Identifying Sharing Rules in Collective Household Models

an overview

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What percentage of a married couple's expenditures are controlled by the husband?

How much money does a couple save on consumption goods by living together versus living apart?

What share of household resources go to children?

How much income would a woman living alone require to attain the same standard of living that she'd have if she were married?

Goals: 1. Empirically tractible. 2. Identify resource shares (bargaining), joint consumption, household member's indifference curves and indifference scales. 3. Avoid untestable cardinalization assumptions.

Overview - What We Assume Households Do

For now two household members, f female and m male (will add children later).

1. Household buys a bundle of goods z.
2. Converts z to private good equivalents $x = F^{-1}(z)$.
3. Divides bundle x into $x = x^f + x^m$ (Pareto efficient).
4. Each member gets utility $U^f(x^f)$, $U^m(x^m)$.

F is the "consumption technology function"
If good j is purely private, then $z_j = x_j = x_j^f + x_j^m$
If good j is purely public, then $z_j = x_j^f = x_j^m$
More generally, goods are partly shared. Example: A couple rides together in their car 30% of the time. Then for consumption of gasoline j, $z_j = x_j/1.3$, so $x_j^f + x_j^m = 1.3z_j$.

Models

Start from Becker (1965, 1981). Assume Pareto efficient households (rules out many noncooperative models).

Define:
d = distribution factors = observables that only affect allocations between members, not utility of either.
μ = Pareto weight function.
U^f, U^m utility functions of women and men, respectively.

Couples will maximize $\mu U^f + U^m$.

Pareto weight μ interpreted as relative bargaining power, but depends on how utility is cardinalized.

Will later define resource share η that doesn't depend on unknowable cardinalizations.

Models

Model C (Chiappori 1988, Bourguignon and Chiappori 1994, Browning, Bourguignon, Chiappori, and Lechene 1994, Browning and Chiappori 1998). Every good is either purely private (private bundles z^f, z^m) or purely public (public good bundle X) $z = \left(z^f + z^m, X\right)$, $p = (p_z, p_x)$.

$$\max_{z^f, z^m, X} \mu(p, y, d) U^f(z^f, X) + U^m(z^m, X) \text{ such that } p_z' \left(z^f + z^m\right) + p_x' X = y$$

Solution demands: $X = X(p, y, d)$, $z = z(p, y, d)$

Model BCL (Browning Chiappori Lewbel 2006). Goods are private or partly shared (general consumption technology F), no money illusion,

$$\max_{x^f, x^m, z} \mu(p/y, d) U^f(x^f) + U^m(x^m) \text{ such that } z = F\left(x^f + x^m\right), \quad p'z = y$$

Solution demands: $z = z(p, y, d)$.

Econometrician observes solution demand functions.

Sharing Rule Definitions

Model C: Define sharing rule = wife's (conditional) share: $\eta(p, y, d) = p_z' z^f / p_z' \left(z^f + z^m\right)$. With no public goods, η is monotonic and one to one with μ.

Model BCL: Define sharing rule = wife's (private equivalents) share: $\eta(p, y, d) = p_z' x^f / p_z' \left(x^f + x^m\right)$. In BCL, for any regular consumption technology function F, η is monotonic and one to one with μ.

Both definitions are generally monotonic in Pareto weights, and so may be interpreted as measures of bargaining power in models where a bargaining game is assumed (assuming the game has efficient outcomes).

Earlier Sharing Rule Identification Results

Main identification result (general form: Chiappori and Ekeland 2009): In model C with or without public goods, given just the household demand function $z = z(p, y, d)$, resource shares $\eta(p, y, d)$ are not identified, but $\partial\eta(p, y, d)/\partial d$ is identified.

Application/variant of this result: Lechene and Attanasio (2010) see a cash transfer to households that leaves food shares unchanged. Transfer changes y but could also be a d. Not explicitly identifying $\partial\eta/\partial d$, but infer it must be nonzero to offset transfer's effect through y on shares.

Without further assumptions, is also true for BCL that $\eta(p, y, d)$ is not identified, since not identified in the purely private goods model, which is a special case of both BCL and C.

Problem: many welfare/policy calculations depend on identifying η, not on just $\partial\eta/\partial d$, e.g., poverty rates, inequality measures, indifference scales.

Goal - Identify Sharing Rule Levels η, not just $\partial\eta/\partial d$

Methods for identifying:

1. Collect extensive intrahousehold consumption data: Cherchye, De Rock and Vermeulen (2010), Menon, Pendakur, Perali (2012).

2. Obtain bounds on shares: Cherchye, De Rock and Vermeulen (2011), Cherchye, De Rock, Lewbel and Vermeulen (2012).

3. Restrict individual preferences across households of different compositions: Browning Chiappori Lewbel (BCL 2006), Couprie (2007), Lewbel and Pendakur (LP 2008), Bargain and Donni (2009), Lise and Seitz (2011).

4. Restrict sharing rules and preferences among individuals within a household: Dunbar, Lewbel, and Pendakur (DLP 2012).

5. Restrict sharing rules and use distribution factors: Dunbar, Lewbel, and Pendakur (DLP 2012b).

Bounds

Weak bounds: a lower bound on each individual $j's$ resource share is cost of $j's$ private, assignable consumption divided by total household consumption. We can do better.

Drop distribution factors d for now. Suppose saw purchased bundles $z_1^j,...,z_n^j$ of individual j in price/income regimes $p_1/y_1,...,p_n/y_n$.

If j maximizes utility, then these bundles satisfy revealed preference inequalities derived from Samuelson (1938), Houthakker (1950), Afriat (1967), Diewert (1973), and Varian (1982).

For known vector valued function M^j, write these inequalities as

$$0 \leq M^j \left(\{ z_i^j, p_i/y_i \}_{i=1,...,n} \right)$$

Bounds continued

A single maximizing consumer j satisfies revealed preference bounds

$$0 \leq M^j \left(\{ z_i^j, p_i / y_i \}_{i=1,\ldots,n} \right)$$

Assume model C (leave out public goods for now), and we observe a couple purchase bundles $z_1,\ldots,z_n$ in regimes $p_1/y_1,\ldots,p_n/y_n$. Then there must exist $\{z_i^f\}_{i=1,\ldots,n}$ such that resource shares $\eta_1,\ldots\eta_n$ satisfy inequalities

$$\begin{aligned} 0 &\leq M^f \left(\{ z_i^f, p_i / \eta_i y_i \}_{i=1,\ldots,n} \right) \\ 0 &\leq M^m \left(\{ z_i - z_i^f, p_i / \left(1 - \eta_i\right) y_i \}_{i=1,\ldots,n} \right) \end{aligned}$$

These inequalities give bounds on resource shares $\eta_1,\ldots\eta_n$. Can extend to include public goods; get bounds even if the econometrician does not know which goods are private and which are public.

Further extension: Observe/estimate couple's demand functions $z = z(p, y)$, obtain inequalities for every p/y point to get tighter bounds.

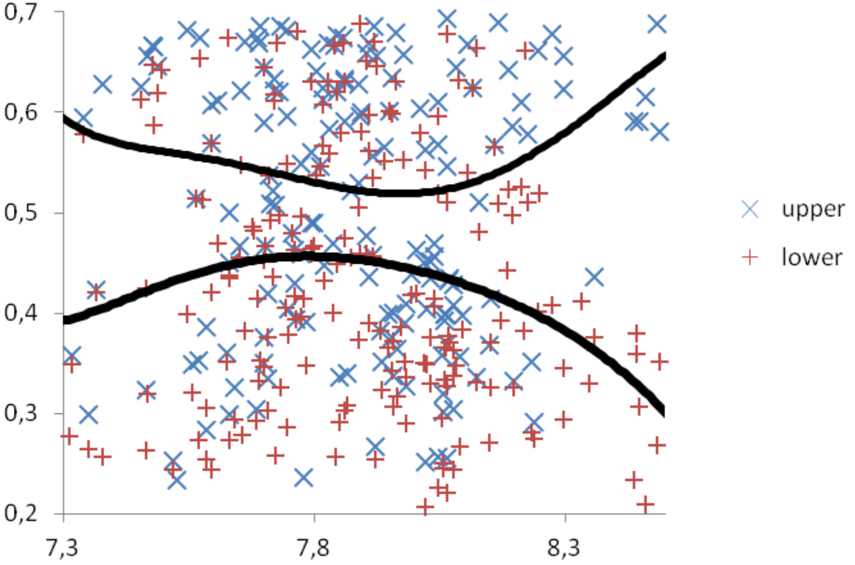
Bounds continued

Next few graphs show some empirical results in Cherchye, De Rock, Lewbel and Vermeulen (2012). 211 Dutch households, from 2009 wave of the LISS (Longitudinal Internet Studies for the Social sciences) panel. Three goods: husband's leisure, wife's leisure, total consumption. Full income is wages times total hours for each plus total consumption. No distribution factors (wages are prices).

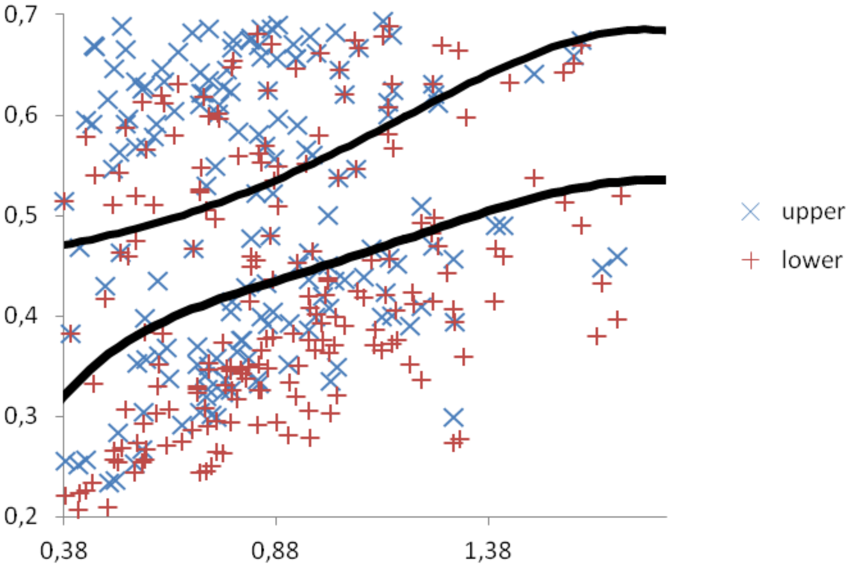
Households sorted left to right by full income or relative wage. The x and + are the estimated lower and upper bound for each household. Dark lines are fitted curves through lower and upper bounds.

Results: Resource shares often around .4 to .6. Shares look uncorrelated to full income, correlated with relative wages.

Sharing rule η bounds (Y-axis) and log of full income (X-axis)



Sharing rule η bounds (Y-axis) and relative wage (X-axis)



BCL Model

Drop distribution factors d.

BCL demands $z = h\left(p/y\right)$ obtained from

$$\max_{x^f, x^m, z} \mu(p/y) U^f\left(x^f\right) + U^m\left(x^m\right) \text{ such that } z = F\left(x^f + x^m\right), \quad p'z = y$$

For $j = f, m$, define $x^j = h^j\left(p/y\right)$ as demands from $\max_{x^j}\left\{U^j\left(x^j\right) \mid px^j = y\right\}$.

Define indirect utility functions $V^j\left(p/y\right) = U^j\left[h^j\left(p/y\right)\right]$.

Working Towards Identification - Duality

BCL show duality: A shadow (Lindahl) price vector $\pi(p/y)$ and a sharing rule $0 < \eta(p/y) < 1$ exist such that

$$x^f(p/y) = h^f\left[\pi/(\eta y)\right], \; x^m(p/y) = h^m\left[\pi/((1-\eta)y)\right], \; h(p/y) = F\left(h^f\right.$$

From $h^j(p/y)$ ordinary duality gives $V^j(p/y)$, and given $\eta(p/y)$ the Pareto weight is

$$\mu = -\frac{\partial V^f(\pi/(\eta y))/\partial\eta}{\partial V^m[\pi/((1-\eta)y]/\partial\eta}$$

So: given demands h, h^f, h^m, if π and η are identified, everything (ordinal) is identified.

Questions:

1. How to identify demand functions h, h^f, h^m?
2. Given h, h^f, h^m, are π, η identified?

Will address second question first.

BCL - Generic Point Identification

Claim: If functions h, h^f, h^m known, then functions x^m, x^f, F, and η are "generically" identified (generic as in Chiappori and Ekeland 2009).

Proof sketch: let $\rho = p/y$. h, h^f, h^m known. Given any $\overline{F} \in \Omega_F$, let

$$\overline{\pi}(\rho) = \frac{D\overline{F}(x)'.\rho}{x'D\overline{F}(x)'.\rho}, \text{ evaluated at } x = \overline{F}^{-1}[h(\rho)],$$

$$\overline{x}(\rho, \eta) = h^m[\overline{\pi}(\rho)/(1-\eta)] + h^f[\overline{\pi}(\rho)/\eta],$$

$$\overline{\eta}(\rho) = \arg \min_{\eta^* \in [0,1]} \max |\overline{x}(\rho, \eta^*) - \overline{F}^{-1}[h(\rho)]|.$$

and define $\widetilde{F}$ by $\widetilde{F}[\overline{x}(\rho, \overline{\eta}(\rho))] = h(\rho)$. This defines a mapping $\widetilde{F} = \mathcal{T}(\overline{F})$. True F is fixed point, and true η is $\overline{\eta}$ with $\overline{F} = F$.

$\mathcal{T}$ might not be a contraction mapping. Loosely, existence of $\mathcal{T}$ shows enough demand functions are identified to generally permit recovery of F and η; are identified as long as the demand functions are not too simple.

BCL - Linear Consumption Technology

Assume a linear consumption technology: $z = F(x) = Ax + a$ makes shadow prices not depend on indirect utility functions:

$$\frac{\pi(p/y)}{y} = \frac{A'p}{y - a'p}$$

gives household demand functions the form:

$$\begin{aligned} z &= h(p/y) = Ah^f\left(\frac{\pi(p/y)}{\eta(p/y)y}\right) + Ah^m\left(\frac{\pi(p/y)}{(1-\eta(p/y))\,y}\right) + a \\ &= Ah^f\left(\frac{A'p}{y-a'p}\frac{1}{\eta(\frac{p}{y})}\right) + Ah^m\left(\frac{A'p}{y-a'p}\frac{1}{1-\eta(\frac{p}{y})}\right) + a \end{aligned}$$

Linear Consumption Technology - Compare BCL to Gorman

BCL with linear consumption technology F is

$$z = h\left(p/y\right) = Ah^f\left(\frac{A'p}{y-a'p}\frac{1}{\eta\left(\frac{p}{y}\right)}\right) + Ah^m\left(\frac{A'p}{y-a'p}\frac{1}{1-\eta\left(\frac{p}{y}\right)}\right) + a$$

Gorman (1976) general linear technology household demand model is:

$$z = Ah^m\left(\frac{A'p}{y-a'p}\right) + a$$

Barten (1964) is Gorman's model with $a = 0$ and A a diagonal matrix.

Gorman had similar motivation for consumption technology as a model of sharing and jointness of consumption, but only in a unitary model. Gorman makes household demands be a scaled function of one individual's demands.

Gorman differs from BCL even if members all have same preferences ($h^f = h^m$) and same equivalent incomes ($\eta = 1/2$).

Nonparametric Identification With Linear Technology

Claim: Given observable demand functions, the functions x^m, x^f, F, and η are generically identified if number of goods is $n \geq 3$.

Take $T \geq n + 10$ price vectors $p^1, ..., p^T$. Then

$$z^t = Ah^f \left(\frac{A' p^t}{y - a' p^t} \frac{1}{\eta^t} \right) + Ah^m \left(\frac{A' p^t}{y - a' p^t} \frac{1}{1 - \eta^t} \right) + a$$

For each t have $(n-1)$ independent equations, total $(n-1)\, T$ equations. The unknowns are A, a, and η^t; total $n^2 + n + T$ unknowns. With $n \geq 3$ and $T \geq n + 10$, have more equations than unknowns, so we have identification as long as the equations are linearly independent (not too simple).

Examples: Identification fails for LES h, Scaling of Barten Technology A not identified for homothetic h^i.

Example: Identification In Almost Ideal Demand System

Claim: Assume h^i is defined by budget shares $\omega^i(p/y^i) = \alpha^i + \Gamma^i \ln p + \beta^i \left[\ln\left(y^i\right) - c^i\left(p\right)\right]$ for $i = m, f$. Assume $\beta^f \neq \beta^m$ and elements of β^f, β^m, and the diagonal of A are nonzero. Then the functions x^m, x^f, F, and η are identified.

Actually substantially overidentifed. Most parameters are identified from demands on just one good. Models that nest Almost Ideal like QUAIDS are also identified.

Proof sketch: Have $\pi = A'p/(1 - a'p)$ and

$$z_k = a_k + \sum_j A_{kj} \left[\frac{\eta}{\pi_j} \omega^f \left(\frac{\pi}{\eta y} \right) + \frac{1-\eta}{\pi_j} \omega^m \left(\frac{\pi}{(1-\eta)y} \right) \right]$$

Intercepts identify a. Coefficients of $\ln y$ identify $\sum_j \left[\eta \beta_j^f + (1-\eta)\,\beta_j^m \right] / \sum_\ell (A_{\ell j}/A_{kj}) p_\ell$. Variation in p and subscripts identifies η, β coefficients and ratios $A_{\ell j}/A_{kj}$. Levels of A_{jk} are identified from the quadratic price terms in $\sum_j A_{kj} \left[\frac{\eta}{\pi_j} c^f\left(\pi\right) + \frac{1-\eta}{\pi_j} c^m\left(\pi\right) \right]$.

Identifying Demand Functions

$z = h(p/y)$ are household demand functions obtained from

$$\max_{x^f, x^m, z} \{\mu(p/y) U^f(x^f) + U^m(x^m) \mid z = F\left(x^f + x^m\right), \, p'z = y\}$$

$h(p/y)$ identified from data on a household's purchases in many p/y regimes. More commonly, estimated from data on many household's purchases, assuming preferences are identical up to observable covariates and ignorable errors. Recent work on unobserved heterogeneity in continuous demand systems (Blundell and Matzkin, Lewbel and Pendakur, many others).

Harder is identifying $x^j = h^j(p/y)$, the demands from $\max_{x^j} \{U^j(x^j) \mid px^j = y\}$.

Identifying Demand Functions

Overly strong BCL assumption: $h^j(p/y)$, the demands from $\max_{x^j}\{U^j(x^j) \mid px^j = y\}$, identified from data on purchases by singles living alone.

Approaches to weakening this assumptions (most discussed below):

Impose constraints on π, η to weaken data requirements on h, h^m, h^f. Examples: π linear or Barten, η independent of y (at some y levels).

Model changes in individual's preferences that occur when change from single to couple.

Impose restrictions on how h^m, h^f vary across people (later SAP).

Impose restrictions on how h^m, h^f depend on shadow prices π, and hence on how their values will vary across households of different types (later SAT).

Exploit combination of η independent of y with presence of distribution factors.

Simplifying the Model

Assume Barten technology: $a = 0$, A diagonal. Let
$w^k(y, p, A)$ = couple's budget share of good k.
$w_j^k(y, p)$ = budget share of good k from utility function $U^j(x^j)$, $j = m, f$.

BCL model then becomes, in budget share form

$$\begin{aligned} w^k(y, p, A) &= \eta(p, y, A) w_f^k \left[\eta(p, y, A) y, A'p\right] + \\ &\quad [1 - \eta(p, y, A)]\, w_m^k \left([1 - \eta(p, y, A)]\, y, A'p\right) \end{aligned}$$

BCL estimate this model, with Quadratic Almost Ideal Demand System QUAIDS model (Banks, Blundell, and Lewbel 1997) for singles.

Simplifying to Engel Curves

Drop prices, write the model in terms of Engel curves. Using data from just one price regime greatly reduces dimensionality and data requirements.

LP (2008), Bargain and Donni (2009) simplify by assuming Independence of Base (IB) (Lewbel 1989, Blackorby and Donaldson 1992), i.e., for each person j, there exists a D_j such that $V_j(A'p, y) = V_j\left[p, y/D_j(A, p)\right]$. Also assume η independent of y, makes household budget share Engel curves simplify to

$$\begin{aligned} w^k(x, A) &= h^k(A) + \eta(A)\, w_f^k\left(y/I_f(A)\right) \\ &\quad + (1 - \eta(A))\, w_m^k\left(y/I_m(A)\right) \end{aligned}$$

Where I_f and I_m are "indifference scales."

These papers simplify to Engel curves, but still use the BCL method of identifying the demand functions w_f and w_m from singles data.

Is η Independent of y reasonable?

Can resource shares η be independent of total expenditures y, as assumed by these "identification just from Engel curves" models. It generates testable restrictions, see DLP (2012).

Empirical evidence Menon, Pendakur, Perali (2012); Cherchye, De Rock, Lewbel and Vermeulen (2012).

Permits η to depend on prices, incomes of each member, wealth, distribution factors, etc. Only assuming η independent of y after conditioning on these other things.

Does not violate Samuelson (1956), who showed resource shares can't be constant for a large class of social welfare functions, since it permits shares to depend on prices.

DLP (2012, online appendix) provides an example of a sensible class of models of utility functions and Pareto weights that yield η independent of y: PIGL or PIGLOG (Meullbauer 1976) utility with weighted S-Gini (Donaldson and Weymark 1980) household social welfare functions.

Some Summary Empirical Results

Use the LP (2008) Engel curve simplification of BCL - singles and couples data. Four models (vary by demographics included and outlier handling). 1990-92 Canadian Family Expenditure Survey, 12 consumption goods, 419 single men, 450 single women, and 332 couples.

Estimated resource share η for median women: 0.36 to 0.46. Small age and education effects (these affect both preferences and shares). Raising proportion of household's income she contributes up by .5 raises η by about .05.

Other estimates: scale-economy measure $p'z/p'x$ should lie between $1/2$ (full sharing) and 1 (completely private). Estimated range 0.70 to 0.78. Indifference scales for women I_f around 1.53; for men I_m around 1.44. A person needs about two thirds, $1/1.53$ or $1/1.44$ of couple's income to reach the same indifference curve living alone that one attains living with a partner.

Dropping the use of Singles data

DLP (2012, 2012b): Drop the IB assumptions, drop the use of singles data.

Keep using Engel curves. Assume there exists a private assignable good for each household member.

Further simplify estimation (and further relax data requirements and modeling constraints) by only using data on the one private assignable good for each household member.

To maintain identification, keep the assumption that resource shares η are independent of total expenditures y (at least for low levels of y). In addition:

DLP (2012) Add SAP or SAT restrictions on demand functions, or,

DLP (2012b) make use of distribution factors.

Focus on Assignable Goods, Bring in Children

Have s children, impose same utility function for each. Household does

$$\max\{\widetilde{U}\left[U^f(x^f), U^m(x^m), U^c(x^c), p/y, s\right] \mid z = F\left(x^f + x^m + sx^s, s\right), p'z =$$

Looking only at private assignable goods (say, clothing), household's budget shares are given by

$$\begin{aligned} W_{cs}(y) &= s\eta_{cs} w_{cs}(\eta_{cs} y), \quad W_{ms}(y) = \eta_{ms} w_{ms}(\eta_{ms} y), \\ W_{fs}(y) &= \eta_{fs} w_{fs}(\eta_{fs} y). \end{aligned}$$

$W_{fs}(y)$ = fraction of y household spends on woman's clothes in a single price regime p. These can be estimated.

$w_{cs}(y)$ = fraction of y spent that would be spent on woman's clothes determined by $\max\{U^f(x^f) \mid \pi_s' z = y\}$, at shadow prices π_s given by Barten technology for household with s children.

η_{fs} = woman's resource share in house with s children.

Similar for man m and child c.

Identification Strategies

$$
\begin{aligned}
W_{cs}(y) &= s\eta_{cs} w_{cs}(\eta_{cs} y), \quad W_{ms}(y) = \eta_{ms} w_{ms}(\eta_{ms} y), \\
W_{fs}(y) &= \eta_{fs} w_{fs}(\eta_{fs} y).
\end{aligned}
$$

We observe W functions, want to identify resource shares η.

LP (2008), Bargain and Donni (2009), extend BCL method by learning functions $w_{ms}(y)$ and $w_{fs}(y)$ from data on singles.

DLP (2012), place semiparametric restrictions (SAP) or (SAT) on the functions $w_{ms}(y)$, $w_{fs}(y)$, and $w_{cs}(y)$. Not restrictions on shape, but restrictions that make some feature of these functions be similar across people (SAP) or similar across household size/types (SAT).

DLP (2012b), combine η not depending on y with distribution factors.

Similar Across People SAP Identification

SAP demand: $w_j(y,p) = h_j(p) + g\left(\frac{y}{G_j(p)}, p\right)$ for $y \leq y^*$, $j = m, f, c$.
Functions h_j, G_j, and g can be anything, but g is the same across people.

Paper gives SAP class of utility functions. SAP, which is similar to but weaker than shape invariance, need apply only to the assignable goods (clothing). Get

$$\begin{aligned} W_{cs}(y) &= \alpha_{cs} + s\eta_{cs}\widetilde{g}_s(\eta_{cs}\gamma_{cs}y), \\ W_{ms}(y) &= \alpha_{ms} + \eta_{ms}\widetilde{g}_s(\eta_{ms}\gamma_{ms}y) \\ W_{fs}(y) &= \alpha_{fs} + \eta_{fs}\widetilde{g}_s(\eta_{fs}\gamma_{fs}y) \end{aligned}$$

Note $\gamma_{js} = G_j(\pi_s(p))$, shadow prices depend on household type/size s, which makes these functions vary by s in the Engle curves. Same for $\alpha_{js} = h_j(\pi_s(p))$ and $\widetilde{g}_s(\cdot) = g(\cdot, \pi_s(p))$.

Similar Across People SAP Identification

SAP Identification overview: By SAP we have

$$\begin{aligned} W_{cs}(y) &= \alpha_{cs} + s\eta_{cs}\widetilde{g}_s(\eta_{cs}\gamma_{cs}y), \\ W_{ms}(y) &= \alpha_{ms} + \eta_{ms}\widetilde{g}_s(\eta_{ms}\gamma_{ms}y) \\ W_{fs}(y) &= \alpha_{fs} + \eta_{fs}\widetilde{g}_s(\eta_{fs}\gamma_{fs}y) \end{aligned}$$

Look at derivatives with respect to y at $y = 0$:

$$\begin{aligned} W'_{fs}(0) &= \gamma_{fs}\eta^2_{fs}\widetilde{g}'_s(0), \quad W''_{fs}(0) = \gamma^2_{fs}\eta^3_{fs}\widetilde{g}''_s(0), \\ W'''_{fs}(0) &= \gamma^3_{fs}\eta^4_{fs}\widetilde{g}'''_s(0) \end{aligned}$$

and same for m and c. Along with $\eta_{fs} + \eta_{ms} + s\eta_{cs} = 1$ gives 10 equations in 9 unknowns η_{fs}, η_{ms}, η_{cs}, γ_{fs}, γ_{ms}, γ_{cs}, $\widetilde{g}'_s(0)$, $\widetilde{g}''_s(0)$, and $\widetilde{g}'''_s(0)$, for each household size s.

Identification only used derivatives at $y = 0$, so only needed restrictions to hold for small y.

Similar Across Types Identification

SAT demand: $w_j(y,p) = g_j\left(\frac{y}{G_t(p)}, \overline{p}\right)$ for $y \leq y^*$, $j = m, f, c$, where $\overline{p}$ are prices only of private goods. Functions h_j, G_j, and g_j can be anything, but the g function only depends on prices through $\overline{p}$.

Again only need to hold for clothing. Get

$$\begin{aligned} W_{cs}(y) &= \alpha_{cs} + s\eta_{cs}\widetilde{g}_c(\eta_{cs}\gamma_{cs}y) \\ W_{ms}(y) &= \alpha_{ms} + \eta_{ms}\widetilde{g}_m(\eta_{ms}\gamma_{ms}y) \\ W_{fs}(y) &= \alpha_{fs} + \eta_{fs}\widetilde{g}_c(\eta_{fs}\gamma_{fs}y) \end{aligned}$$

SAP made $\widetilde{g}_s(\cdot) = g(\cdot, \pi_s(p))$ only have a type s subscript, while SAT makes $\widetilde{g}_j(\cdot) = g_j(\cdot, \overline{p})$ only have a person j subscript.

For identification, look at same derivatives as in SAP, but now combine across a households of a few different types (difference sizes s), using that $\widetilde{g}_j(0)$ doesn't vary by s, to get more equations than unknowns.

Identification by Distribution Factors

With η depending on distribution factors d, and not on y, we have

$$\begin{aligned} W_{cs}(y,d) &= s\eta_{cs}(d)\, w_{cs}\left[\eta_{cs}(d)\, y\right] \\ W_{ms}(y,d) &= \eta_{ms}(d)\, w_{ms}\left[\eta_{ms}(d)\, y\right] \\ W_{fs}(y,d) &= \eta_{fs}(d)\, w_{fs}\left[\eta_{fs}(d)\, y\right]. \end{aligned}$$

We observe W functions, want to identify resource shares η.

Identification: Again look at derivatives wrt y at $y = 0$:

$$W'_{fs}(0) = \left[\eta_{fs}(d)\right]^2 w'_{fs}(0)$$

and similar for m and c. For each value d takes on, this along with $s\eta_{cs}(d) + \eta_{ms}(d) + \eta_{cs}(d)$ gives 4 equations. If d takes on at least 3 different values then we get 12 equation in 12 unknowns: $w'_{cs}(0)$, $w'_{ms}(0)$, $w'_{fs}(0)$, and, for each of 3 values of d, $\eta_{cs}(d)$, $\eta_{ms}(d)$, and $\eta_{cs}(d)$.

Some Estimates - Malawi Data

The Malawi Integrated Household Survey, conducted in 2004-2005. from the National Statistics Office of the Government of Malawi with assistance from the International Food Policy Research Institute and the World Bank.

High quality data: enumerators were monitored; big cash bonuses were used as an incentive system; about 5 per cent of the original random sample in each year had to be resampled because dwellings were unoccupied; (only) 0.4 per cent of initial respondents refused to answer the survey.

We use 2794 households comprised of non-urban married couples with 1-4 children aged less than 15. Private assignable good is men's, women's and children's clothing (including footwear).

Demographics: region, children age summaries, fraction of girls, adult low and high age dummies, education levels of each spouse, distance to a road and to a market, dry season dummy, religion (christian, muslim, animist/other).

Estimated Levels of Resource Shares

		SAP		SAT		**SAP&SAT**	
		Est	*Std Err*	Est	*Std Err*	**Est**	**StdErr**
1 kid	man	0.443	*0.048*	0.378	*0.076*	**0.400**	**0.045**
	woman	0.308	*0.041*	0.368	*0.062*	**0.373**	**0.042**
	kids	0.249	*0.037*	0.254	*0.072*	**0.227**	**0.036**
	each kid	0.249	*0.037*	0.254	*0.072*	**0.227**	**0.036**
2 kids	man	0.423	*0.051*	0.436	*0.090*	**0.462**	**0.051**
	woman	0.222	*0.042*	0.212	*0.056*	**0.221**	**0.043**
	kids	0.355	*0.045*	0.352	*0.100*	**0.317**	**0.045**
	each kid	0.177	*0.022*	0.176	*0.050*	**0.158**	**0.023**
3 kids	man	0.427	*0.057*	0.437	*0.099*	**0.466**	**0.053**
	woman	0.185	*0.046*	0.166	*0.054*	**0.176**	**0.044**
	kids	0.388	*0.050*	0.397	*0.114*	**0.358**	**0.050**
	each kid	0.129	*0.017*	0.132	*0.038*	**0.119**	**0.017**
4 kids	man	0.318	*0.070*	0.352	*0.112*	**0.384**	**0.063**
	woman	0.214	*0.054*	0.168	*0.062*	**0.182**	**0.052**
	kids	0.468	*0.061*	0.479	*0.133*	**0.434**	**0.059**
	each kid	0.117	*0.015*	0.120	*0.033*	**0.109**	**0.015**

Summary of Results - Malawi Data

SAP and SAT accepted, no data on singles needed.

Can't reject constant Father's share of 40%.

Mother share decreases by 5.5% per child.

Girl's get about 90% of what boys get.

Mother education level at 90 percentile instead of median decreases Father's share to 30%, 2/3 of the gain goes to mother, 1/3 to children.

Conclusions

Resource shares are a better measure of household resource allocation and power than Pareto weights (do not depend on a cardinalization of utility).

Many household welfare calculations depend on resource share levels, not just how they vary with distribution factors.

An example is the calculation of indifference scales, which unlike equivalence scales, can be identified by revealed preference.

A variety of alternative identifying strategies are proposed to point identify or to bound resource shares.

One Next step: identify distribution of resource shares across households allowing for unobserved hetereogeneity in preferences and power.