.27	5	3.42	849
15	5.79	5	3.65	110	5.92	5	3.51	105	5.38	4.5	3.54	221

Number of observations in parenthesis

Means and Medians are calculated on non-zero observations

Units: hours per week

Patterns

TV watching

Table: Time Spent Watching TV in a Typical Week

	Black Children				Hispanic Children				White Children			
Age	Mean	Median	S.D	N	Mean	Median	S.D	N	Mean	Median	S.D	N
12-15	24.49	22	14.84	1280	20.06	17	13.28	1058	16.37	14	11.25	2716
12	23.39	21	14.19	289	18.59	16	12.28	244	16.72	14	11.82	622
13	24.38	21	15.5	408	20.35	18	13.48	341	16.25	14	10.67	884
14	25.88	25	14.4	447	20.79	18	13.83	344	16.28	14	11.33	943
15	22.53	18	15.34	136	20.11	17	13.01	129	16.25	14	11.58	267

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Means and Medians are calculated on non-zero observations

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Multiplicity of equilibria

- Under (I) and (U) all the SPNE of the game have the (PS) property: there exists a matrix of cutoffs

$$\underbrace{c}_{(n-1)\times T} = (c_1, c_2, \ldots, c_T)$$

such that

- $\rho_t(k_{t-1}) = \rho_i \Leftrightarrow c_{i-1} \leq k_{t-1} \leq c_i$

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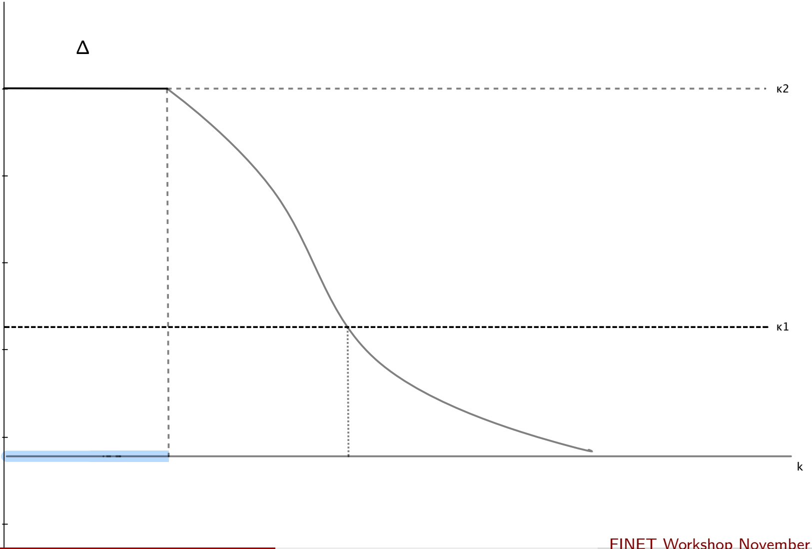
- Given $a(K, \tau)$ the matrix $\mathbf{c}$ is not unique due to possible corner solutions and because of the dynamic aspect of the problem solved by the child

Figure: Multiple Equilibria



Two Step Estimation

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  - First step: solve the child's problem numerically and estimate F, G δ, u together with **c** (child's response is a function) through SML using measurement error
  - Second step: given **c** recover w_t using a GMM estimator based on the equilibrium conditions

First Step

Measures and Choices

The NLSY97 contains

- 2 measures of children's cognitive skills: GPA and PIAT math test scores
  - I use a linear factor model assuming that they are proxies for the state variable of the model

$$GPA_t = n_0 + n_1 k_t + \epsilon_t^G$$
$$PIAT_t = m_0 + m_1 k_t + \epsilon_t^P$$

with ϵ_t^G and ϵ_t^P being normally distributed

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- 2 measures of time allocation : time spent watching TV and doing homework which are part of the choice set of the child
  - I adopt an hurdle model for the measurement equation to deal with both the intensive and the extensive margin:

$$g(e^o) = \begin{cases} \rho_e & \text{if } e^o = 0 \\ \frac{1-\rho_e}{1-G(0|e)} g(e^o|e) & \text{if } e^o > 0 \end{cases}$$

    where
    - e^o denotes the observed effort (time spent doing homework), while e the optimal effort implied by the model
    - G is the CDF of a normal $g(\cdot|e)$ with mean $\lambda_{0,e} + \lambda_{1,e} e$ and variance σ_e^2

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  - An analogous model is adopted to model TV viewership
- I use the responses on the limits to construct three binary indicators (1 if the the parent decides alone, 0 otherwise). Each response is allowed to be misclassified with positive probability

$$\Pr(\rho^o(j) = 1|\rho(j) = 1) = E_j + (1 - E_j)\Pr(\rho(j) = 1)$$

First Step

Sample and Data-Model Map

To map the model to the data I

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In the sample I include

- Children who have no siblings living in the household at the time of the interview (treat in and out moves as exogenous)
- High school graduates (do not model dropping out decisions) with a normal grade progression

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Estimation Algorithm

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- there are missing variables for my measures of human capital
- computing conditional probabilities requires to integrate out all the possible realizations of the state variables
- to overcome the computational burden I implement the method developed by Keane and Wolpin (IER 2001) which
  - only requires to simulate "outcome" histories, i.e. only needs unconditional simulations
  - uses the densities of the measurement/classification errors to reconcile the predictions of the model with the observations in the data

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Data Limitations, Heterogeneity and Identification

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Key identification arguments

Conditional on a matrix $\mathbf{c}$:

- the observed variation in effort as a function of k is informative about the parameters entering $\delta(k)$

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Conditional on a matrix **c**:

- the observed variation in effort as a function of k is informative about the parameters entering $\delta(k)$
- the parameters entering the payoff functions are identified by unconditional observed variations of time allocation choices
- Given a stock of human capital, the variations in the effort conditional on an observed parenting style pins down the parameters entering $G(\tau|\rho)$

- The matrixes **c** are identified by the observed variations in the parenting style choices conditional on the human capital of the child (analogous to an ordered logit with heterogeneous cutoffs)

Second Step

- The goal of the second step is to recover the parameters into Π and the costs κ_i
  - The estimated matrix $\hat{\mathbf{c}}$ provides, for each parental type at most $T \times (n-1)$ moments where

$$\int W(K) dF(K | a(\hat{c}_i, \rho(\hat{c}_i)), \hat{c}_i) = \kappa_{i-1} - \kappa_i > 0$$

    where in the last period $W = \Pi$. Let $\#\Pi$ denote the number of parameters entering into Π

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Parametrizations

Preferences

$$v(\alpha_t) = \begin{cases} (1 - e_t)^\eta & \text{if } t < T + 1 \\ \Xi(k_T) & \text{if } t = T + 1 \text{ (the game is over)} \end{cases}$$

with $\eta \in (0, 1)$

$$\Xi(k_T) = \frac{1}{\xi}\Big\{1 - \exp\Big[-\xi\Big(\frac{k_T^{1-\sigma} - 1}{1 - \sigma}\Big)\Big]\Big\}$$

with $\sigma > 0$ and $\xi > 0$

$$\delta(k) = \frac{\vartheta_0}{\vartheta_0 + \exp(-\vartheta_1 k)}$$

with $\vartheta_0 > 0$

First Step

Parametrizations

Technologies

- $$G(\tau|\lambda) = \text{Beta}[d(\rho_t), 1] \quad \text{with} \quad \tau \in [0,1], \lambda_1 > 0$$
- I parametrize $d(\rho_t)$ as follows
$$d(\rho_t) = \frac{\lambda_0 \rho_t}{1 + \lambda_0 \rho_t}$$
there exists a positive probability of misclassification
- The human capital production is the sum of two power density production functions
$$F(K|e,k) = \frac{K^{a(e)}}{2} + \frac{K^{b(k)}}{2} \quad \text{with} \quad K \in [0,1]$$
with
$$a(e) = \frac{\theta_0 + e^{\theta_1}}{1 + \theta_0 + e^{\theta_1}}$$
$$b(k) = \frac{\exp(\theta_2 k)}{1 + \exp(\theta_2 k)}$$
with $\theta_0 > 0$, $\theta_1 \in (0,1)$ and $\theta_2 > 0$.

Estimates

Table: Estimates

parameter	Estimate	S.E.
η	0.24	0.003
σ	0.14	0.002
ξ	24.52	8.64
ϑ_0	0.48	0.009
ϑ_1	0.57	0.04
λ_0^{TV}	0.23	0.008
λ_0^{curfew}	0.68	0.004
$\lambda_0^{friends}$	0.02	0.001
θ_0	7.82	1.14
θ_1	8.12	2.34
θ_2	1.45	0.07

Model Fit

Homework Doing

Table: % of Children Studying

age	Data	Model
12	91.24	92.48
13	89.37	92.14
14	87.65	88.42
15	87.14	89.24

Table: Average Study Time

age	Data	Model
12	4.55	4.82
13	4.53	4.72
14	4.45	4.62
15	4.57	4.81

Model Fit

Limits

Table: Proportion of Parents Setting the Curfew

age	Data	Model
12	68.74	71.12
13	66.59	70
14	60.77	64.72
15	53.22	58.83
16	48.27	52.72

Model Fit

Limits

Table: Proportion of Parents Setting Limits on TV shows

age	Data	Model
12	39.21	42.65
13	32.03	34.71
14	21.62	24.98
15	13.91	15.32
16	11.16	13.21

A Policy Question

- Governments intervention in disciplining children is subject to debate :

  > *In the early days of the Labour government there was much discussion in the media about where the boundary lay between interferences and appropriate involvement. There are some who view the family as a private institutions and believe that, except in the extreme cases, it should be largely free from state interferences. On the other hand there are those who take the position that government does have a role in enhancing support for the family - Coleman and Roker*

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- No matter what: you stay home, no TV, no friends
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- However the are important distributional effects

Table: Distributional Effect on GPA

$P1$	$P2$	$P3$	$P4$	$P5$	$P6$	$P7$	$P8$	$P9$	$P10$
19%	17.6%	12.4%	8.7%	4.2%	2.9%	0.5%	−4.7%	−7.2%	−11.8%

Table: Distributional Effect on PIAT

$G1$	$G2$	$G3$	$G4$	$G5$	$G6$	$G7$	$G8$	$G9$	$G10$
10.1%	8.3%	5.1%	2.3%	1.9%	0.1%	−0.3%	−0.4%	−2.1%	−1.6%

Counterfactuals

Complete Freedom

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$P1$	$P2$	$P3$	$P4$	$P5$	$P6$	$P7$	$P8$	$P9$	$P10$
−14.6%	−11.8%	−9.6%	−7.5%	−4.8%	−3.9%	−1.7%	−0.8%	−0.3%	−0.1%

Table: Distributional Effect on PIAT

$P1$	$P2$	$P3$	$P4$	$P5$	$P6$	$P7$	$P8$	$P9$	$P10$
−9.1%	−7.3%	−5.8%	−5.6%	−3.9%	−2.1%	−1.3%	−0.4%	0.1%	0.5%

Road Ahead

- Welfare Analysis

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- Reshuffling: optimal assignment problem

Longer Term Research Agenda:

- Monetary Incentives

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