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Assignment Models of the Distribution of Earnings

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I. *Introduction*

RELATIVE WAGES are changing. Over the last decade or so, earnings of high school graduates have declined relative to college graduates, and earnings of young adults have declined relative to older adults; as a result, the distribution of earnings has become more unequal.[1] These relative changes are hard to explain in the context of models where the return to education is fixed by the long run supply behavior of individuals or in which the productivity and earnings of individuals are the result of their education and experience, independent of the availability of jobs in the economy. While changes in the industrial and occupational mix of the economy are routinely incorporated into ad hoc explanations of shifts in the distribution of earnings, they are absent from most formal models of the distribution.

This paper reviews models that explain the distribution of earnings as arising from the market economy's solution to the problem of assigning workers to jobs. The amount a worker can contribute to production typically depends on which job the worker performs. This occurs because jobs require many different tasks, and human performances at those tasks are extremely diverse; because industrial sectors use different technologies that rely on different combinations of human skills; or because jobs vary in the amounts of resources combined with labor. For the economy as a whole, total output then depends on how workers are assigned to jobs, i.e., which worker performs which job.

The existence of an assignment problem implies that workers face a choice in their job or sector. Their earnings are not determined by their performance in

[1] Frank Levy and Richard J. Murnane (1992) analyze recent tends in the distribution of earnings. Using Current Population Survey data, they show that earnings of workers with 16 years schooling have increased relative to earnings of workers with 12 years schooling between 1979 and 1987 and that (for workers with 12 years schooling) earnings of workers aged 45–54 have increased relative to workers aged 25–34 during the same time period (Table 7). Levy and Murnane (1992, Table 4) also report results of several authors showing that earnings inequality for all earners and for males increased between 1979 and 1987, using various inequality measures.

one sector alone: if they do poorly at one job or sector, they can choose another. Choice of job or sector creates an intermediate step between individuals' characteristics and their earnings. The observed relationship is constructed from worker choices.

Income or utility maximization guides workers to choose particular jobs over others. Higher wages for workers with some characteristics then play an allocative role in the economy rather than simply being rewards for the possession of particular characteristics. Workers found in a given sector are not randomly drawn from the population as a whole. Instead workers' locations in sectors or jobs are based on the criterion that their choices maximize their income or utility.

The models discussed here are characterized by the presence of an assignment problem, together with the consequences of worker choice and nonrandom selection. Despite outward differences, the models discussed in Section III have in common that they specify the jobs or sectors available to workers, the relevant differences among workers, the technology relating worker and job characteristics to output, and the mechanism that assigns workers to jobs.

These models generally proceed by first describing the assignment problem present in the economy. Then one can derive the wage differentials that are consistent with an equilibrium assignment of workers to jobs. The equilibrium wage differentials are those that yield equality between amounts of labor supplied and demanded in each submarket of labor. By providing a general equilibrium framework for studying inequality, assignment models reveal a rigorous route by which demand factors influence inequality and correctly specify the relation between the distribution of individual characteristics and inequality. The earnings function is no longer a directly observable relationship but instead is the equilibrium outcome to the solution of the assignment problem.

Explicit consideration of the economy's assignment problem provides a unity to seemingly separate topics. Wage differentials, occupational choice, organization of hierarchies, unequal skill prices and self-selection bias are topics that have been studied by themselves but which arise as consequences of the assignment problem. The existence of many labor market phenomena, such as search, mobility, hierarchy tournaments, unemployment, and specialized labor markets, can be motivated as labor market responses to the problem of assigning workers to jobs.

Although not generally recognized as a subcategory of income distribution theories, assignment models have a fairly long history. They can be said to begin with Jan Tinbergen's model (1951) with continuous distributions of workers and jobs and A. D. Roy's sectoral model (1951) with workers choosing between two or more occupations. These models differ in a number of ways but share the feature that the distribution of earnings can be explained through the assignment problem.

Empirical modeling of the distribution of earnings requires the econometric specification of worker alternatives, even though only the chosen sector or job is observed. This generates a set of econometric problems that have been addressed in applications of Roy's and Tinbergen's models.

Probably most economists would agree to the basic premises underlying assignment models, that both supply and demand are relevant and that individual performances vary from job to job. But there may be some disagreement regarding the implications of those premises for the conduct of research on the distribution of earnings. This survey emphasizes the implications of assignment models for the earnings functions, the human capital approach and the decomposition of inequality.

A. *Dog Bone Economy*

Many distribution theories achieve results by ignoring or trivializing the assignment problem. This leads to misinterpretation of empirical relations such as the earnings function. As an example of what can go wrong when the assignment problem is ignored, consider the following dog bone economy. The agents in this economy are n dogs kept in a pen. These dogs vary by weight, teeth, muscles, and tenacity, all observable. At the beginning of the day, a dump truck arrives with n bones, differing in size. The bones are dumped in a neighboring area. The Hicksian Day begins when the gate opens and the dogs go after the bones. At this point a nontâtonnement allocation process beings in which each dog can only hold onto one bone, losing the bone to any dog able to take it away. Equilibrium arises when each dog has a bone that is not wanted by any dog that could take it away, and when each dog prefers its own bone to any bone that it could take away from another dog. Hierarchical ordering of dogs and bones would eliminate cycling and guarantee the existence of equilibrium, but this assumption is unnecessary for the story.

After the dust has settled, an economist appears on the scene and collects data on the dogs and their bones. Bones can be rated by their value. The economist then runs a regression of the bone values as a function of dog characteristics and finds a strong relationship (let us say the R^2 is 0.80). This relationship is an earnings function, with the value of bones as earnings. Pleased with the results, the economist uses the estimated earnings function to predict the distribution of earnings the following day, when the dump truck will bring a new load of bones. For each dog, the economist can predict the dog's bone value on the basis of the dog's characteristics (weight, teeth, muscles, tenacity). From the distribution of dog characteristics, the economist tries to infer the distribution of bone values.

Will the economist succeed in predicting the next day's distribution of bone values? After all, the earnings function is fairly accurately estimated.

Of course, the distribution of bone values the next day does not in any way depend on the distribution of dog characteristics; it depends simply on what the dump truck brings. But if there is no relation between dog characteristics and the distribution of bone values, how did the economist achieve such an accurate relationship between bone values and dog characteristics? The earnings function estimated by the economist merely describes the assignment that arises between dogs and bones. This assignment is a temporary equilibrium that depends on the given distributions of dogs and bones. The predictive content is limited to identifying the bone that will be found with a given dog, given the distributions of bones and dogs. We can explain why one dog got one of the bones and why another dog got a different bone but we cannot draw any conclusions about the causal or technological relation between dog characteristics and bones.

The most important feature of this story is the illusion created by the success of the regression of bone values against dog characteristics. The nature of the earnings function is not apparent, and the influence of the distributions of bones and dogs on the earnings function is invisible. The dog bone economy presents an extreme case in that the bones are exogenously determined by what the dump truck brings rather than on any characteristics of the dogs themselves.

Yet the results of this economy conform to the economist's prior beliefs about the determinants of the bone distribution. The bone "earnings function" is consistent with a model in which a dog's characteristics determine the bone size it "earns." The existence of an assignment problem lying behind the empirically observed relationship is completely invisible.

This invisibility provides an example of the fallacy of composition. In thinking about the distribution of earnings, it would seem natural to begin with the explanation of a single individual's earnings. Given the economy, including the rewards for education, training, and other characteristics, this individual's earnings will depend only on his or her own characteristics. With the observed relationship between the individual's earnings and his or her characteristics, it would be possible to predict a change in earnings from any change in the individual's characteristics. In aggregating individual earnings to get the distribution of earnings, however, the economy, including returns to education and training, cannot be taken as given. The economy's rewards for various characteristics are endogenously determined and must themselves be explained by any distribution theory. In particular, the consequences for the earnings distribution of a change in the distribution of worker characteristics cannot in general be predicted from the change for a single worker. What constitutes a theory of the individual's earnings cannot automatically be extended to a theory of the distribution of earnings.

A first requirement of an earnings distribution theory is therefore to avoid the fallacy of composition involved in going from the earnings function to the distribution of earnings, or else to specify the conditions under which it is legitimate to do so. It is unnecessary to use an assignment model to avoid the fallacy of composition. But by specifying the determinants of the earnings function, assignment models accurately represent the interaction between supply and demand elements in shaping the distribution of earnings.

B. *Relation to Other Approaches*

Assignment models are closely related to other approaches to the study of inequality. They are consistent with structuralist theories in sociology, in which wage structures influence the wages associated with particular jobs (Mark Granovetter 1981; Arne L. Kalleberg and Ivar Berg 1987). As in assignment models, earnings depend on the characteristics of both the worker and the job. However, the structuralist theories do not assume competitive access to jobs. A major question then concerns how workers are matched to jobs (Aage B. Sørensen and Kalleberg 1981). Noncompetitive access to jobs, for example through rationing or segmentation, provides a route through which institutional structures can influence the distribution of earnings. Lester Thurow (1975) develops a similar model in which the wage rate is determined mainly by the job. This leads to an assignment in which workers queue for jobs based on trainability.

Sherwin Rosen (1974) develops a model of the determination of implicit prices of product characteristics. The resulting relationship between price and product characteristics, called an hedonic price function, is an envelope of buyer and seller offer curves. As in assignment models, this price function assigns consumers to producers in a market with heterogeneous products. The earnings function generated by the assignment problem (e.g., in Tinbergen's model) is essentially an hedonic price function in a labor market context—an hedonic wage function. Studies of hedonic wage functions are mainly directed towards estimating compensating wage differentials for job characteristics such as risk (Robert E. B. Lucas 1977b; Robert S. Smith 1979; Rosen 1986a). Ronald Ehrenberg and Smith (1991) provide an accessible exposition of how compensating wage differentials for risk (pp. 266–74) and education (pp. 314–18) are determined using an hedonic model.

An alternative expression for assigning workers to jobs is matching. Boyan Jovanovic (1979) develops a model in which the output from a specific worker-job match is distributed as a random variable that is initially unknown to the employer or worker. The model is used to explain turnover as information about productivity is revealed during job tenure. The matching literature is primarily concerned with ex post differences in the outputs obtained from worker-firm matches, whereas the assignment models discussed in this article emphasize ex ante differences among workers and firms. As productivities are not explicitly related to ex ante characteristics of workers or jobs in matching models, the approach is less useful in explaining the distribution of earnings, although it has been applied to the question of wage growth over a worker's career (Jacob Mincer and Jovanovic 1981), turnover, and unemployment (Jovanovic 1984) and returns to on-the-job training (John M. Barron, Dan A. Black, and Mark A. Loewenstein 1989).

Arguments about mismatches in the labor market, either with respect to location or skills, are based on simplified forms of assignment models (John D. Kasarda 1988; Levy and Murnane 1992, Section VII.B, review mismatch models). Technological change, in particular the advance of information-based industries, has shifted the skill requirements of jobs. At the same time, entering workers are failing to acquire these skills, leading to a mismatch between supplies and demands. The mismatch arguments implicitly regard skill requirements and supplies as unresponsive to economic incentives, at least in the short run, so that planning and intervention are necessary. In assignment models, these supplies and demands are not rigidly determined but respond to wage differentials. Steeper wage differentials would then resolve the mismatches that have been observed and forecast.

Assignment models tend to be highly abstract and mathematical, often using simplifying and unrealistic assumptions about workers and jobs to achieve analytical results. They have not so far generated a set of easily identifiable questions which can be answered by accessible empirical procedures. Further, they may divert attention from issues of household composition, income transfers, discrimination and social problems that have a more direct impact on poverty and inequality. However, they point the way to the steps necessary to incorporate demand and job choice in empirical models of earnings.

C. *Contents*

The next section discusses the extent of the assignment problem in the economy and the way that decisions of workers and employers generate assignment patterns. Section III presents three basic types of assignment models, depending on whether characteristics of workers and jobs are continuous or discrete. These types are the linear programming optimal assignment problem, the differential rents model, and Roy's sectoral model. Section IV compares these models with regard to the choices available to workers, wage determination, self-selection, and comparative advantage. Section V considers implications of assignment models for the decompositions that are used to study the distribution of earnings. These decompositions include analyzing the distribution by industrial or occupational sector, use of an earnings function, and human capital models. The conclusions in Section VI review the relations between assignment problems, self-selection, and comparative advantage. The section indicates the most important extensions of assignment models and explanations for changes in wage differentials as well as relevant research questions.

II. *The Economy's Assignment Problem*

A. *Existence*

What would the economy be like without an assignment problem? With only a single, observable skill, a worker would be able to get the same wage no matter which job he or she took. No specific training, education, diversity in skills or preferences would limit in any way the jobs that one would seek. Finding a job would be reduced to locating a firm with a vacancy.[2] Firms would be indifferent as to which workers they employed. Hiring would be reduced to the trivial problem of taking the first worker that came along. Unemployment would only arise if the number of workers exceeded the number of jobs. Wage differences among workers could arise, but all labor could be expressed in terms of the amount of an average or standard worker it was equivalent to. Professors at universities could be replaced by a sufficient number of high school graduates, presumably all lecturing at the same time. This very article could have been written by anybody, perhaps in less time.

But of course the economy does have an assignment problem. The size and importance of the assignment problem can be seen from the resources expended to solve it. Unemployment imposes large costs through forgone production, nonpecuniary costs, and uncertain incomes. Much of this unemployment arises from workers seeking jobs better than those readily available at the lowest wages, at least when depression conditions are absent. Firms spend substantial amounts through personnel departments in advertising positions and interviewing candidates. After employment, firms collect information about workers to facilitate later assignment within the firm through internal labor markets. Quits and layoffs by agents seeking better matches impose losses of specific training. Expenditures on screening and signals may arise because of the advantage of some assignments over others; they may also interfere with efficient assignments. The formation of specialized labor markets may arise in order to reduce the costs of assignment. Occupational segregation and segmentation, by distorting the assignment, impose efficiency losses on the economy as well as inequities in the treatment of individuals and groups.

Much empirical work supports the existence of an assignment problem. Joop Hartog (1985, 1986a, 1986b, 1988) estimates earning functions that include both individual and job characteristics, using data from the Netherlands and elsewhere. Hartog compares three models. Some versions of human capital models suggest that only individual characteristics should effect earnings, while job characteristics are the major determinants in segmented labor market theories. In assignment models, both sets of variables would be significant (in the ab-

[2] Job search by itself does not imply that an assignment problem exists. It is conceivable that there is only one skill, with marginal products proportional to the skill, but that this skill is imperfectly observed. Workers would search for employers who rated their skills more highly, and firms would search for workers whose skills were underrated. Changes in employment would affect the distribution of earnings but (for a given level of unemployment) not output.

sence of an exact correspondence between individual and job characteristics). Hartog finds that both individual and job characteristics affect earnings. Further, there are significant interactions between them, supporting the existence of an assignment problem.[3] Hartog (1977, 1980, 1981b) and R. E. B. Lucas (1974) identify significant ways in which jobs differ. Sattinger (1978) establishes the existence of comparative advantage among individuals using data on mechanical aptitude tests taken by secondary school students. The ratios of performance of pairs of individuals are computed for four tasks. For each pair, these ratios are then ordered from highest to lowest. In the absence of any systematic comparative advantage, one ordering will be as likely as any other. Using a chi-square goodness-of-fit test, the hypothesis of no systematic comparative advantage is rejected. Heckman and Guilherme Sedlacek (1985), in estimating extensions of Roy's model, show that differentials for education and experience are larger in manufacturing than in nonmanufacturing. Also, in a later paper (1990), they reject a simpler model with no assignment problem in which worker earnings would be the same in all market sectors.

Unequal wage structures among economic sectors provide indirect evidence of an assignment problem. Heckman and Jose Scheinkman (1987) establish that worker characteristics receive unequal rewards in different sectors of the economy, so that workers face a nontrivial choice problem. William T. Dickens and Lawrence F. Katz (1987) and Alan B. Krueger and Lawrence H. Summers (1988) also conclude that wage structures vary among industrial sectors.

Like other major allocative problems of the economy (such as what, how, and for whom), the assignment problem is not apparent to individual agents who are simply solving their own utility or profit maximizing problems. Employed and unemployed workers in an economy engage in job search, eliciting job offers until they find a satisfactory one. Employers typically interview a number of candidates for a job, seeking the most appropriate candidate. But out of these activities arises an assignment of workers to jobs. An assignment of workers to jobs can be defined as a listing of each worker together with the job he or she performs.[4] The next section examines how the decisions of individual agents solve the assignment problem facing the economy.

B. *Comparative Advantage*

Now consider the reasons why some assignments occur instead of others. One reason is comparative advantage.[5] Consider a fixed-proportions technology in which employers need to have a fixed set of tasks performed to yield a given level of production. Suppose workers do not have preferences for some tasks over others. Each job is associated with a particular task. Let a_{ij} be the number of

[3] As a specific example of interactions, Hartog (1985) estimates an earnings function with dummy variables for each combination of job level (or required education) and worker education. He tests and rejects a specification in which education and job level contribute independently to earnings. Educational differentials therefore depend upon the job.

[4] The analysis of this paper takes jobs as given. Rosen (1978) considers the subproblem of how employers arrange into jobs the tasks that they need performed.

[5] Application of comparative advantage to the analysis of labor markets is commonly attributed to Roy. Roy does not analyze his model in terms of comparative advantage but comments (1951, p. 145), "It should be apparent that the analysis attempted in this article bears some sort of affinity to the theory of comparative advantages. A situation has been examined in which individuals' comparative advantages in various activities differ widely." Sattinger (1975) applies comparative advantage to the study of the distribution of earnings and Rosen (1978) develops a general analysis of comparative advantage in labor markets.

times that worker i can perform job j's task per period. If

$$\frac{a_{11}}{a_{21}} > \frac{a_{12}}{a_{22}} \tag{1}$$

then worker 1 is said to have a comparative advantage at job 1 and worker 2 has a comparative advantage at job 2 (note that if (1) holds, then $a_{22}/a_{12} > a_{21}/a_{11}$).

Comparative advantage determines the assignment in a market system with this technology as follows. Suppose that in equilibrium the wage rate prevailing for worker i is w_i. The employer offering job j will seek to minimize the cost of getting the job's tasks performed, taking the wage rate as given.[6] Using worker i, the cost would be w_i/a_{ij}. Employer j will prefer to hire worker 1 instead of 2 whenever $w_2/a_{2j} > w_1/a_{1j}$, or

$$\frac{a_{1j}}{a_{2j}} > \frac{w_1}{w_2}. \tag{2}$$

From (1), it follows that we would never observe employer 2 hiring worker 1 when employer 1 hires worker 2: it is impossible for w_1/w_2 to be simultaneously greater than a_{11}/a_{21} and less than a_{12}/a_{22}. Depending on the wage rates, it is possible that both employers would prefer worker 1, or that both prefer worker 2. But the only assignment in which the employers prefer different workers is when employer 1 prefers worker 1 and employer 2 prefers worker 2. With this technology, the equilibrium assignment must be consistent with the comparative advantage relations as given by (1).

This example also shows how knowledge of the equilibrium assignment can explain wage differences. Suppose in equilibrium worker 1 is observed in job 1 while worker 2 is in job 2. Then the ratio of wages for the two workers must lie between the workers' trade-offs at the first job (i.e., the ratio of their performances) and their trade-offs at the second job:

$$\frac{a_{11}}{a_{12}} \geq \frac{w_1}{w_2} \geq \frac{a_{12}}{a_{22}}. \tag{3}$$

In this way, ratios of performances in the two jobs set limits within which the wage differential must fall.

The term "comparative advantage" is used in different ways by various authors. As defined using (1), comparative advantage arises whenever ratios of outputs for two workers are not identically equal in every job. Comparative advantage then establishes the existence of an assignment problem but one would need to know the direction of the inequality in (1) to determine which particular assignment comes about. An alternative relation is absolute advantage, which arises when a worker is better at a job than other workers. In terms of the outputs in (1), worker 1 has an absolute advantage at job j compared to worker 2 if $a_{1j} > a_{2j}$. If each worker has an absolute advantage at his or her own job, compared to any other worker and that worker's job, then comparative advantage must also be present in the sense defined in (1).[7] The

[6] Alternatively, fees could be offered for the performance of tasks, and workers could maximize their incomes. The resulting equilibrium wage rates for workers would still satisfy (2) and (3). In models in which an inexact assignment occurs (because of imperfect information) or in disequilibrium, the wage for a worker may depend on both the worker and job characteristics.

[7] While in simple economies it is possible that absolute advantage determines assignment (with each worker employed at the job at which he or she is best), this becomes unreasonable in large economies. For example, if there are one million workers, there would need to be at least one million different jobs, and in each job a worker would need to be better than nearly a million other workers. Even so, only for a very special set of wages would each worker choose the job at which he or she was best. However, Glenn MacDonald and James T. Markusen (1985) describe a technology with two activities in which absolute advantage (in the form of absolute skill levels) results in assignments that are not completely determined by comparative advantage, as in the scale of operations effect in the following section.

significance of comparative advantage is that a worker can still get a job even though he or she is worse at all jobs than other workers, i.e., even though absolute advantage is absent for that worker. Some economists find it useful to restrict comparative advantage to the case where absolute advantage is absent for some worker; this will be referred to as the standard comparative advantage case.[8]

C. *Scale of Operations Effect*

Some economists may believe that comparative advantage is the only production principle underlying the assignment of workers to jobs, but this is incorrect. As a counterexample, consider an economy in which a job is associated with the use of a particular machine that can be used by only one person at a time. Suppose the possible values of output (price times quantity) obtained per hour from the two workers at two jobs are as follows:

	Job 1	Job 2
Worker 1	\$35	\$20
Worker 2	\$20	\$10.

Here, worker 1 has a comparative advantage at the second job and worker 2 has a comparative advantage at the first job, because $\$35/\$20 < \$20/\$10$. However, the maximum value of output, \$45, is obtained when worker 1 is employed at job 1 and worker 2 is employed at job 2. In an eight hour day with this assignment, output would be \$280 at job 1 and \$80 at job 2.

Suppose we tried to reallocate labor according to comparative advantage. Suppose we put worker 2 in job 1 for eight hours and worker 1 four hours at job 1 and four hours at job 2. If this were possible, it would yield a net increase of \$20 from job 1. But it would require twelve hours of labor in job 1 during an eight hour day, which is ruled out by the assumption that only one worker at a time can be employed at a job. A worker occupies a job (or the machine associated with the job), preventing the reassignments indicated by comparative advantage.

The reason comparative advantage does not indicate the optimal assignment in this case is that earnings from a job are no longer proportional to physical output at the job. With cooperating factors of production (either explicit in the form of a machine or implicit via a scarcity in the jobs available), an opportunity cost for the cooperating factor must be subtracted from the value of output to yield the earnings.[9] Ronald H. Tuck (1954, p. 1) describes the resulting problem facing the economy as one of

> . . . assigning each individual member of the economy to work of an appropriate level of responsibility, and of doing this in such a way that the best possible use is made of the available human talent and experience.[10]

Basically, more resources (in the form of more capital, labor or greater responsibility) are allocated to workers with

[8] This restricted case is consistent with the example of comparative advantage worked out by David Ricardo in the context of trade (1951, p. 135). In that example, Portugal has an absolute advantage over England in the production of both wine and cloth. But there are still gains from trade because England has a comparative advantage in the production of cloth. The importance of comparative advantage is that it explains trade even when one country has an absolute advantage at both goods. If each country had an absolute advantage at one good, trade would be obvious.

[9] Some authors determine comparative advantage on the basis of earnings in different jobs or at different educational levels rather than on the basis of physical output. With cooperating factors, this approach is ambiguous.

[10] Tuck (1954) explains the distribution of firm sizes in terms of the distribution of productive resources among entrepreneurs. Robert E. Lucas, Jr. (1978) and Walter Oi (1983) also develop theories of the size distribution of firms that involve the assignment of resources to heterogeneous entrepreneurs.

greater abilities because the resources have a greater effect on output for those workers. In turn, with greater resources, output is more sensitive to the abilities of workers, raising wage differentials for workers with greater abilities.

The principle affecting the distribution of earnings has been developed in a number of contexts. Thomas Mayer (1960) and Melvin Reder (1968) use the term "scale of operations effect" in their models of the distribution of earnings (see also discussions by Reder (1969, pp. 219–23) and Sattinger (1980, pp. 32–35)). Mayer uses the term to describe the potential value of output, while Reder uses it for the value of resources under a person's control. Rosen (1981) applies the scale of operations effect to the incomes of superstars. George Akerlof (1981) develops an analogy between jobs and dam sites to explain why some workers might be unemployable. A productive dam that does not fully utilize a dam site may not be chosen because it prevents more productive dams from being used at the site. The dam site carries an opportunity cost that must be subtracted from a dam's output to determine whether it is suitable for the site. Stephen J. Spurr (1987) discusses how the scale of operations effect results in larger claims being assigned to lawyers of higher quality.

The scale of operations effect is also related to theories of compensation within hierarchies developed by Herbert A. Simon (1957) and Harold Lydall (1959; 1968, pp. 125–29). In a hierarchical model developed by Guillermo A. Calvo and Stanislaw Wellisz (1979), the effect of a supervisor shirking is that workers under the supervisor also shirk. This increases the sensitivity of the firm's output to workers' abilities as supervisors and leads a firm to place more able workers at higher levels in the hierarchy. In Calvo and Wellisz' model, the scale of operations effect provides a link between assignment models and efficiency wage models, in which firms pay above market wages in order to influence workers' productivities. Differences in efficiency wages can then be explained in terms of differences in the scale of operations rather than differences in costs of monitoring workers.[11]

With the scale of operations effect, the wage ratio for the two workers will not lie between the ratios of outputs as in the comparative advantage case because of the presence of opportunity costs from the use of a machine or the filling of a position or job. Consider now how wages are determined for workers in the context of a model in which the cooperating factor is capital, in the form of heterogeneous units called machines. Assume only one worker at a time can be combined with a machine. Let p_j be the price of a unit of output from machine j, and let a_{ij} be the output produced per period by worker i at machine j. Let w_i be the wage rate for worker i. The owner of machine j takes the wage as given and chooses the worker that maximizes the residual $p_j a_{ij} - w_i$ instead of the output values $p_j a_{ij}$ appearing in the example at the beginning of this section. If the owner of machine 1 is observed to choose worker 1 while the owner of machine 2 chooses worker 2, $p_1 a_{11} - w_1 \geq p_1 a_{21} - w_2$ and $p_2 a_{12} - w_1 \leq p_2 a_{22} - w_2$. Therefore:

$$p_2(a_{12} - a_{22}) \leq w_1 - w_2 \leq p_1(a_{11} - a_{21}). \quad (4)$$

The difference in wages must lie between the difference in the value of output produced by the two workers on machine 1, and the corresponding difference on machine 2. The assignment of worker 1

[11] Rosen (1982), Michael Waldman (1984b), David Grubb (1985) and Peter F. Kostiuk (1990) develop additional models that relate versions of the scale of operations effect to assignment and earnings within hierarchies.

to machine 1 and worker 2 to machine 2 can come about only if $p_2(a_{12} - a_{22}) \leq p_1(a_{11} - a_{21})$. If $p_2(a_{12} - a_{22}) > p_1(a_{11} - a_{21})$, only the opposite assignment could be observed in equilibrium (i.e., worker 1 at machine 2 and worker 2 at machine 1).

Alternatively, one could begin by assuming workers choose machines. Let r_j be the rental cost for machine j. Then worker i chooses j to maximize $p_j a_{ij} - r_j$. Again, if worker 1 is observed to choose machine 1 while worker 2 chooses machine 2, $p_1 a_{11} - r_1 \geq p_2 a_{12} - r_2$ and $p_1 a_{21} - r_1 \leq p_2 a_{22} - r_2$. Therefore:

$$p_1 a_{21} - p_2 a_{22} \leq r_1 - r_2 \leq p_1 a_{11} - p_2 a_{12}. \quad (5)$$

In this way, differences in wages and rents are determined symmetrically by the problem of assigning workers to jobs.

The technologies leading to the comparative advantage and scale of operations cases are very different. In the comparative advantage case discussed above, the tasks from different jobs are needed in fixed proportions and cannot be substituted for each other, but a particular job could be filled by an indefinite number of workers. Opposite circumstances hold in the scale of operations case. Output from different jobs is simply added together, so that there is perfect substitutability among the outputs of different jobs. However, with only one worker per job, more workers cannot be added to a job to make up for low output levels.

Clearly, there are many potential technologies relating worker and job characteristics to aggregate production. In these cases, the optimal assignment would not be determined by simple bilateral comparisons as in (1) and (2).

D. *Preferences*

In addition to production-relevant characteristics, preferences may also guide the assignment. In Tinbergen's original model (1951), workers prefer jobs with effort requirements that are close to their effort capabilities.[12] These requirements and capabilities are unrelated to production in the model. Let h_j be the effort requirement of job j and let g_i be the effort capability of worker i. Worker trade-offs between the wage and the effort requirement h_j are described by the following family of wage offer or indifference curves:

$$w_i = w_{0i} + a(g_i - h_j)^2. \quad (6)$$

Along a given wage offer curve, the lowest wage acceptable to a worker, w_{0i}, occurs when the effort requirement equals the effort capability of the worker, i.e., $g_i = h_j$. If the effort requirement is higher or lower than g_i, the worker must receive a higher wage in order to achieve the same level of utility. Higher values of w_{0i} yield higher wage offer curves and higher levels of utility, so that the worker chooses h_j to maximize $w_i - a(g_i - h_j)^2$.

With this assumption regarding preferences, workers with higher effort capabilities will always end up in jobs with higher effort requirements. This type of assumption (in which workers and jobs are matched on the basis of distance between characteristics) is useful in generating hierarchical assignments in other contexts, for example marriage (Gary S. Becker 1973; Lam 1988). The assumption in Tinbergen's model can be contrasted

[12] Tinbergen (1956) extends his model to multidimensional worker and job characteristics. He further considers a generalization in which the production side of the economy can be incorporated into the determination of the wage function (1956, pp. 170–71). In this case, wage differentials combine productivity differences and compensating wage differentials. In related work, Tinbergen develops a normative theory of income distribution in which a tax function is found that maximizes social welfare (1970); estimates an empirical model with discrete categories of labor distinguished by educational level (1975a, 1977); and estimates elasticities of substitution among educational levels as a means of explaining educational differentials (1972, 1974, 1975b).

with one in which workers all uniformly prefer jobs with higher values of some characteristic (or else all prefer lower values). For example, in the compensating wage literature, all workers may dislike a particular job feature such as riskiness, noise or distance to work but have different valuations of those characteristics. The unequal valuations lead to an assignment of workers to jobs.

Wage differentials in Tinbergen's model also differ in an important regard from wage differentials in the comparative advantage and scale of operations cases. If the distribution of worker characteristics exactly matches the distribution of job characteristics (so that $h_j = g_i$ if worker i gets job j), wage differences would be eliminated. Further, if workers end up in jobs with effort requirements below their capabilities, wages will need to be a decreasing function of capabilities in order to induce workers to take the jobs. This result would not arise with comparative advantage or the scale of operations effect as long as the worker characteristic contributes to production.

This section has shown how the profit or utility maximizing decisions of workers or employers generate an assignment of workers to jobs. The aggregate assignment problem will typically be invisible to individual agents, but their decisions may lead to a pattern of assignment that prevails throughout the economy. The next step is to examine how the problem of assigning workers to jobs generates wage differentials and the distribution of earnings among workers.

III. *Alternative Assignment Models*

The three assignment models developed in this section seem very different. The linear programming optimal assignment problem is a model of the conditions for an efficient assignment. The differential rents model explains wage differentials. Roy's model explains self-selection into occupations. The point common to all three models is that they explicitly formulate the assignment problem that must be solved in the economy. This problem enters as an intermediate step in the connection between worker characteristics and earnings. Because they are all linked by the explicit presence of an assignment problem, all three models exhibit common phenomena (such as conditions for an efficient assignment, wage differentials that depend on job assignments, and self-selection effects), although in different forms and with different emphasis.

A major difference in the models considered here is in their description of worker and job characteristics. In the linear programming optimal assignment problem, workers and job characteristics take discrete values, and in the differential rents model they are continuously distributed. In Roy's model, jobs are discrete (in the form of sectors), while worker characteristics are continuously distributed, so that many workers will end up in the same sector. These differences in modeling account for the outward differences in results.

The starting point will be the linear programming optimal assignment problem. This problem provides a very general model with which to analyze the economy's assignment of workers to jobs and its results have many features that are common to all assignment models.[13]

[13] Dale Mortensen (1988) reviews matching problems related to the assignment problem. In this literature, matches are formed through the voluntary actions of agents rather than as the solution to an aggregate maximization problem. David Gale and Lloyd Shapley (1962) analyze equilibrium in a matching market and present an adjustment process that would lead to a stable market structure based on preferences of agents on both sides of the market. Shapley and Martin Shubik (1972) and Becker (1973) analyze the same problem when one agent in the match can compensate the other for forming the match, for example through the wage in a labor contract. Alvin Roth and Marilda A. Oliveira Sotomayor (1990) review game-theoretic analysis of two-sided matching problems.

No restrictive hierarchical assumptions are made regarding workers or jobs. That is, there are no explicit parameters describing workers that would allow one to rank them with regard to skills. Further, no continuity assumptions are made regarding distributions of workers and jobs. Each worker's wage depends in a complex way on the outputs obtained from alternative assignments rather than on the marginal increase in output obtained by using more labor or slightly different labor. On the other hand, the linear programming assignment problem imposes some restrictive assumptions: there are equal numbers of workers and jobs, and they must be combined in fixed proportions, with one worker per job. By altering conditions in the model, one can generate the differential rents model that will be discussed in Section III.B. or Roy's sectoral model that will be discussed in Section III.C. This procedure will facilitate a comparison of various models.

A. *Discrete Workers and Jobs: The Linear Programming Optimal Assignment Problem*

Tjalling C. Koopmans and Martin Beckmann (1957) consider a linear programming optimal assignment problem in which economic activities are assigned to locations. The dual prices in the solution of the assignment problem then correspond to market determined profits and land rents. By changing the context of the assignment problem, one can consider how wages and machine rents (or profits associated with a job) are determined.

A linear programming optimal assignment problem arises as follows. Suppose there are n workers and n machines (with each machine corresponding to a job), and let a_{ij} be the value of output obtained by worker i at machine j. The problem is to find the assignment, with one worker per machine, that maximizes the sum of output values. The assignment problem is a special case of the general linear programming problem of maximizing a linear objective function subject to inequality constraints. As part of the simplex method of solving this problem there are simplex multipliers or dual prices associated with each worker and machine. Let w_i be the dual price for worker i and let r_j be the dual price for machine j. These dual prices have the following properties. If worker i is assigned to machine j in the optimal solution, then $w_i + r_j = a_{ij}$; otherwise $w_i + r_j \geq a_{ij}$. With the optimal solution, the dual prices exhaust the product. If workers obtain their income by renting machines at the prices given by the r_j's, then (ruling out ties) they would be led to select the machines they are assigned to in the optimal solution. The maximum income of worker i would be w_i. Similarly, if machine owners hire workers at the factor prices w_i, they would choose the workers assigned to their machines in the optimal solution, and the maximum income for the owner of machine j would be r_j. The dual prices w_i and r_j distribute income in such a way that the assignment problem is solved through the income maximizing behavior of individual agents. These dual prices perform as market prices and could arise from a competitive solution.[14]

In the solution of the optimal assignment problem, there is no expression showing the relationship between dual prices and any explicit characteristics of workers or machines, a relationship that would be analogous to an earnings function. However, it is possible to apply factor analysis to the matrix A formed from the outputs a_{ij} in order to infer character-

[14] Gerald Thompson (1979) investigates the relation between the prices generated by auctioning or bidding and the dual prices of the assignment problem.

istics of workers and machines.[15] With this factorization, outputs from matches can be represented as:

$$a_{ij} = \Sigma_{k=1}^{R} \lambda_k p_{ik} q_{jk} \tag{7}$$

where R is the rank of the matrix formed from the outputs a_{ij}, p_{ik} is the amount of the k-th latent property of worker i, q_{jk} is the amount of the k-th latent property of machine j, and λ_k is the weight for the k-th property. With this factorization, the k-th property of workers interacts only with the k-th property of machines in the determination of outputs.[16]

Suppose in the optimal assignment that worker i is matched with machine j and that worker c is matched with machine d. Then from the condition that the owner of machine d would not prefer worker i, $a_{id} - w_i \leq a_{cd} - w_c$, and from the condition that the owner of machine j would not prefer worker c, $a_{cj} - w_c \leq a_{ij} - w_i$. Combining these inequalities and using (7) yields:

$$\begin{aligned} \Sigma_{k=1}^{R}\lambda_k(p_{ik} - p_{ck})q_{dk} &\leq w_i - w_c \\ &\leq \Sigma_{k=1}^{R}\lambda_k(p_{ik} - p_{ck})q_{jk}. \end{aligned} \tag{8}$$

The inequalities in (8) show the upper and lower limits for the wage differences between worker i and worker j. The limits depend on the differences between the latent properties of the two workers, i.e., $p_{ik} - p_{ck}$ appears on both sides of (8). But the limits also depend on the machine properties q_{dk} and q_{jk}, which enter as weights, increasing the importance of some worker properties and decreasing the importance of others. The effect of worker properties on wages therefore depends on which jobs are performed in equilibrium. This result illustrates one of the central points about assignment models: a change in either the workers or jobs in the economy alters the assignment and the wage differentials that are observed.

The determination of limits for machine rents is exactly symmetric to the determination of wage limits:

$$\begin{aligned} \Sigma_{k=1}^{R}\lambda_k p_{ck}(q_{jk} - q_{dk}) &\leq r_j - r_d \\ &\leq \Sigma_{k=1}^{R}\lambda_k p_{ik}(q_{jk} - q_{dk}). \end{aligned} \tag{9}$$

In this expression, worker properties enter as weights for the importance of various machine properties.

The dual prices from the solution of the optimal assignment problem exhibit two forms of indeterminacy. Because agents choose partners on the basis of relative rewards, it is possible to shift all wages up by a given amount and all rents down by the same amount (or else all wages down and all rents up). In (8) and (9), limits are placed only on differences between wages or rents. In this model, the problem of assigning workers to machines determines relative wages and machine rents but not their absolute levels. The absolute levels of wages and rents are determined outside the assignment problem, perhaps by the availability of idle machines or workers. A second indeterminacy arises because individual wages and rents can increase or decrease within the limits in (8) and (9) while still leading to the same assignment. With continuous distributions of workers and jobs, as in the differential rents model

[15] See Sattinger (1984). Factor analysis refers to the factorization of matrices, i.e., representing a matrix as the product of other matrices. In the term factor prices, factor refers to factors of production such as labor or land.

[16] It is possible that R is less than the number of workers or machines, in which case the factorization represents the complete data more compactly in terms of the R underlying properties of machines or workers. Alternatively, one could use the factorization to approximate the data using fewer than R properties. Arrange the λ_k in decreasing value, and set $b_{ij} = \Sigma_{k=1}^{S} \lambda_k p_{ik} q_{jk}$. Then the matrix $B = (b_{ij})$ is the best rank S estimator of A, in the sense that among all rank S matrices with the same numbers of rows and columns, B minimizes the sum of the squares of differences between the entries of A and the entries of B. The main purpose of the factorization used here, however, is that it permits one to relate differences in wages to worker and machine properties.

of the following section, this indeterminacy disappears because the bounds for wage differences approach each other in the limit. The particular wages and rents that arise depend on the adjustment process and institutions that lead the economy to equilibrium.[17]

B. *Continuous Distributions of Workers and Jobs: The Differential Rents Model*

In Ricardo's analysis of rent (1951, p. 70), the difference in rents for two nearly similar tracts of land with unequal fertility will equal the difference in output on the two tracts, holding labor and capital constant. The absolute level of rents can be calculated from the condition that no rent is paid on marginal land for which cultivation yields only enough to pay for the capital and labor used. These principles can also be applied in the labor market. The wage differential associated with a particular worker characteristic can be calculated from the increase in output from changing that characteristic, holding everything else the same. While land is heterogeneous in Ricardo's differential rents model, though, both labor and jobs (or capital) are heterogeneous in the labor market. The wage differential therefore depends on the assignment of workers to jobs.

The differential rents model (Sattinger 1979, 1980) arises when the output in the optimal assignment problem depends on a single explicit characteristic of the worker and a single explicit characteristic of the job.[18] Under certain conditions, a hierarchical assignment arises in which more skilled workers perform jobs with greater resources. Hierarchical models of this type are interesting because they direct attention to an important feature of market systems, the tendency to reinforce and exaggerate differences among workers. With heterogeneous jobs, more skilled workers (who would perhaps have gotten higher earnings anyway) have their earnings boosted by being assigned to jobs with more capital, responsibility, or subordinates.

By imposing some conditions on the model considered in the previous section, it is possible to obtain the differential rents model discussed in this section.[19] Suppose that each job is associated with a unit of capital, called a machine, and suppose that each machine can be described by a single characteristic, its size, which measures the amount of resources or capital associated with the job. Let $a_{ij} = f(g_i,k_j)$, where g_i is a measure of worker i's skill (alternatively, capability, education, or ability), k_j is a measure of the size of machine j, and production $f(g,k)$ is an increasing function of g and k and has continuous first and second order derivatives. (It is not necessary for $f(g,k)$ to take the same functional form as the linear factorization in (8).) Now suppose that the numbers of workers and machines increase indefi-

[17] Vincent Crawford and Elsie Knoer (1981) and Alexander Kelso and Crawford (1982) analyze an adjustment mechanism in which employers make offers and workers then accept or reject them. Then the resulting solution is best from the point of view of the employers. Alvin Roth (1984, 1985) shows that the best allocation for one side of the market is the worst for the other.

[18] The assumption that workers can be described by a single skill or ability is counterfactual. Individuals have extremely diverse abilities at various tasks, and these abilities are only partially correlated. A measure of averages like an IQ may be stable but it will be a poor predictor of an individual's performance at any given task. Unfortunately, the simplifying assumption of a single worker characteristic may be confused with arguments about the existence of IQ, which are irrelevant to the issues considered here. Tinbergen (1956) constructs a model with continuous distributions of workers and jobs in which workers are described by multiple characteristics.

[19] Tinbergen's models (1951, 1956) and Sattinger's models of comparative advantage (1975) and compensating wage differences (1977) also assume continuous distributions of workers and jobs but cannot be derived from the same optimal assignment problem discussed in III.A.

nitely so that the values of g and k cover intervals. Let $G(x)$ be the proportion of workers with skill levels less than or equal to x, and let $K(x)$ be the proportion of machine sizes that are less than or equal to x.[20]

In this economy, aggregate output is obtained by summing the production from each match of a worker with a machine. In the absence of preferences, the efficient assignment will be the one that maximizes this aggregate production.

Consider now how the production function $f(g,k)$ together with the distributions $G(x)$ and $K(x)$ determine the relationship between wages and the skill level g. Let this relationship be represented by $w(g)$. The owner of a machine of size k^* will attempt to maximize the profits obtained from that machine. If the owner hires a worker of skill g, profits will be given by $f(g,k^*) - w(g)$. To decide whether this skill level maximizes profits, the owner would compare the increases in production from using a worker of greater skill with the increase in wages. If the increase in production is greater, the owner would choose a higher skill level. If the increase in production is lower than the wage increase, the employer would choose a less skilled worker. The owner has found the right skill level when the increase in production equals the increase in wages. Formally, maximization of profits for the machine owner implies the first order condition

$$w'(g) = \frac{\partial f(g,k^*)}{\partial g} \qquad (10)$$

where $w'(g) = dw/dg$. The term $w'(g)$ is simply the wage differential, the increase in wages from a given increase in the worker's skill level. The term $\partial f(g,k^*)/\partial g$ is the increase in output from using a worker of a higher skill level, holding machine size constant at k^*. This method of calculating wage differentials is similar to Ricardo's calculation of differential rents. Also, (10) is analogous to the familiar competitive labor market condition that the wage equals the marginal revenue product, only with an increment in skill replacing an increment in the number of workers.

The first order condition (10) does not by itself determine the wage function $w(g)$. In this economy, the effect of an increase in the worker's skill level, and the size of the wage differential, depend on which job the worker performs. For each value of g at which we wish to calculate the wage differential $w'(g)$, we would need to know the size of the machine k^* of the employer who hires that labor. This information is contained in the economy's assignment of workers to jobs.

Usually, to find the general equilibrium of an economy, one must determine simultaneously the prices and quantities that satisfy the equilibrium condition. In the context of an assignment model, this means finding both the wage function $w(g)$ and the assignment at the same time. As employers choose workers on the basis of the wage function $w(g)$, this would be analytically very difficult in the general case.

However, a number of simplifying assumptions make it possible to determine the assignment without first knowing the wage function. First, in the time period under consideration, the distribution of jobs or machines does not depend on the wage function $w(g)$. The number of jobs does not increase or decrease in response to a high or low profit. Because workers and jobs are each described by only one variable, the production function $f(g,k)$

[20] In this model, the distribution of machine sizes is taken as given. Akerlof (1969) considers a model in which capital is allocated to workers. Some workers are then structurally unemployed because their output will not cover the cost of capital.

may be such that only a simple hierarchical assignment can arise, i.e., one in which more skilled workers are employed at jobs with larger machines (an alternative would be that more skilled workers are employed at jobs with smaller machines).

The procedure for determining equilibrium is as follows. First, a tentative assignment is assumed (based on what one would expect from the technology). Then this assignment is used to derive the wage function $w(g)$. Finally, the tentative assignment together with $w(g)$ are checked to see whether they satisfy the second order conditions and whether any other assignment could arise.

In the model developed here, the tentative assignment is that more skilled workers will be employed at jobs with larger machines. With this assumption, the top n jobs will go to the top n workers. The n-th worker, in order of decreasing skill g, will be employed at the n-th machine, in order of decreasing machine size. The number of workers with skill greater than or equal to some level g is $1 - G(g)$. Similarly, the number of jobs with machine size greater than or equal to k is $1 - K(k)$. Setting these two amounts equal yields a relationship $k(g)$ which describes the machine size for the job assigned to a worker of skill g under the tentative assumption. Suppose, for example, that g_0 is such that thirty percent of workers have skill levels greater than g_0. Then thirty percent of jobs will have machine sizes greater than $k(g_0)$.

With the assignment determined, it is now possible to use (10) to find the slope of the wage differential $w'(g)$. Suppose we are interested in the slope of the wage differential at skill level g_0. From the tentative assignment, $k(g_0)$ is the machine size for the owner who chooses to hire the worker with skill level g_0. From (10), the slope $w'(g_0)$ equals the partial derivative of production with respect to skill, calculated at $k^* = k(g_0)$.[21] Formally,

$$w'(g_0) = \left[\frac{\partial f(g_0,k^*)}{\partial g_0}\right]_{k^*=k(g_0)}. \qquad (11)$$

This expression corresponds to the limits for wage differences in the optimal assignment problem in (8). With continuous distributions and only one characteristic for workers, it shows very simply that the wage differential for a worker with skill level g_0 depends on the assignment of workers to jobs. One needs to know the machine size $k(g_0)$ assigned to that worker in order to calculate the wage differential from (11).

One feature of this model is that machine rents are determined simultaneously with the wage function $w(g)$. Let $r(k)$ be the rent for a machine of size k. The machine rent is given by the residual obtained by subtracting the wage from production: $r(k) = f(k,g) - w(g)$. This factor price is treated as a rent instead of profits as it could be determined in a manner exactly symmetric to the wage function.

The validity of the tentative assumption can now be checked. The employer's second order condition for profit (or rent) maximization is that profits should be a concave function of the skill level g, i.e.,

$$\frac{\partial^2 f(g,k^*)}{\partial g^2} - w''(g) < 0 \qquad (12)$$

for $k^* = k(g)$, where $w''(g) = d^2w/dg^2$. It can be shown that this condition holds if the mixed partial derivative $\partial^2 f(g,k)/$

[21] It is important to realize that the partial derivative on the right-hand side in (10) is taken treating k^* as a constant. The value of the partial derivative is then found by substituting $k(g_0)$ for k^*. It would be incorrect to substitute $k(g)$ for k^* and then take the derivative since the condition (10) is derived for an employer with a fixed machine size k^*.

$\partial g \partial k$ is positive.[22] In turn, a positive mixed partial derivative arises when the effect of skill on production is greater at larger machines, i.e., $\partial f/\partial g$ is an increasing function of k.

The expression in (11) yields the slope of the wage function $w(g)$. One must integrate with respect to g to obtain the wage function itself. The resulting expression includes a constant of integration, an arbitrary parameter that determines the absolute level of all wages. The labor market process in which employers choose workers determines only relative wages (i.e., the wages of one worker in relation to wages of another worker) and not their absolute level. For example, in this model all wages could be shifted up by one dollar and the first order condition in (10) would continue to be satisfied. Because of the fixed proportions technology, in which one worker can only be used in combination with one machine, the marginal products of workers and machines are not defined. The share of output between workers and employers must therefore be determined by other phenomena.

In the model developed here, reserve prices of labor and capital determine absolute levels of wages and rents.[23] The reserve price of labor, p_w, is the minimum amount that workers must receive in order to be willing to work. If wages are below p_w, workers choose to remain idle or engage in some other activity rather than work. Similarly, owners of machines must receive p_r or else they will withhold their machines from production.

As with Ricardo's differential rents, the absolute levels of the wage and rent functions are determined by the conditions that hold for the last or marginal match. As one moves down the list of workers in order of decreasing skill, the machine size assigned to that worker in equilibrium declines, along with the level of production from the match, $f(g,k)$. In one possible outcome, the level of production declines to the sum of the reserve wage and reserve rent, $p_w + p_r$, while there are still workers with lower skill levels and machines of smaller size. Suppose the skill level when this occurs is g_m and the corresponding machine size is $k_m = k(g_m)$. If the wage $w(g_m)$ were greater than p_w, unemployed workers would bid the wage down to p_w. If the wage were less than p_w, then the rent $r(k_m)$ would be greater than p_r. Employers with idle machines would offer higher wages and accept lower rents until the wage again equaled p_w. The outcome of this adjustment process is that $w(g_m) = p_w$ and $r(k_m) = p_r$. These conditions are sufficient to determine the absolute levels of the wage and rent functions.[24] The conditions

[22] From (11),

$$w''(g) = \left[\frac{\partial^2 f(g,k^*)}{\partial g^2}\right]_{k^*=k(g)} + \left[\frac{\partial^2 f(g,k^*)}{\partial g \partial k^*}\right]_{k^*=k(g)} \left[\frac{dk^*}{dg}\right],$$

so rearranging yields:

$$-\left[\frac{\partial^2 f(g,k^*)}{\partial g \partial k^*}\right]_{[k^*=k(g)]} \left[\frac{dk^*}{dg}\right] = \left[\frac{\partial^2 f(g,k^*)}{\partial g^2}\right]_{[k^*=k(g)]} - w''(g).$$

The right side of this expression should be positive for the employer's second order condition for profit maximization to be satisfied. If $\partial^2 f(g,k)/\partial g \partial k$ is positive for all workers and machines, then the tentative assignment with $dk/dg > 0$ satisfies the employer's second order condition. Also, the second order condition would not hold with the reverse assignment, in which $dk/dg < 0$.

[23] Reserve prices of labor could be represented in the linear programming optimal assignment problem by the presence of extra "null machines" for which output equals labor's reserve price. A surplus of such machines would force their rents to zero. Machine reserve prices could be represented in a similar manner.

[24] Two other possible cases could arise. First, suppose that there are more workers than machines, and that production $f(g,k)$ is sufficient so that all machines can be used (i.e., $f(g,k)$ is always greater than the sum of the reserve prices, $p_w + p_r$). Suppose the smallest machine size is k_m and the corresponding

guarantee that output equals the sum of the wage and machine rent for the last or marginal match. It can then be shown that wages and rents exhaust the product for the nonmarginal, more productive matches.[25]

By making assumptions regarding the functional forms for production and the distribution of workers and machine sizes, it is possible to draw specific conclusions regarding the shape of the wage function and earnings inequality. For example, suppose $f(g,k)$ takes the Cobb-Douglas form $g^{\alpha}k^{\beta}$, and suppose skills and machine sizes are lognormally distributed with variances of logarithms σ_g^2 and σ_k^2, respectively. Then using (11) the wage function $w(g)$ takes the form

$$w(g) = Ag^{(\alpha\sigma_g+\beta\sigma_k)/\sigma_g} + C_w. \qquad (13)$$

where A is a constant and C_w is the constant of integration obtained when $w'(g)$ is integrated. This function will be concave, linear, or convex depending on whether $(\alpha\sigma_g + \beta\sigma_k)/\sigma_g$ is greater than, equal to or less than one. If $\alpha + \beta = 1$ and if $\sigma_k > \sigma_g$ (i.e., machine sizes are more unequally distributed than skills), then $w(g)$ will be convex.[26] The quantity $w - C_w$ will be lognormally distributed with variance of logarithms $\alpha\sigma_g + \beta\sigma_k$, a linear combination of the inequalities in skill and machine size distributions. If both workers and machines are unemployed, then (from the condition that labor and capital factor prices equal their reserve prices in the marginal match), C_w can be calculated as

$$C_w = \frac{\beta\sigma_k p_w - \alpha\sigma_g p_r}{\alpha\sigma_g + \beta\sigma_k}. \qquad (14)$$

This amount could be positive or negative. A larger value of C_w reduces earnings inequality. In this way, the separate influences of the production function, the distributions of skills and machine sizes, and the reserve prices can be found. As mentioned in the introduction, these influences will be unapparent. Only the wage function $w(g)$ will be observed, so that wages will appear to depend only on g.

The differential rents model and related hierarchical models explain why the

worker's skill level is g_m. Then unemployed workers bid the lowest wage rate down to p_w, so $w(g_m) = p_w$. The owner of machine size k_m gets the residual, so $r(k_m) = f(g_m,k_m) - p_w$. These two conditions determine the absolute levels of $w(g)$ and $r(k)$. In the other case, there are more machines than workers. At the last match, the machine owner gets p_r while the worker gets the residual.

[25] For any level of skill g_0, the total differential of output is:

$$df = \frac{\partial f}{\partial g_0}dg_0 + \frac{dk}{dg}dg_0\left[\frac{\partial f}{\partial k}\right]_{[k=k(g_0)]}.$$

Using (11) and the analogous expression for $r(k)$,

$$df = w'(g_0)\,dg_0 + r'(k(g_0))\,dk/dg)dg_0$$

so that the change in output equals the change in factor payments. With output equal to factor payments for the marginal matches, output will also be exhausted for matches involving higher values of g_0. Some other technical questions related to the differential rents model are whether the equilibrium exists, is unique and is efficient, in the sense of maximizing production net of reserve prices of labor and capital. Existence and uniqueness are established by construction: given the production function and distributions of workers and machines, the equilibrium is actually found, including the assignment $k(g)$ and the wage function $w(g)$. The efficiency of the assignment can be established from the assumption that production from a match is an increasing function of skill and machine size and that the mixed partial derivative is positive. The proof proceeds by supposing that another assignment maximizes production net of reserve prices. Then because the assignment is different, two workers and their machines can be found such that the more skilled worker is using the smaller machine. By switching machines for these two workers, even greater production can be obtained, contradicting the assumption that the alternative assignment maximizes production net of reserve prices. The contradiction proves that the hierarchical assignment is efficient.

[26] The wage function derived by Tinbergen (1951, 1956, 1970) displays the same flexibility. The slope of the wage function and whether it is concave or convex can be related to the distributions of worker and job characteristics. John Pettengill (1980) uses a similar methodology to examine the effects of unionization on earnings inequality.

distribution of earnings differs in shape from the distribution of abilities. A. C. Pigou (1952, p. 650) raises the question (now known as Pigou's paradox) of why the distribution of earnings is positively skewed when abilities are symmetrically distributed. This paradox presumes that earnings ought to be proportional to some single-dimensional measure of abilities. It can be resolved by recognizing that workers are engaged in many different activities: there is no single measure of ability that determines a worker's earnings. In the context of the differential rents model, one obtains a different distribution of abilities among workers depending on which machine is used. If every worker used the same type of machine, the distribution of earnings would take the same shape as the distribution of abilities, defined by worker outputs.[27] With unequal machine sizes, however, workers with greater skill levels are assigned to larger machines. This boosts their earnings above what they would be if everyone used the same machine. Because of the positive mixed partial derivative $\partial^2 f/\partial g \partial k$, differentials for skill (i.e., $\partial f/\partial g$) will be greater for more skilled workers. The distribution of earnings will not resemble the distribution of outputs at any one machine and instead will be positively skewed relative to any such distribution.

The model presented here incorporates some important elements of the economy. These include an explicit assignment problem and the role of cooperating factors, in this case capital as represented by separate machines. The main point is the expression for the wage differential in (11). But there are also several shortcomings of the approach. It is essentially a short-run model, taking the distributions of workers and jobs as given.[28] It assumes a very restrictive production technology in which only one worker can be combined with a machine. More generally, one would expect a production function in which there could be variable numbers of workers combined with the capital. The model relies heavily on calculus and on continuity assumptions that allow one to work with derivatives. Most importantly, because of the absence of any stochastic element, the model predicts an exact correspondence between skill and machine size, which is inconsistent with observed assignments.

C. *Discrete Jobs or Occupations: Roy's Sectoral Model*

The model developed in this section (commonly called Roy's model) differs from the previous section's model in that workers choose among only a few jobs or occupations instead of a continuum of jobs. Rather than each job being filled by only one worker, a subset of all workers can be found in a given job. Because of worker self-selection of jobs, the distribution of workers in a given job will differ systematically from the labor force as a whole.[29] These effects of worker choice

[27] Each worker's earnings would be the value of output minus the machine rent, which would be the same for all workers. The distribution of earnings would be the same as the distribution of output values but shifted to the left by the amount of the machine rent. These conclusions can be derived directly by setting $\sigma_k = 0$ in (13) and (14).

[28] The assumption of rigid distributions of workers and jobs is the same for both mismatch arguments and differential rents models. However, unlike mismatch arguments, there are no excess supplies or demands in differential rents models, as these are eliminated by the wage differentials.

[29] In an earlier article, Roy (1950) attempts to show that abilities combined multiplicatively, so that outputs and hence incomes are lognormally distributed. He uses data from manual occupations such as chocolate packing to test this hypothesis. The 1951 article arises from the realization that workers ". . . engaged in a particular occupation tend to be selected in a purposive manner from the working population as a whole" (1951, p. 135). Then the workers found in a specific occupation would not have the same distribution of abilities as the population as a whole would have in that occupation.

or self-selection make Roy's model particularly interesting to econometricians dealing with self-selectivity issues.

In the basic two-sector model developed by Roy, members of a simple economy must choose between catching rabbits or fishing for trout. A worker's income in a sector is proportional to the number of rabbits or trout caught. Unlike the model developed in the previous section, there is no restriction on the number of workers in a job or occupation. Workers can move from one job to another depending on the price of trout in terms of rabbits. The model is very rich in yielding a wide range of outcomes depending upon the relation between abilities in the two sectors. The prediction of observable distributions renders the model more applicable to empirical work than the previous section's model.

Roy's model can be represented as a special case of the linear programming optimal assignment problem discussed in Section III.A. With two sectors, the output value entries a_{ij} for worker i would be the same for all jobs in a given sector. In order for all output to go to the worker, with no subtraction of rent for the machine or job, it is necessary for the opportunity cost of filling a given job to be zero. This can be accomplished by assuming that there are more jobs in each sector than there are workers. ("Null workers," with zero outputs in each sector, can be added so that the number of workers equals the number of jobs.) Then the rent for each job will be zero and the wage rate for a worker choosing a job would equal the value of output. The matrix of output values (a_{ij}) will have rank 2. This representation demonstrates an important difference between Roy's and the differential rents models: with no scarcity of jobs in either sector, taking a job in either sector entails no opportunity cost so that assignment is based entirely on comparative advantage.

In Roy's model, there is no simple expression for the wage differential as in (11), showing the relevance of the assignment. However, as in that model, a worker's income is not a simple function of a skill measure but depends on which job the worker performs (i.e., rabbits or trout). In both models, aggregate output depends on the assignment of workers to jobs, and the assignment problem is solved by the income-maximizing decisions of agents. A shift in demand (e.g., an increase in the price of trout in terms of rabbits) can influence the distribution of earnings by increasing the relative earnings of trout fishers and leading some workers to move from catching rabbits to fishing for trout.

This section presents a graphical treatment of Roy's model.[30] Suppose there are two sectors, rabbits and trout. Suppose that if the entire population chose one of the sectors, the distribution of outputs (in terms of rabbits or trout caught) would be lognormal. Let σ_1^2 and σ_2^2 be the variances of logarithms of outputs in the rabbit and trout sectors, respectively, and let ρ be the correlation between the logarithms of a worker's outputs in the two sectors. Without loss of generality, suppose that amounts are more unequally distributed in the second sector (trout), so that $\sigma_1^2 < \sigma_2^2$. This corresponds to Roy's assumption that trout are more difficult to catch than rabbits. Three basic cases arise depending on the correlation between the two sectors.

i. *Case of* $\sigma_1/\sigma_2 \leq \rho \leq \sigma_2/\sigma_1$.[31]

[30] Heckman and Sedlacek (1985, 1990), James Heckman and Bo Honore (1990), and G. S. Maddala (1977, 1983) develop statistical versions of Roy's model. According to comments made to this author by Michael Farrell, Roy had developed a complete statistical model as the basis for his conclusions but did not include it in the 1951 paper. This is consistent with the detailed conclusions he reaches.

[31] Cases can also be described in terms of the covariance, given by $\sigma_{12} = \rho\sigma_1\sigma_2$. In Case i, $\sigma_1^2 \leq \sigma_{12} \leq \sigma_2^2$, while in Case ii $\sigma_{12} \leq 0$ and in Case iii $0 \leq \sigma_{12} \leq \sigma_1^2 \leq \sigma_2^2$.

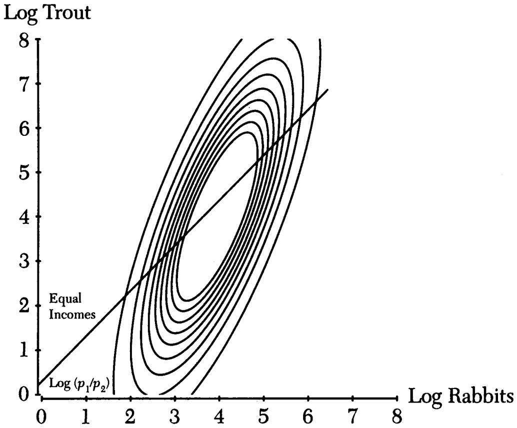

Figure 1. Contour Plot of Density of Sector Performances

This is the standard comparative advantage case. In this case, outputs are highly correlated, so that workers with higher levels of output in one occupation are also very likely to have higher levels of output in the other sector.

The effect of selection on the distribution of outputs in the two sectors and on the distribution of income can be seen in Figures 1 through 5. Figure 1 shows a contour plot of the distribution of worker performances in the two sectors. In this figure, it is assumed that the variance of logarithms of rabbits caught by the population, σ_1^2, is 1, while the variance of logarithms of trout, σ_2^2, is 4. The means of the logarithms of rabbits and trout are both 4.[32] Also, the correlation between sector outputs is assumed to be 0.75. The points on a given contour line in Figure 1 correspond to combinations of rabbits and trout such that the density of workers is the same.

Assume that the price of a rabbit is

[32] Although the logarithmic means are equal to 4 for both trout and rabbit skills, the means themselves are unequal. The mean of a lognormal distribution is given by $e^{\mu+\sigma 2/2}$, where μ and σ^2 are the mean and variance of logarithms (John Aitchison and J. A. C. Brown 1957). The means for rabbit and trout skills are thus 90 and 403.

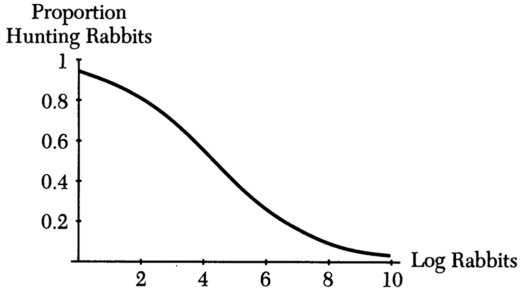

Figure 2. Proportion of Workers Choosing to Hunt Rabbits

1.2 while the price of a trout is 1, so that one rabbit is worth 1.2 trout. A worker chooses to hunt rabbits whenever 1.2 times his or her rabbit catch is greater than one times his or her trout catch. The 45-degree line sloping upward from the logarithm of the price ratio, 0.182, on the vertical axis shows all combinations of rabbits and trout that yield the same income. Any worker with a combination of rabbits and trout below this line would make greater income hunting rabbits. Any worker with a combination above this line would choose to fish for trout.

Figures 2 through 4 show how this assignment mechanism affects the distributions of workers observed hunting rabbits and fishing for trout. Figure 2 shows, from among workers who can catch a particular number of rabbits, the proportion that choose to hunt rabbits. In this case, the proportion hunting rabbits declines as the number of rabbits increases. The mean logarithm of rabbits caught among those choosing to hunt rabbits will therefore be less than the population mean of 4. Workers who can catch many rabbits are likely to have a comparative advantage at trout fishing, and therefore choose that sector. A better rabbit hunter would get only a small income advantage from his or her superior catch because the inequality in number of rabbits caught is relatively small. The number of trout

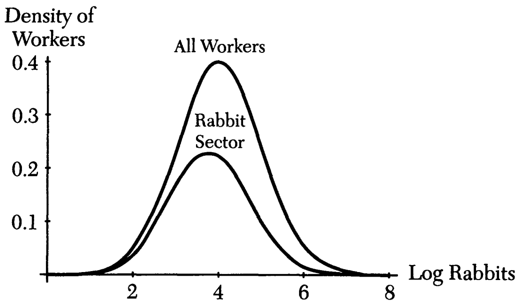

Figure 3. Rabbit Hunting Abilities Among All Workers and Among Rabbit Hunters

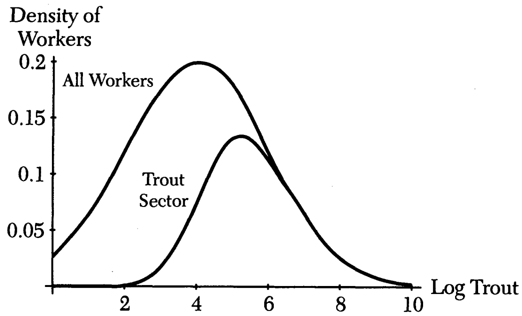

Figure 4. Trout Fishing Abilities Among All Workers and Among Trout Fishers

caught are more unequally distributed, so an above average performance in that occupation will yield a much higher income.

In Figure 3, the upper curve shows the distribution of rabbits caught by all workers (the vertical axis is the density of workers who catch a given number of rabbits). This upper curve is a lognormal distribution, so that the logarithm of rabbits is normally distributed with mean 4 and variance 1. The lower curve shows the density of workers by rabbits caught for those choosing the rabbit sector (this curve is not normalized; the area under the curve is 0.55, equal to the proportion of all workers choosing the rabbit sector).[33] At higher numbers of rabbits, workers are less likely to choose the rabbit sector because their income may be larger in the trout sector. Workers catching low numbers of rabbits, however, are likely to choose the rabbit sector.

Figure 4 makes the same comparison with respect to the number of trout caught. The upper curve shows the distribution of trout caught by all workers. The logarithms of trout are normally distributed with mean 4 and variance 4. The lower curve shows the density of workers by trout caught for workers choosing the trout sector. Nearly all those with high trout catches choose the trout sector, while those with low trout catches select the rabbit sector.

The distribution of income in the economy can be found by combining the lower curves in Figures 3 and 4, setting 1.2 trout equal to one rabbit and expressing income in rabbits. This is done in Figure 5. The upper curve is the distribution of incomes in the economy, obtained by summing the densities of workers by income for the two sectors. The lower curve on the left arises from the rabbit sector, while the lower curve on the right arises from the trout sector. This figure shows that the upper tail of the income distribution comes from workers in the trout sector, while the lower tail comes from workers in the rabbit sector. There is, however, substantial overlap in incomes from the two sectors: the assignment of workers to sectors is not entirely hierarchical, in the sense that some workers in the trout sector are

[33] This density is obtained by multiplying (for each number of rabbits caught) the proportion choosing the rabbit sector, given in Figure 2, times the density of all workers who can catch a given number of rabbits, given by the upper curve in Figure 3. Let $n(x_1,x_2;\sigma_1,\sigma_2,\rho)$ be the joint probability density function for the bivariate lognormal distribution, where x_1 is the number of rabbits, x_2 is the number of trout, and σ_1, σ_2 and ρ are the standard deviations for rabbits and trout and the correlation, respectively. Those who choose rabbits are workers for whom $x_2 \leq 1.2x_1$, so that at x_1 the height of the lower curve in Figure 3 is given by

$$\int_{x_2=-\infty}^{1.2x_1} n(x_1,x_2;\ \sigma_1,\sigma_2,\rho)\ dx_2.$$

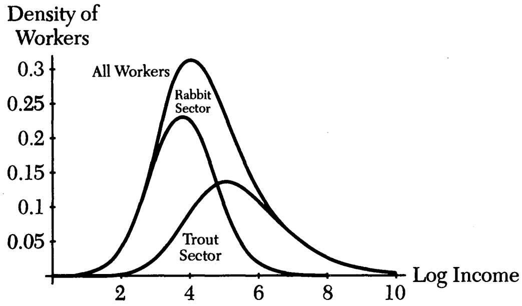

Figure 5. Aggregate and Sectoral Distributions of Income

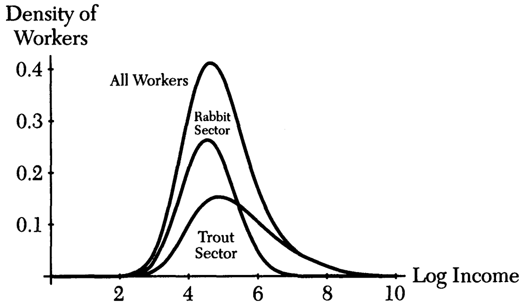

Figure 6. Aggregate and Sectoral Distributions of Income, Case ii with $\rho_{12} < 0$

less productive than some workers in the rabbit sector.

The aggregate income distribution tends to be more skewed to the right than a lognormal distribution as the right tail, from the trout sector, is more elongated than the left tail. The mean logarithm of income is 4.48, greater than the mean would be if there were no choice and all workers had to stay in one sector. Overall, income inequality, as measured by the variance of logarithms, is 2.03, greater than the population inequality in rabbit catches, $\sigma_1^2 = 1$, but less than the population inequality in trout catches, $\sigma_2^2 = 4$. The distribution of income resembles neither the distribution of abilities in catching rabbits nor the distribution of abilities in catching trout. As illustrated, it will tend to be more highly skewed than either of the ability distributions.

In this case, workers are assigned to sectors on the basis of comparative advantage. Workers who do well in a sector (i.e., rabbits) do not necessarily select that sector; instead they may select the other sector because they have a comparative advantage in it. Workers may select a sector (rabbits) even though they do badly in it because they have a comparative advantage in that sector. There is an implicit ranking of the sectors in that trout fishing is the sector at which better workers tend to have a comparative advantage.

ii. *Case of* $\rho < 0$.

This case arises when performances in the two sectors are negatively correlated, i.e., the better rabbit hunters tend to be the worse trout fishers. Those with worse performances in an occupation are more likely to choose the other occupation to earn their living. In this case, the assignment is roughly described by absolute advantage, which arises when workers in an occupation are better at that occupation than workers choosing the other occupation. Workers in an occupation tend to have higher outputs in that occupation than workers choosing the other occupation, although there will be exceptions. Worker choices lead to a simple assignment pattern: each occupation tends to be filled with the best workers in that occupation. The workers with the highest incomes will tend to be those with extreme performances, good and bad, rather than those with average or above average performances in both occupations.

Figure 6, corresponding to Figure 5, shows the aggregate and sectoral distributions of income in the case where the correlation between sector performances is -0.5, everything else the same.

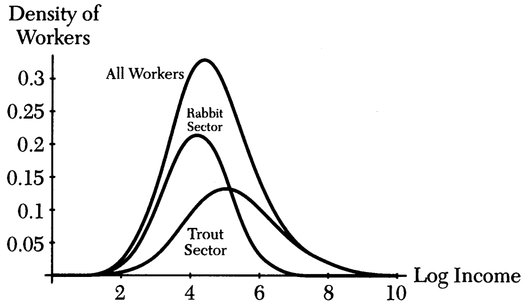

Figure 7. Aggregate and Sectoral Distributions of Income, Case iii with $0 < \rho_{12} < \sigma_1/\sigma_2$

The mean logarithm of income is 4.97 and the variance of logarithms is 1.31. Compared to case i, there are virtually no workers with logarithms of income below 2. As in Figure 5, though, the upper tail is dominated by workers in the high variance trout sector.

iii. *Case of* $0 \leq \rho < \sigma_1/\sigma_2$

This intermediate case arises when outputs in the two occupations are positively correlated but not as much as in the standard comparative advantage case in i above. Workers with better performances in the first sector are more likely to choose that sector, even though they also tend to be slightly better in the second sector. The importance of this case is that a positive correlation between sector performances does not necessarily generate the standard comparative advantage case.

Figure 7, corresponding to Figures 5 and 6, shows the aggregate and sectoral distributions of income for this case, assuming $\rho = 0.25$. In this case, the mean logarithm of income is 4.71 and the variance of logarithms is 1.76.

Comparison of Figures 5 through 7 for the three cases reveals a number of common features. In all three cases, the upper tail is dominated by workers in the high variance sector, trout. This effect stands out clearly because the variances in the two sectors were arbitrarily chosen to be so far apart. The aggregate distribution of income takes the same general shape in all three cases, with the largest inequality (as measured by the variance of logarithms) in the case where ρ is the highest. The lower tail is dominated by workers in the low variance rabbit sector. However, in the case with $\rho < 0$, some low income workers are also in the trout sector. Despite the assumption that a trout is worth less than a rabbit and that mean logarithms of performances are the same, average incomes are higher in the trout sector. The higher average income arises because of the high incomes going to workers in the upper tail of the trout ability distribution. The unequal variance between the sectors appears to play at least as strong a role as correlation between sector performances in shaping the distribution of income.

The listing of cases in this section shows that a variety of outcomes is possible depending on the correlation ρ. In particular, the standard comparative advantage case in i is not inevitable and is a special case of Roy's model.

Roy's model can also be used to illustrate how demand can influence the distribution of earnings and the division of workers between sectors. Table 1 shows the effects of changing the price of trout in terms of rabbits in case i. As the price of trout goes up, the proportion of workers selecting the rabbit sector declines, mean earnings increases and the variance of logs increases. Workers originally in the trout sector find their earnings boosted by the price increase, relative to workers in the rabbit sector, who have lower earnings on average. The effects can be seen in Figure 8, showing the distribution of earnings for two prices of trout, 0.5 and 1. The lower tail is unaffected because it arises from workers who stay in the rabbit sector. However, the upper tail is shifted to the right as work-

TABLE 1
EFFECTS OF TROUT PRICE CHANGES ON SECTORS

Price of Trout in Terms of Rabbits	Proportion Hunting Rabbits	Mean Earnings	Variance of Logarithms	Mean Rabbit Skill	Mean Trout Skill
0.5	0.688	229	1.66	73	1133
0.83	0.551	350	2.03	67	833
1.00	0.500	411	2.18	65	757

ers from the trout sector, who account for the upper tail, experience an increase in earnings from the higher trout price. In this case, the increase in the price of trout raises earnings inequality as measured by the variance of logarithms of earnings.

As the price of trout doubles from 0.5 to 1.0, average earnings of workers in the trout sector will not double. The reason sector earnings are not proportional to prices is that a nonrandom selection of workers moves from the rabbit sector to the trout sector in response to a trout price increase. This is demonstrated for Case i in Figures 9 and 10, which show worker movements in response to an increase in the price of trout from 0.83 rabbits (corresponding to 1.2 trout per rabbit) to 1.0 rabbits. This price change would be represented in Figure 1 by a shift in the "Equal Incomes" line downward from a vertical intercept of log (1.2/1) to log (1/1), through the origin. Figure 9 shows the proportion of workers leaving the rabbit sector as a function of the number of rabbits they can catch. As shown, workers with greater rabbit skills are more likely to leave that sector, thereby lowering the average skill level in the rabbit sector. Figure 10 shows how the movers compare with the workers already in the trout sector. The ratio of the number of entrants to current workers in the trout sector is greater at lower numbers of trout caught. After the price change, average skill levels in the trout sector will be lower. The average wage in the trout sector increases less than the price of trout.

The response of average skill levels to changes in rabbit or trout prices demonstrates an important feature of Roy's model, the aggregation bias that arises because of movements between sectors (Heckman and Sedlacek 1985, pp. 1107–10). Changes in average wage rates do not accurately reflect changes in the wage

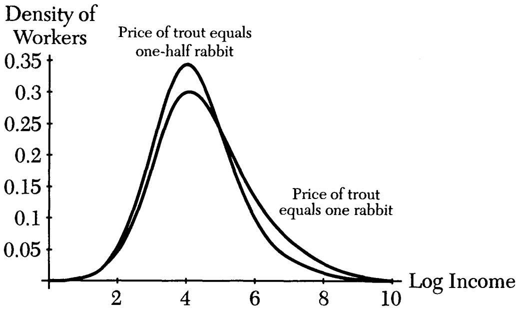

Figure 8. Shift in Earnings Distribution from Change in Price of Trout

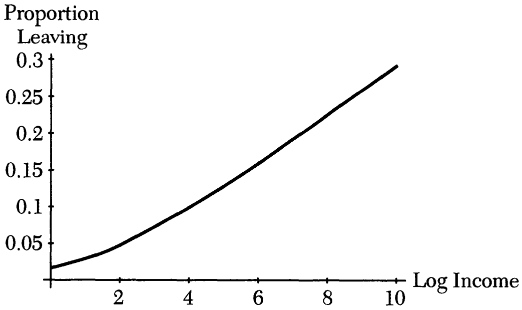

Figure 9. Proportions of Workers Leaving Rabbit Sector

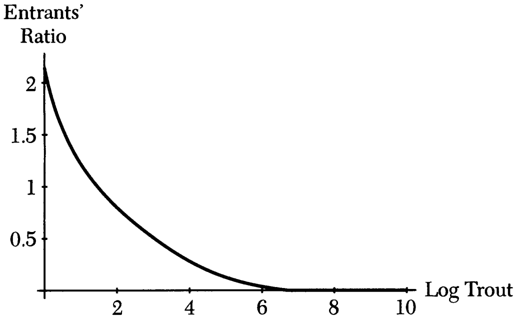

Figure 10. Ratios of New Entrants to Current Trout Fishers

rates for workers with given skill levels.

Heckman and Sedlacek (1985) estimate an empirical version of Roy's model using Current Population Survey data from 1968 to 1981. They assume that workers choose between two sectors, manufacturing and nonmanufacturing, justifying this division because manufacturing has been the focus of so much previous empirical work. This model is rejected by two test criteria that they propose. If wages are the product of tasks and task prices, and if the relation between tasks and skills does not change over time, the coefficients of skills in estimates of the logarithms of wages in successive cross-sections should be the same. This is referred to as the proportionality hypothesis and fails to hold for the estimated model. The second test is whether the residuals in the loglinear wage equation are distributed normally as assumed, and this is rejected using a chi-square goodness-of-fit test.

In response, Heckman and Sedlacek consider a multi-sector generalization but reject it because of the expense of estimating such models. Instead they extend Roy's model in four other ways. First, individuals are assumed to maximize utility instead of money incomes. Second, earnings are decomposed into hourly wage rates and hours of work that are freely chosen. Third, Heckman and Sedlacek develop a general non-normal model for the distribution of residuals which has Roy's lognormal assumption as a special case. Fourth, individuals are assumed to have a nonmarket or household production sector as an alternative to market work.[34]

With the assumption of utility maximization, preferences influence assignment and earnings in the extended model by leading workers to choose sectors that do not necessarily maximize earnings. In this way, the distribution of utilities, which generates sectoral choice, can differ from the distribution of task performances, which generates the earnings distribution.

One cautionary note concerning preferences in Heckman and Sedlacek's model is that their presence may be necessary to mimic sector-specific training. In the model, workers at each point in time are assumed to choose between the manufacturing, nonmanufacturing and nonmarket sectors. It seems likely that workers who have experience and training in a sector would have their productivity raised in that sector relative to what it would be in another sector. The training would be specific to the sector, just as specific training raises productivity only in the firm that gives it. An experienced worker in a sector could expect to get less in the other sector than indicated by the estimated task-skills relationship. The worker would then be much less likely to switch sectors than on the basis of predicted earnings alone. A suitable distribution of preferences would correct for the absence of assumed sector specific training.

Preferences could also be present in

[34] In a later article, Heckman and Sedlacek (1990) try to determine which extensions to Roy's model are most important in improving its goodness of fit. They find that the existence of a nonmarket sector is more important than allowing for departures from lognormality.

lieu of search by workers. With sectoral choice made through search (and with many sectors), workers do not usually end up in the sector that absolutely maximizes their earnings. In the absence of assumptions that workers find sectors through search, the distribution of preferences could substitute for the random wage outcomes of search.

Heckman and Sedlacek find that education has twice as strong an effect in manufacturing as in nonmanufacturing. Wages grow much more rapidly with work experience in manufacturing than in nonmanufacturing. These results show that it would be incorrect to apply one earnings function to all workers independent of sector.[35] The model leaves unexplained the shapes of the wage functions in the two sectors. What aspect of the manufacturing sector causes its wage function to differ from the wage function in the nonmanufacturing sector? The wage functions in each sector could themselves be hedonic wage functions, arising from assignment problems within each sector. It seems unlikely that all workers in the manufacturing sector are perfect substitutes for each other, as required for the efficiency units assumption. Also, it is not clear that workers choose manufacturing versus nonmanufacturing sectors instead of occupations.

The variance of the error term in nonmanufacturing is greater than in manufacturing. Heckman and Sedlacek find this to be consistent with greater heterogeneity in the industries classified as nonmanufacturing. Because preferences and not earnings determine sectoral choice, the results of the extended model do not conform exactly to one of the cases discussed earlier in this section. In particular, the correlation ρ is not identified in this model because of the presence of unobservables. However, education and experience positively affect both manufacturing and nonmanufacturing tasks, so performances in the two market sectors would be positively correlated in the absence of other skill related variables.

Heckman and Sedlacek estimate the effect of self-selection on inequality in Roy's model by comparing the observed earnings inequality with the level that would arise if workers chose sectors randomly. They find that self-selection decreases the variance of logarithms of wages within each sector, moves the mean wages in the two sectors closer together, and reduces the variance of logarithms of wages in the economy by 11.6 percent.[36]

In a later article, Heckman and Sedlacek (1990) examine extensions to the Roy model in more detail. Heckman and Honore (1990) analyze statistical properties and the empirical content of Roy's model. Although the statistical analysis used in all of these models appears to be very difficult, the work establishes Roy's model and its extensions as a practical way to apply assignment models empirically.

IV. *Comparisons and Extensions*

A. *Choices*

The three models discussed in the previous section share a number of features in common. First, of course, is the existence of multiple sectors. Workers therefore face a choice of sectors, or else employers in each sector face a choice of workers. From the point of view of

[35] Heckman and Sedlacek (1990, p. S353) explicitly test whether the observed wage distribution could be explained by a model with only a single production market, so that workers only decide whether to work or not and get the same wage no matter where they work. Such a model is rejected.

[36] James Heckman and Bo Honore (1990) prove that in the Roy model, self-selection reduces inequality compared to random assignment to sectors.

the economy as a whole, the existence of multiple sectors entails an assignment problem.

In the simplest case, where workers are described by a single characteristic, multiple sectors arise because the output in some jobs is more sensitive to that characteristic than others. In the differential rents model, the effect of worker skill on output, $\partial f/\partial g$, is an increasing function of the size of the machine. As in other scale of operations models, an efficient assignment requires that more skilled workers be assigned to jobs with more resources, either capital or responsibility. Alternatively, jobs may differ by a single parameter of difficulty, which measures the sensitivity of the job to the worker skill. This sensitivity to worker skill is also present in Roy's model. If performances in the two sectors were perfectly correlated (so that one could predict performance in one sector from the performance in the other sector), workers would still face a choice between rabbits and trout. With unequal variances of performances, better rabbit catchers would choose to fish for trout as that sector is more sensitive to their skill levels.

Even if jobs did not differ in their sensitivity to worker skills, multiple sectors would arise because of the great variety of tasks performed in the production of goods and services and the diversity of human performances at those tasks. Multiple sectors provide workers with choices. Both the existence of choice and the features of those choices affect the distribution of earnings. The discussion of Roy's model in Section III.C. emphasizes the features of those choices, such as the unequal variances and correlation between the two occupational performances. An alternative way to view Roy's model is by comparing its outcome with the outcome of a single sector model, in which the distribution of outputs would be identical in shape to the distribution of earnings. Suppose the second sector has the same distribution of outputs as the first sector (equal variances and zero correlation). Then the addition of a second sector provides workers with a choice and alters the distribution of earnings from its shape in the single sector case. It is clear that the addition of more sectors would alter the distribution further. But the effect of additional choice is not apparent in Roy's model as in the alternative cases considered in Section III.C., the number of sectors stays the same at two. Benoit Mandelbrot (1962) and Hendrik S. Houthakker (1974) develop models that partly explain how choice among many sectors, in the absence of unequal variances, affects the distribution of earnings.

Mandelbrot seeks to show how several occupations can have different exponents for Pareto upper tails. As a simplified version of Mandelbrot's model, suppose that workers have vectors of aptitudes given by $(y_1, \ldots, y_n)$. Assume each aptitude follows a Pareto distribution, with probability $(y/y_0)^{-\alpha}$ that income is greater than or equal to y, where y_0 is the lowest aptitude and α is the same for all aptitudes.[37] If the sector's offers are proportional to only a single aptitude, then the offers will follow the Pareto distribution. Accepted offers will differ from the Pareto distribution because lower offers are likely to be rejected. But the higher offers are more likely to be accepted, so in the upper tail accepted offers will again resemble the Pareto distribution. Now suppose a sector weights two aptitudes heavily. To be specific, suppose a firm offers a wage equal to $A\hat{y}$ whenever the lower of the two aptitudes is $\hat{y}$, where

[37] Instead of the Pareto distribution, Mandelbrot adopts a somewhat weaker assumption regarding aptitudes. They are assumed to follow the weak law of Pareto, so they asymptotically resemble the Pareto distribution in the upper tail.

A is some constant.[38] Multiplying the two cumulative distribution functions together, the likelihood that an offer equal to or exceeding $A\hat{y}$ is made is $(\hat{y}/y_0)^{-2\alpha}$. The distribution of wages for this occupation will asymptotically resemble a Pareto distribution but with parameter 2α instead of α. Similarly, a sector for which the offer depends on c aptitudes being simultaneously large will have a wage distribution which in the upper tail will resemble a Pareto distribution with parameter $c\alpha$. In this way different sectors can have wage distributions which are asymptotically Paretian but with different parameters.

In this economy, the very high offers go to workers in sectors that weight a single aptitude highly. Sectors requiring two or more aptitudes do not make many high offers. The workers getting the highest wages are those who are extremely good at a single skill that is crucial to a sector rather than workers who have a high average of aptitudes. The distribution of offers with the lowest Pareto coefficient (corresponding to the greatest inequality) will come to dominate the upper tail of the entire earnings distribution, which will then resemble a Pareto distribution with coefficient α.

Houthakker's model is similar to Mandelbrot's in that a worker has a vector of n occupational aptitudes and chooses the occupation that yields the highest income. Houthakker shows how the distribution of earnings can be derived from the joint distribution of aptitudes. The individual has a vector of occupational aptitudes $(y_1, y_2, \ldots, y_n)$ which varies randomly with cumulative density function $F(y_1, y_2, \ldots, y_n)$. An individual selects the occupation that maximizes income, z. If the prices of aptitudes are each unity, the cumulative density function of incomes will be given by $F(z, z, \ldots, z)$, the probability that each aptitude will be less than or equal to z. The resulting distribution of income will differ in form from the distributions of individual aptitudes, and Houthakker illustrates this general result with examples using the bivariate exponential and bivariate Pareto distributions.

B. *Wage Differentials*

In a single competitive labor market, the wage rate is determined by the familiar condition that quantity supplied equals quantity demanded. The supply and demand curves for a single labor market, however, do not show their dependence on substitution of labor or jobs from other labor markets. With many closely related labor markets, as in assignment models, the demand curve for a particular type of labor is determined by the cost of hiring alternative types of labor. Wage determination in assignment models generally takes the form of conditions imposed on wage differentials. These conditions are expressed differently in the three assignment models that have been considered but are all essentially generated by trade-offs in production that arise from varying skill levels. The conditions are shown in (3) for the comparative advantage case and (4) for the scale of operations case. The role of trade-offs in production is clearest in the differential rents model. In (11), the wage differential equals the effect of an increase in the skill level g on output, holding the size of machine constant at the level corresponding to the equilibrium assignment. In the optimal assignment problem, the limits in (8) are the differ-

[38] This functional form for the offer would arise if the worker's productivity is a fixed-proportions function of the worker's aptitudes. For example, if a job required a worker to perform two tasks simultaneously, the productivity would be proportional to the minimum of the two aptitudes. Mandelbrot derives his results under much more general conditions and assumes that offers are a nonhomogeneous form of the independent aptitude factors (1962, p. 61).

ences in the two workers' outputs at machines d and j.

The role of trade-offs in production is less clear in Roy's two-sector model as there are no wages associated with workers. However, by defining workers' wages as equal to their incomes, it is possible to derive an analogous result.[39]

An important conclusion arising from the determination of wage differentials in all three assignment models is that prices or values of worker characteristics are not uniform across the economy. Finis Welch (1969) raises the issue of uniform skill prices and develops a model in which production costs depend on aggregate combinations of skills, so that skill prices can be equalized across sectors. Rosen (1983) and Heckman and Scheinkman (1987) describe conditions under which uniform skill pricing will not arise.

Assignment models provide a direct explanation of unequal skill prices in an economy. For example, in Roy's model, suppose the two skills are rabbit hunting and trout fishing. If a worker chooses to fish for trout, the price of the rabbit skill is zero and the price of the trout skill is given by the trout price. A worker catching rabbits similarly receives a zero reward for any trout fishing skill. In the differential rents model, the value of an increment in the worker skill g depends on the size of the machine assigned to that worker in equilibrium. In the optimal assignment problem, the value placed on differences in worker characteristics in (8) depends on the characteristics of workers' machines. Unequal wage structures in sectors of the economy are therefore a direct outcome of the existence of an assignment problem.

C. *Self-selection*

In all three of the assignment models considered here, self-selection is the mechanism that is used to bring about the assignment. Workers select a sector or job, and thereby assign themselves to it, when it offers them greater income or utility than any other sector. This selection criterion results in a distribution of performances within the sector that differs systematically from the distribution for the population as a whole. These selection corrections have been worked out for the two-sector Roy model but not for the general multisector case. Self-selection is not a necessary feature of assignment models, however, and is only one of a few mechanisms that may operate in the economy to assign workers to jobs. Self-selection requires that a worker have complete information about potential earnings or utility in each sector. This is reasonable when there are only two sectors. But the assumption of complete information becomes unreasonable when there are many sectors with little guide to the worker as to which one is most suitable. Once one abandons full information and self-selection, the information structure plays a role in determining the feasible assignment mechanism, the resulting assignment and wage differences.

Studies of worker behavior suggest that workers engage in search to find jobs. In the standard search model, workers need to know only the distribution of wage offers among jobs and not the

[39] Let p_j be the price of output in sector j and let a_{ij} be the output of worker i in sector j. Let $w_{ij} = p_j a_{ij}$, the amount that worker i would get if he or she chose sector j. If worker 1 chooses sector 1, then $w_{11} = p_1 a_{11}$ and $w_{12} = p_2 a_{12} \leq w_{11}$. If worker 2 chooses sector 2, then $w_{22} = p_2 a_{22}$ and $w_{21} = p_1 a_{21} \leq w_{22}$. Then

$$\frac{p_1 a_{11}}{p_1 a_{21}} = \frac{w_{11}}{w_{22}} \geq \frac{w_{11}}{w_{22}} \geq \frac{w_{12}}{w_{22}} = \frac{p_2 a_{12}}{p_2 a_{22}}$$

$$\text{so that } \frac{a_{11}}{a_{21}} \geq \frac{w_{11}}{w_{22}} \geq \frac{a_{12}}{a_{22}}.$$

This is the same relation as in (3) in Section II.B., even though Roy's model uses a different technology and assumes workers choose sectors instead of employers choosing workers.

wage corresponding to each job. A worker selects a reservation wage and accepts the first job offer with a wage that equals or exceeds it. The selection criterion under search would appear to be much simpler than under self-selection. A worker observed to be in a sector (or job) gets a wage that exceeds the reservation wage but that does not necessarily exceed the wage in every other sector (or job). Then the income or utility in a given sector needs to satisfy only one bilateral comparison, i.e., with the worker's reservation income or utility.[40] An alternative approach is to assume that in each industry or sector, workers make a choice between that sector and a composite sector consisting of all other sectors. Maddala (1983, pp. 275–78) considers additional methods that rely on multiple binary-choice rules or on applications of order statistics.

With search present, the process of assigning workers to jobs itself generates some additional sources of inequality (Sattinger (1985) analyzes the distributional consequences of job search). Workers suffer unequal amounts of unemployment in their search, contributing to inequality in earnings and welfare. Workers with identical characteristics may get unequal wage rates because of the random outcomes of search, further contributing to inequality. Workers can influence their expected wage rates and likelihood of unemployment through their choice of reservation wages, providing an alternative source of inequality. On the other hand, search alters the assignment in such a way that higher skilled workers end up, on average, with fewer resources than in an exact, self-selection assignment, possibly reducing their earnings and overall inequality.

A further consequence of departing from the mechanism of full information self-selection is that there will be a demand for information about worker or job characteristics. MacDonald (1980) develops a model of person-specific information in which there are two types of workers and two types of firms. The type of a worker cannot be directly observed, but workers can invest in generating information about their own types. Neither worker type has an absolute advantage at both types of firms, so any information leads to improved worker-firm matches. The value of information in the labor market leads firms to offer higher wages to workers who can provide information about their types. All workers then receive a return to information investment. In later articles, MacDonald (1982a, 1982b) considers how information enters the production function through the assignment. A continuum of tasks is assigned to workers of two types based on the quality of information, and output rises when the quality of information increases.

Models of signaling, filters, and screening (Kenneth Arrow 1973; A. Michael Spence 1973) show that worker investment in information about themselves could simply give them a competitive advantage in the labor market. Workers get a private return to informational investments that exceeds the social returns. With an assignment problem in the economy, however, informational investment can yield a social return that equals or exceeds the private return, even though it does not change worker productivity (e.g., Arrow's model with two types of jobs; 1973, p. 202). In MacDonald's model (1980), there are no externalities from information about worker

[40] The degree of simplification depends on how the reservation wage is determined. If it is exogenous (an unknown function of the worker's characteristics, the same for all workers, plus a random element), then the simplification is straightforward. If the reservation wage is determined endogenously, as the solution to the worker's optimizing problem, then the econometrics could be as complicated as the self-selection procedures.

types as in signaling or screening models so that the social returns equal private returns. Waldman (1984a) and Joan E. Ricart i Costa (1988) develop models in which the worker's assignment itself acts as a signal to other firms, leading to possible inefficient assignments.

Investment in information generates a pattern of mobility over time as well as effects on the life-cycle earnings profile. Hartog (1981a) develops a two-period model in which wages in the first period are based on signals and in the second period on capabilities. He shows that dispersion in signal classes increases over time, and more capable individuals experience higher earnings growth.

The structure of information also affects the amount of earnings inequality. Michael Rothschild and Joseph Stiglitz (1982) develop a model in which a worker's output is maximized when placed in a job level corresponding to the worker's ability. However, the worker's ability depends on both observed and unobserved characteristics, so the job placement depends on the worker's expected ability level. A given observed characteristic is then related both directly to production and indirectly, through its correlation with unobserved characteristics. Firms may be unable to distinguish direct and indirect effects. When few characteristics are observable, expected ability levels do not vary greatly, and workers are placed in narrowly varying job levels. When many characteristics are observable, expected ability levels vary greatly and workers are placed in widely varying job levels. With more observable characteristics, both the expected wage and the variance of wages are greater.

Tournaments may be regarded as a mechanism of assigning workers to hierarchical levels in a context in which worker abilities are revealed through competition (Edward Lazear and Rosen 1981; Rosen 1986b). As performances depend on effort, large prizes are required for workers in the top ranks to maintain incentives to compete.

Labor markets provide another mechanism for assigning workers to jobs. Instead of one big labor market, submarkets arise based on observable characteristics of workers and jobs. Without an assignment problem (at least spatially), the existence of submarkets would serve no economic purpose. Screening, job market signaling, dual labor markets, and occupational segregation may determine assignment through restricted access to jobs for some workers, resulting in assignments that are less efficient than possible given the constraints of costly and incomplete information (Dickens and Kevin Lang 1985; Insan Tunali 1988; T. Magnac 1991).

D. *Comparative Advantage*

Comparative advantage is not necessarily present in the three models considered here, and it does not necessarily determine the assignment. In the linear programming optimal assignment problem, comparative advantage will be absent if the matrix of output values (a_{ij}) has rank one. Then each column will be a scalar multiple of any other column. The ratios of output values for any two workers, a_{1j}/a_{2j}, will be the same no matter which machine they use. Despite the absence of comparative advantage, an optimal assignment will arise in which more productive workers are assigned to more productive machines, according to the scale of operations effect. If the matrix has rank two or more, then comparative advantage must arise but it will not be the only determinant of the assignment.

In the differential rents model, comparative advantage will be absent whenever $f(g,k)$ is multiplicatively separable (i.e., it can be written as a function of g times a function of k). For example, suppose $f(g,k) = g^{\alpha}k^{\beta}$ as in the Cobb-Douglas production function. Then the ratio of output values for workers 1 and 2 will be $(g_1^\alpha/k^\beta)/(g_2^\alpha/k^\beta) = (g_1/g_2)^\alpha$, an amount that does not depend on the machine size. Comparative advantage will therefore be absent. The optimal assignment will require larger machines to be combined with more skilled workers as in the scale of operations effect. If $f(g,k)$ is not separable (e.g., as in the constant elasticity of substitution production function with elasticity unequal to one), then comparative advantage will arise.

Only in Roy's model does comparative advantage determine the assignment of workers to jobs. Because the value of output in each sector equals the worker's earnings, worker self-selection leads to an assignment that is consistent with comparative advantage in the sense defined in (1). With this general definition, no restrictions are placed on the correlation between performances in one sector and in the other. The standard comparative advantage case, in which absolute advantage is absent (case i in Section III.C.), is only one of three possible cases.

The basic reason comparative advantage determines assignment in Roy's model but not in the others is that cooperating factors such as capital play no significant role. In the linear programming optimal assignment problem and the differential rents model, the value of output is divided between labor and the employer so that wages are no longer proportional to output. In Roy's model, however, the value of output goes entirely to the worker.

The absence of any role for cooperating factors of production is not an inherent feature of Roy's model. Capital can be incorporated into Roy's model as follows. Suppose that within a sector, workers perform tasks that are an input together with capital. Suppose output in a sector is given by

$$Q = Q(n,K,T) \qquad (15)$$

where Q is total output per period in the sector, n is the number of workers, K is the amount of capital, and T is the total number of tasks performed. Assume Q has continuous first and second order derivatives. If n does not appear as an argument of Q in (15), then the marginal product of a worker will be $t_i\partial Q/\partial T$, where t_i is the number of tasks performed by the worker (this is essentially Heckman and Sedlacek's assumption (1985, p. 1080) in their derivation of Roy's model). Potential earnings in a sector will continue to be proportional to tasks as in Roy's standard model.

However, suppose n appears in (15) and suppose further that Q has constant returns to scale in all three arguments. Then

$$\begin{aligned} Q(n,K,T) &= nQ(1,K/n,T/n) \\ &= nh(\rho,\gamma), \end{aligned} \qquad (16)$$

where $\rho = K/n =$ capital per worker, $\gamma = T/n =$ average tasks per worker, and $h(\rho,\gamma) = Q(1,K/n,T/n) =$ output per worker. Then the marginal product of a worker who performs t_i tasks per period is

$$MP_i = h(\rho,\gamma) - \rho h_\rho - \gamma h_\gamma + t_i h_\gamma, \qquad (17)$$

where MP_i is the marginal product, $h_\rho = \partial h/\partial\rho$ and $h_\gamma = \partial h/\partial\gamma$.

Unless $h(\rho,\gamma)$ is homogeneous of degree one, the intercept in (17) will be positive or negative so that the marginal product is no longer proportional to the number of tasks performed.[41] It can be shown that inequality in marginal products and wages in a sector will be greater or less than inequality in tasks performed

[41] If $h(\rho,\gamma)$ is homogeneous of degree one, $h(\rho,\gamma) = \rho h_\rho + \gamma h_\gamma$ by Euler's Theorem, so that $MP_i = t_i h_\gamma$ and potential earnings in a sector are proportional to tasks performed. This reduces to the case where Q is a function of K and T only as $Q = nh(\rho,\gamma) = h(n\rho,n\gamma) = h(K,T)$.

depending on whether the intercept is negative or positive. Depending on the functional form for $h(\rho,\gamma)$, a change in either ρ or γ will alter the relation between wages and tasks within a sector. Movement of workers from one sector to another, with no movement of capital, raises the capital to labor ratio in the sector they move from and lowers it in the sector they move into. These changes alter relative wages between and within sectors.

In this way, capital can be incorporated into Roy's model. This results in a much more complex model that would be more difficult to estimate econometrically. But this extension is necessary in order to investigate how growth, capital accumulation, and business cycles affect the distribution of earnings and why wage structures vary from sector to sector.

V. *Strategies in Studying the Distribution of Earnings*

In analyzing any complex research question, a standard approach is to decompose the problem by breaking it up into smaller questions that can be more easily explained. Approaches to the distribution of earnings can be understood in terms of the decompositions used to analyze it. These decompositions include breaking the economy up into sectors, use of an earnings function, or perfectly elastic supply or demand curves. Assignment models demonstrate the invalidity of the ceteris paribus assumptions which lie behind these decompositions. This section discusses the decompositions used in various approaches, the problems revealed by analyzing the economy's assignment problem, the solutions suggested by existing models, and strategies for further work.

A. *Sectoral Decompositions*

A seemingly natural way to study the distribution of earnings is to break the population down into subgroups based on demographic, occupational, or industrial categories. At any point in time, one can then study earnings inquality in terms of differences within and between groups. With this disaggregation or decomposition, one would expect to be able to calculate the consequences for inequality of a change within a sector, e.g., from the number in that sector or the distribution of earnings within that sector. However, this calculation requires the ceteris paribus assumption that the composition of the other sector remains the same, and Roy's model shows directly why this assumption does not hold.

In Roy's model, a change in the number of workers, mean earnings or variance of logarithms of earnings within a sector does not occur in isolation. Shifts of workers from one sector to another occur because of changes in the relative prices of output in the two sectors. When the price of output of one of the sectors increases, a nonrandom selection of workers in the second sector move into that sector. This movement alters the means and distributions in both sectors. Figures 9 and 10 and Table 1 in Section III.C. demonstrate the consequences of changes in relative sector prices. In Table 1, as the price of trout in terms of rabbits goes up from 0.83 to 1.00, the proportion hunting rabbits declines from 0.551 to 0.5. If one used this result to predict the effects of the price change on inequality, though, one would miss the self-selection effects of the shift on the composition of workers within sectors. As shown, both mean rabbit skills and mean trout skills decline, and other changes occur within the sectors. Thus the number or distribution in one sector cannot be taken as given as the other sector changes. From the assignment perspective, the source of the change in distribution lies in the reassignment of workers from one sector to another in response to sector price changes, rather than in the separate changes within each sector. By incorporating the selection decision, Roy's model allows one to predict the consequences of changes in sector prices.

Roy's model is appropriate any time the population is divided into two groups based on individual choice. For example, consider the decision to participate in the labor market. This context provided much of the early work on self-selection corrections (Reuben Gronau 1974; H. Gregg Lewis 1974; Heckman 1974, 1979), and Heckman and Sedlacek estimate models with a nonmarket or household sector (1985, 1990). The decision to participate in the labor market divides the population into two sectors, the paid labor market and the nonmarket or household sector. A decomposition in which one looked at only the paid labor force would be misleading. From the perspective of Roy's model, workers in the paid labor market are a selection of all potential workers. As the task price in the paid labor market goes up, a selection of individuals will move from the nonmarket to the paid labor market sector. Changes in average wages will not be proportional to changes in task prices, and average worker productivity will be affected, depending on the parameters of the task distributions. An empirical version of Roy's model can be used to examine these and other effects which occur along with changes in labor force participation.

Measures of earnings inequality may be used to compare alternative distributions among a fixed population but may inaccurately indicate changes in inequality when the number of earners grows or contracts. For example, an increase in the paid labor market task price could draw in predominantly low wage workers, making them better off while raising measured earnings inequality. The appropriate correction is to include workers outside the paid labor market in the measure of inequality, so that movement of a worker in or out of the labor force would not by itself cause changes in inequality. Heckman and Honore (1990, Theorem 6, p. 1135) derive results relating inequality in one of the sectors in Roy's model to overall inequality under the assumption of lognormality.

A related application arises in studying the effect of development on inequality. One sector in Roy's model would be the market (perhaps urban) sector and the other a nonmarket sector (perhaps rural, agricultural, or subsistence). Development generates a higher task price for labor producing the market good relative to the nonmarket good. The effects on the selection of workers in the market sector could then be derived and related to average earnings, productivity, and observed inequality as development proceeds.

International trade provides another potential empirical application of Roy's model. A classic question in trade theory, reflected in the Hecksher-Ohlin and Samuelson-Stolper theorems, is the effect of trade on factor payments and income distribution. Roy's model could be applied to this question by assuming that within a country, workers (or producing units) are divided between export and import-competing sectors. Results from Roy's model could then be used to examine effects of terms of trade on production in the two sectors, average earnings and productivity in each sector, and the overall distribution of earnings.

In addition to their own work, Maddala (1983, p. 289) and Heckman and Honore (1990, p. 1121) discuss further applications of Roy's self-selection corrections, which include labor force participation, returns to education, retirement, union wage differentials, migration, occupational choice, movement between regions, and marital status. More recently, George Borjas (1990) examines the effects of self-selection on the skill composition of immigrants. Charles Brown (1990) analyzes the operation of self-selection in army retention. Also, Gary Solon (1988) and Robert Gibbons and Katz (1992) consider the earnings of industry changers, Lazear (1986) examines choice of piece rate versus salary pay structures as a consequence of sorting between firms, and Tunali (1988) and Magnac (1991) extend Roy's model to examine segmented labor markets.

Further applications of Roy's model are possible if it can be extended to more than two sectors. A current research question concerns the reasons why wages differ among industrial sectors or establishments. Levy and Murnane (1992, Section VI.C) review this literature in relation to the distribution of earnings, and James D. Montgomery (1991), Lang (1991), and Sattinger (1991) analyze capital intensity as a source of wage differences. Other possible reasons include efficiency wages, unobserved abilities, union threats, and involuntary unemployment. A first step in analyzing the relation between wages and industrial characteristics is to correct for sectoral selection biases within industries. However, the econometrics of Roy's model with self-selection would appear to place a barrier of only two or three sectors that can be estimated as a worker's income or utility in a sector must exceed the income or utility in every other sector. The extension to many sectors would appear to be possible if workers are assumed to engage in search instead of self-selection to find jobs, as discussed in Section IV.C.

B. *Earnings Function*

A second approach to decomposing the distribution of earnings is to express separately the prices and quantities of worker characteristics that contribute to earnings. This approach uses an earnings function to describe the prices for worker characteristics, i.e., the relation between worker characteristics and earnings. This earnings function can be combined with the distribution of worker characteristics to generate the distribution of earnings. As discussed in the introduction, this approach neglects that what is exogenous to the determination of an individual's earnings is endogenous to the determination of the distribution of earnings. The approach therefore involves a form of the fallacy of composition.

The problems that arise from using an earnings function can be demonstrated using the following simple model. Suppose at any one point in time that workers differ by a single characteristic x and that the logarithm of earnings is related to x by the following earnings function:

$$\ln y_i = a + b \ln x_i + e_i \qquad (18)$$

where e_i is a mean zero random error term uncorrelated with x_i. From the perspective of supply and demand models, it seems reasonable to suppose that the return to the skill variable x depends on both demand and supply variables. The demand for x, according to the capital-skill complementarity hypothesis, would depend on the economy's capital to labor ratio ρ. Suppose therefore that the coefficient b depends positively on the capital to labor ratio ρ and negatively on the average population value of the worker characteristic x.

Now taking variances on both sides of (18),

$$\mathrm{Var}(\ln y) = b^2 \mathrm{Var}(\ln x) + \mathrm{Var}(e). \qquad (19)$$

This relationship will hold tautologically whenever (18) holds. If a single worker experiences a change in his or her own characteristic from x_i to x_i', the expected logarithm of earnings for that worker would change from $a + b \ln x_i$ to $a + b \ln x_i'$. But suppose all worker characteristics increase by 10 percent. Then (19) will incorrectly predict the consequences of this change for earnings inequality, as measured by the variance of the logarithms of earnings. Var(ln x) will increase by $1.1^2 = 1.21$, but Var(ln y) will not increase by 1.21 b^2: as b depends negatively on the average worker characteristic, b^2 will decline. Further, if one attempts to estimate (19) directly (using for example time series data), the estimated coefficient of Var(ln x) will not equal b^2; it will instead confound changes in Var(ln x) with changes in b^2.[42]

Use of an earnings function obscures the influence of demand on the distribution of earnings. Demand variables such as the capital to labor ratio appear to play no role in the determination of individual earnings in (18) but that is an illusion. The influence of the aggregate capital to labor ratio on earnings would be invisible in any single period empirical estimation of (18) as it would be the same for each observation. Then in the expression for earnings inequality in (19), demand variables do not appear explicitly, suggesting that earnings inequality does not depend on them. But while the coefficient b can be regarded as a constant in a single period estimate of (18), it will vary from one period to another in (19).

With an assignment problem present in the economy, the earnings function is no longer a direct relationship arising from the contributions of worker characteristics to production. The assignment problem introduces an intermediate step between worker characteristics and earnings. The observed earnings function is generated from the supply and demand decisions of workers and firms. The hedonic wage and price literature develops econometric procedures for estimating how the wage varies with worker attributes and for the identification of supply and demand functions for worker characteristics (Rosen 1974; Dennis Epple 1987; Timothy Bartik 1987). An immediate application of this approach in the area of the distribution of earnings is compensating wage differentials for job characteristics (R. E. B. Lucas 1977b; Smith 1979; Greg J. Duncan and Bertil Holmlund 1983; John H. Goddeeris 1988; Mark Killingsworth 1986). In Killingsworth's analysis, workers have heterogeneous preferences for a given job characteristic so that the compensating wage differential will depend on the distribution of worker preferences. Killingsworth applies the analysis to differentials between white and blue collar labor. Causes of compensating wage differentials are relevant to issues of segmentation, discrimination and comparable worth (Killingsworth 1987).

Pettengill (1980) applies related procedures to the question of how labor unions affect skill differentials and inequality. Unionized firms adjust to higher negotiated wages by employing higher quality workers. The effect of unionization is then to shift demands for workers to higher quality levels, leading to greater skill differentials and inequality in the economy. Pettengill also extends the analysis to credentialism, discrimination, absenteeism, cyclical changes in productivity and wages, and minimum wages.

A major reason for interest in the earnings function is that it describes the earnings that a worker with a given set of characteristics can obtain in the labor market. With an assignment problem, however, there will no longer exist a single expected wage associated with a given set of worker characteristics, as implied by the traditional earnings function. Instead, the worker will face a distribution of potential wages and job characteristics

[42] In approaches derived from a solution to the assignment problem, the coefficient b in (20) is endogenously determined, and the influence of the distribution of individual characteristics, Var(ln x) in (21), on earnings inequality is correctly specified (for example in the earnings function solved by Tinbergen 1956, p. 168).

from alternative jobs or sectors. The purpose of describing the alternatives facing workers would be better served by estimating the wage offer distributions for workers with given sets of characteristics.[43] If job characteristics such as risk or satisfaction vary from sector to sector, the joint distribution of wages and job characteristics should be estimated.

Tinbergen (1975a, 1977) and Hartog (1986a, 1986b, 1988) estimate models in which earnings depend on both worker and job characteristics. Such models are relevant to questions of overeducation (Mun C. Tsang and Henry M. Levin 1985; Russell W. Rumberger 1987; Nachum Sicherman 1991) and mismatches. An important outcome from such models is that the return to education varies according to the job placement of the worker, e.g., whether the job requires more or less education than the worker has.

C. *Human Capital Models*

Human capital models of the distribution of earnings also rely on decompositions. These models structure the determination of earnings in such a way that the influences of supply and demand can be separated. The decompositions used are inconsistent with the existence of an assignment problem but are not essential features of human capital models of individual behavior (as distinct from models of the distribution of earnings). Robert J. Willis and Rosen's (1979) intertemporal extension of Roy's model shows that the human capital and assignment models can be combined.[44]

In the model developed by Mincer (1958, 1974), workers choose a level of schooling based on the maximization of the present discounted value of lifetime earnings. With continuous discounting, the earnings function generated by this assumption in the long run is

$$\ln Y_s = \ln Y_0 + rs, \tag{20}$$

where Y_s is the yearly income with s years of schooling beyond the minimum, Y_0 is yearly income with the minimum schooling level, r is the discount rate and s is the number of years of schooling beyond the minimum. If Y_s and s satisfy the relation in (20), the present discounted values will be equalized for each level of schooling.

At each level of schooling, there is in the long run a perfectly elastic supply of labor at the yearly income determined by (20). If the yearly income for a given level of schooling yields a higher present discounted value than other levels, new workers will choose that schooling level. The amount of labor will continue to increase until the yearly income is pushed down to a level which yields the same present discounted value as all other schooling levels.

With horizontal supply curves, the location of demand curves cannot influence the yearly incomes of workers in the long run. Under the assumptions of Mincer's model, the equilibrium earnings function is invariant to changes in the demands for labor. The coefficient of the schooling variable in (20) is the discount rate and does not depend on demand variables.

However, in this model, the distribution of earnings depends both on the yearly incomes for workers at each schooling level and the numbers at each schooling level. With a horizontal supply curve for each level of schooling, the number of workers with that level depends on the location of the demand curve. Although demand does not enter explicitly anywhere in the model, it plays a central role in determining the distribution of earnings.

[43] Christopher J. Flinn and Heckman (1982) and Nicholas Kiefer and George Neumann (1979a, 1979b) estimate wage offer distributions facing workers engaged in search.

[44] Mark Blaug (1976), Lucas (1977a), Rosen (1977), Sattinger (1980) and Willis (1986) discuss alternative interpretations of human capital earnings models.

This human capital model provides a simple decomposition of the determination of the earnings distribution. Worker supply behavior completely determines the earnings function, which remains the same in the long run as long as the discount rate is the same. Demands for workers do not influence this earnings function. With horizontal supply curves, the location of demand curves completely determine the numbers of workers at each schooling level.

This decomposition is possible because of the absence of any assignment problem. All workers are identical, so any worker can obtain any schooling level. In the long run each worker is indifferent as to which schooling level to obtain. If instead workers had preferences for some schooling levels or if they faced different costs of obtaining schooling, the decomposition would break down. The supply of workers to a given schooling level would no longer be horizontal. As the demand for workers at a particular schooling level increases, the yearly income would need to be higher to compensate for cost and utility differences. Shifts in demand would then alter the earnings function.

The terms in the earnings function (20) can be reinterpreted to yield a very different model, one that yields an explicit decomposition of earnings inequality. From a worker's point of view, a rate of return to an investment in schooling can always be calculated. In Mincer's model, this rate of return is exactly equal to the discount rate in the long run equilibrium. Now suppose that the rate of return is instead a random variable.[45] The earnings function can then be written as

$$\ln Y_{si} = \ln Y_0 + r_i s_i, \tag{21}$$

where Y_{si} is the yearly income for worker i, r_i is the average rate of return to schooling for worker i, and s_i is the number of years of schooling beyond the minimum for worker i. In this model, the variables r_i and s_i are determined exogenously so that a separate model is needed to explain a worker's level of schooling and rate of return (see Becker 1975, pp. 94–117, for a theory of investment behavior that explains the rate of return and schooling level). So far, the earnings function in (21) describes the generation of a worker's earnings, taking the economy as given. The next step is to extend the earnings function in (21) to an explanation of earnings inequality by taking variances on both sides of (21). Under the assumption that the rate of return r_i and level of schooling s_i are independently distributed,

$$\mathrm{Var}(\ln Y) = \bar{r}^2\,\mathrm{Var}(s) + \bar{s}^2\,\mathrm{Var}(r) + \mathrm{Var}(s)\mathrm{Var}(r), \tag{22}$$

where $\mathrm{Var}(\ln Y)$ is the variance of logarithms of earnings, and $\bar{r}$ and $\bar{s}$ are the average rate of return and level of schooling for all workers.

This procedure appears to provide a neat decomposition of earnings inequality into rate of return and schooling sources, but unless restrictive assumptions hold, the decomposition has no predictive content.

Willis and Rosen (1979) develop a model that can be used to show why the existence of an assignment problem interferes with the use of the decomposition in (22). In their development, the assignment problem is to allocate individuals to schooling levels on the basis of tastes, talents, expectations, and parental wealth. Individuals base their decision on human capital considerations but because of heterogeneous tastes and talents they are not indifferent among

[45] Lucas (1977a) analyzes different interpretations of the interest rate in human capital models. Mincer (1974, p. 27), Becker and Barry Chiswick (1966), and Chiswick and Mincer (1972) use human capital models of the earnings function in which the rate of return to schooling is a random variable.

schooling levels. Rates of return calculated from observed relationships between schooling and wages will not reflect the rates of return facing individuals. Willis and Rosen argue that individuals who stop with a high school education earn more than college educated individuals would if they had stopped with a high school education. On average, then, individuals have an absolute advantage in terms of earnings at the schooling level they choose: they earn more than the rest of the population would at that level.[46] In response to a wage increase for a particular schooling level, only a subset of individuals switch to that level. The supply of individuals to a schooling level is therefore upward sloping rather than perfectly elastic as in Mincer's model. The wages and rates of return for workers at a schooling level depend on both the supply and demand for workers with that amount of schooling as well as observed and unobserved worker skills.

Now consider what happens when the distribution of schooling shifts. We may suppose that changes in parental wealth or tastes lead more workers to get a college education. Moving down the demand curve for college educated labor, the rates of return for college educated labor declines. The change in the distribution of schooling causes a change in the distribution of rates of return.[47]

This connection between schooling and rates of return makes (22) unusable. To keep things simple, suppose $\bar{s}$ goes up while Var(s) stays the same. The expression in (22) would predict that earnings inequality would go up by $2\bar{s}$Var(r) times the increase in $\bar{s}$. This prediction would be incorrect as $\bar{r}$ and Var(r) also change. The ceteris paribus assumption that is needed to use comparative statics does not hold. Instead of earnings inequality increasing as workers get more education, it may decline if the rate of return falls enough.[48]

Early assignment models trivialize human capital issues by assuming that worker characteristics such as schooling levels are exogenously determined. Early human capital models trivialize assignment issues by assuming that workers all face the same investment returns independent of sector. But assignment and human capital models are not inherently competing theories of the distribution of earnings. In both, behavior is motivated by wealth maximization (or, in extensions, utility maximization). In both, workers are assigned to sectors or educational levels on the basis of their own choices rather than rationing. The

[46] According to Willis and Rosen (1979), this result argues against a large positive correlation between a worker's productivity with a high school education and with a college education, ruling out simple rankings of individuals. This corresponds to cases ii and iii in the Roy model (III.C) and is inconsistent with the one-dimensional ability or skill models discussed in III.B. In addition to absolute advantage, comparative advantage is also present in the sense defined in (1).

[47] To be caused by shifts in the distribution of schooling, the recent increases in the returns to schooling (noted in Section I) would require that fewer workers choose to get a college education.

[48] This difficulty can be avoided by supposing that a worker's rate of return does not depend on the sector or job chosen. Each worker's labor could be expressed as a multiple of some standard worker's labor, independent of occupation or job. This is known as the efficiency units assumption as all labor can be expressed in terms of a common measure (see discussions by Rosen 1977; Sattinger 1980, pp. 15–20; and Paul Taubman 1975, pp. 3–6). It is equivalent to the absence of any assignment problem. With wages proportional to a worker's efficiency units, employers are indifferent as to which labor to use in a job. For example, an employer is indifferent between hiring a worker with two efficiency units at a wage rate w and hiring two workers of one efficiency unit apiece at a wage rate of $w/2$ each. This assumption would insure that the distribution of rates of return is unaffected by shifts in the distribution of schooling, so that the standard comparative static analysis can be applied to (22). However, the efficiency units assumption seems an unreasonable restriction in models designed to explain the economic consequences of human diversity.

advantage of viewing investment in education as an assignment to schooling levels is that it more accurately describes the choices available to individuals. As in Willis and Rosen's analysis (1979), a model that specifies the unobserved alternatives for workers allows one to examine the issue of ability bias and estimate the real return to individuals for investment in schooling.

D. *Assignment Models*

In their structuring of the determination of earnings, assignment models also rely on a decomposition of the distribution of earnings. The existence of an assignment problem introduces an intermediate decision stage between workers' characteristics and their earnings. In many of the theoretical models, the assignment problem can be solved first and then used to determine the wage differentials. This occurs, for example, in Tinbergen's model, the differential rents model, and the linear programming optimal assignment problem. In general, however, assignment and the determination of earnings occur simultaneously.

In empirical studies, the information that would be needed to solve the economy's assignment problem is unavailable. The current assignment can be observed but not the alternatives facing individual agents. The relevant characteristics of workers and firms may not even be observable. Complete specification of the assignment problem, prior to the determination of earnings, is therefore unrealistic in empirical estimates.

Tinbergen's empirical study of the distribution of earnings structures the problem using a supply and demand approach (1975a, pp. 29–30). In a single labor market, the structural supply and demand equations can be solved to yield two reduced form equations, one for the price (wage) and the other for the quantity (employment). These reduced form equations are functions of both supply and demand factors. Extending this approach to multiple labor markets, the earnings function and measures of inequality should be regarded as reduced form equations that depend on the distribution of both supply and demand factors. This approach does not incorporate self-selection corrections but carries the simple, practical injunction that both supply and demand factors need to be included in empirical estimates of the earnings function or inequality measures. Tinbergen (1975a, 1977) applies an empirical decomposition of the determination of earnings in which the labor market is broken down into compartments based on discrete values of some worker characteristic such as education. In this approach, the separate effects of supply and demand (in the form of job requirements) can be estimated. Tinbergen's separate estimates of labor market compartments suggest a very pragmatic, disaggregated, partial equilibrium approach to studying the distribution of earnings. First, one isolates a sector or segment of the economy of interest (for example, wholesale and retail trade, which employs many younger workers at low wages). Then one analyzes the market in terms of supply and demand, fully specifying the alternatives that are available to employers and workers (in order to account for self-selection phenomena) and institutions in the labor market. This approach is often followed in practice but without reference to the income distribution developments that lie behind it.

If the problems of extending Roy's model to many sectors can be solved, then an empirical general equilibrium method of studying the distribution of earnings can be developed in which the conditions in each sector can be considered separately. However, it is difficult to describe how to formulate such an empirical, multi-sector model without actually doing so.

VI. *Conclusions*

A. *The Importance of Choice*

This survey has examined three potential reasons why the existence of an assignment problem affects the distribution of earnings. The first reason is comparative advantage, which is measured by bilateral comparisons of output of two workers at two jobs. In technologies with cooperating factors of production, as in the linear programming optimal assignment problem and differential rents model (Sections III.A and III.B), comparative advantage does not necessarily determine the assignment; instead, the scale of operations effect may influence the assignment. In technologies where comparative advantage does determine the assignment, as in Roy's model, it is consistent with all cases, in the trivial sense that the ratios of outputs for two workers varies from sector to sector. The standard comparative advantage case, in which there is a large positive correlation between sectoral outputs, yields predictable consequences for the distribution of earnings but does not necessarily arise in Roy's model. In short, saying there is comparative advantage tells us very little about the distribution of earnings other than that it doesn't resemble the distribution of abilities in any one sector.

Self-selection is another phenomenon that is often closely associated with assignment models. Although all three models in Section III use self-selection as a means of choosing among sectors or jobs, assignment models do not require the self-selection mechanism. Instead, workers could engage in search to find the sector that maximizes their income or utility. This is particularly appropriate when there are many sectors or jobs to choose from, making the assumption of full information about jobs less reasonable.

The major point common to all assignment models is the existence of choice among jobs, occupations, or sectors for a worker. With choice, a worker's earnings or utility are not determined by performance within a single area of endeavor. Instead, the worker can avoid the consequences of a bad performance in one sector by choosing another sector. Comparative advantage is significant in describing relationships among opportunities in different sectors. Self-selection describes how decisions may be made. But the underlying feature is the variability of worker output among sectors or jobs. This variability arises from the different sensitivity of jobs to worker abilities (i.e, the difficulty of jobs), the large dispersion in tasks performed in different jobs throughout the economy, and the diversity and lack of correlation among individuals' performances of those tasks. Variability of output from worker-firm matches generates a problem in choosing jobs or sectors, from the worker's point of view, and a problem of assigning workers to jobs, from the perspective of the economy as a whole.

B. *Extensions*

Through the work of Heckman and Sedlacek (1985, 1990), Roy's model provides the most promising route to estimate assignment models. They find that two extensions of Roy's model are essential to explain the distribution of wages in the U.S. economy. First, workers choose sectors on the basis of utility maximization instead of income maximization. Workers have sector-specific preferences that alter the assignment of workers to jobs relative to what one would expect on the basis of income maximization. Second, Heckman and Sedlacek find that nonparticipation in the labor market is an important choice facing workers.

Workers participating in either the manufacturing or nonmanufacturing sectors are therefore a selection from all potential workers in the population.

This article suggests that two additional extensions are important. First, the interactions between capital and demands for labor should be adequately specified and estimated. In Heckman and Sedlacek's models, changes in capital intensity or energy prices affect demands for workers and the division of labor through task prices. With their assumptions, earnings are proportional to tasks, which serve as measures of sector specific abilities. Then wage differentials for worker characteristics within a sector would not change over time. However, if capital intensity affects marginal products as in (17), earnings will not be proportional to tasks. Then changes in capital per worker or tasks per worker could influence the demand for workers in more complicated ways. Incorporation of the role of capital will provide a link between observable conditions of demand and the distribution of earnings.

The second extension concerns the number of sectors. With self-selection, the econometrics appear to place a barrier of only two or three sectors that can be estimated because a worker's income or utility in a sector must exceed the income or utility in every other sector. The extension to many sectors would appear to be possible if workers are assumed to engage in search instead of self-selection to find jobs. Then the income or utility in a given sector needs only to satisfy one bilateral comparison, i.e., with the worker's reservation income or utility.

With many sectors, the relation between sector characteristics and demands for workers can be examined. Characteristics of industries lead firms to seek different mixes of workers. In the context of a search model firms must pay higher wages to gain more acceptances from workers with desired characteristics. Such wage premia are consistent with absence of monopsony when search assigns workers to jobs. Wage differences between industries can persist without all workers going to the higher wage industry. With search assigning workers to jobs, the extension of assignment models to industrial sectors can be used to explain interindustry differences in wage structures.

C. *Relation to Other Distribution Theories*

The distribution theory arising from assignment models is distinct from established theories. In perfectly competitive neoclassical models, wages in a labor market are determined from intersecting supply and demand curves, where the demand curve is derived from firm production functions. In equilibrium, wages equal the marginal revenue product, the product price times the marginal product of labor. The expression for the wage differential in the differential rents model, (11), provides a comparable condition. In neoclassical models, wages are found by varying the quantity of labor, whereas in assignment models, the wage differential is found by varying the assignment (i.e., changing the type of labor used in a particular job). In cases where workers and capital are combined in fixed proportions (the differential rents model and the linear programming assignment problem), the marginal product of labor will not be defined. Then the wage differentials will be consistent with alternative absolute levels of wages and profits. This indeterminacy is absent in neoclassical models but instead characterizes classical models, in which factor prices are determined exogenously. Classical models, however, are usually concerned with factor shares and do not explain relative wage rates.

The major difference between assignment and human capital models is in the interpretation of the earnings function. In the earnings function in Mincer's model, the coefficient of schooling is the discount rate. This coefficient would remain unchanged in response to changes in the distributions of jobs or workers. In the alternative approach, as developed by Becker and Chiswick, the coefficient of schooling is a rate of return which does not explicitly depend on supply or demand variables. In these models, auxiliary assumptions combined with the human capital model of investment behavior allow one to decompose the distribution of earnings. The existence of an assignment problem is inconsistent with these auxiliary assumptions. In assignment models, the earnings function is interpreted as an hedonic wage function, a reduced form relationship instead of a structural relationship. A change in the distribution of either workers or jobs would lead to a new equilibrium, with a new coefficient of schooling. With an assignment problem, the earnings function cannot be used to predict the consequences of changes in jobs or workers. There is no reason, however, why the human capital model of individual behavior cannot be combined with sectoral choice or set in the context of an assignment problem, as in Willis and Rosen's model.

Because abilities enter so importantly, assignment models may appear to be akin to theories that relate the distribution of earnings to the distribution of abilities. This link is perhaps fostered by the simplifying assumption in some models of a single parameter describing workers. However, the abilities considered in assignment models are the outputs or performances in various jobs, not IQ's or other measures of innate ability. Further, the point of assignment models is just the opposite of ability models: the distribution of abilities cannot by itself explain the distribution of earnings. In particular, using one task to identify abilities, the distributions of earnings and abilities will not have the same shape. Assignment models emphasize the diversity of human performances from one task to another and the roles of choice and demand in placing higher or lower values on particular abilities. Earnings inequality depends both on differences among workers and the extent to which the economy exaggerates or moderates those differences through the assignment of workers to different tasks.

D. *Relative Wage Changes*

Assignment models provide several explanations for why relative wages change. In the linear programming assignment problem, a shift in jobs would produce changes in wage differentials. In (8), if new jobs have machine properties which are greater for those jobs that have higher values of the λ's, then wage differentials would increase. However, this model is too abstract to relate its results to observed changes in skill and age differentials.

The differential rents model of Section III.B provides a realistic explanation of how differentials can change over time. In (13), the wage function derived using specific assumptions about functional forms can be expressed by $w(g) = Ag^{(\alpha\sigma_g+\beta\sigma_k)/\sigma_g} + C_w$, where A is a constant and C_w is a constant determined by the reserve prices of workers and machines. The exponent of the skill variable g in this expression is an increasing function of σ_k, which measures the inequality in the distribution of capital among jobs. As capital becomes more unequally distributed among jobs, the wage function $w(g)$ becomes more concave. With more capital per worker among the most skilled workers, their wage differentials increase. With less capital per worker among the less skilled, their wage differentials decline. The quantity $w - C_w$ is lognormally distributed with variance of logarithms $\alpha\sigma_g + \beta\sigma_k$, and this measure of inequality is an increasing function of σ_k.[49]

Using an extended version of Roy's model, Heckman and Sedlacek (1985, p. 1107) estimate that the price of the manufacturing task declined 22 percent from 1976 to 1980, while the price of the nonmanufacturing task rose 21 percent in the same time period. In their analysis, this produces a movement of workers from manufacturing to nonmanufacturing, which is the sector with the lower education and experience differentials. This movement alone would tend to reduce aggregate differentials. However, in the nonmanufacturing sector, the greater task price raises skill and experience differentials, while in the manufacturing sector these differentials decline.[50] Workers moving from manufacturing to nonmanufacturing tend to be at the bottom of the task distributions in the two sectors. Their movement raises average worker quality in manufacturing while reducing it in nonmanufacturing. The average wage in manufacturing declines both because of the lower task price and the lower average worker skill levels. In nonmanufacturing, the task price and average worker quality have opposite effects on the average wage and the net effect is ambiguous. Heckman and Sedlacek do not indicate what the net effect of all these movements would be on aggregate educational and experience differentials. If the economy is not described by the manufacturing versus nonmanufacturing dichotomy, then movements within sectors will also contribute to changes in relative wages. The extensions to Roy's model to incorporate capital, discussed in Section IV.D, would explain how differentials could change within sectors independently of task price changes.

[49] Sattinger (1980) finds that inequality in the distribution of capital is significantly related to earnings inequality. The measure of inequality in the distribution of capital is derived from industry capital to labor ratios. Dickens and Katz (1987, p. 66) review studies that have found a positive relationship between capital to labor ratios and average industry wage rates. They also find a positive relationship in their own study (p. 78).

[50] That is, the slope of the skill-earnings relationship increases in nonmanufacturing and decreases in manufacturing. Note, however, that under the assumptions used by Heckman and Sedlacek, elasticities of earnings with respect to skills are constant.

E. *Research Questions*

Each approach to the distribution of earnings suggests a set of relevant research questions. In the human capital approach, for example, important phenomena are decisions to invest and the returns to those investments. The existence of an assignment problem suggests a different set of questions.

First, how do workers differ in ways that are relevant to employment? What choices do workers face between occupations and industrial sectors? How do the choices facing workers differ by educational level? How many sectors do workers choose among? How do worker preferences affect choice of sector, wage differentials and earnings inequality?

On the employer side, how does the technology in each sector relate worker characteristics to output? How do these technologies generate different demands for workers? How are demands for workers related to features of the industry or occupation such as capital intensity or hierarchical span of control? Do workers occupy capital in the sense discussed in Section II.C? How has the mix of jobs changed over time, how is it related to shifts between manufacturing and nonmanufacturing, and how does trade affect the mix? What wage differentials do firms need to offer to attract a labor force with a given set of characteristics?

What mechanisms operate in the economy to assign workers to jobs? What combinations of self-selection, search, and separated labor markets do workers use to find jobs? What are the costs of these assignment mechanisms, what are the efficiency properties of the assignments they bring about, and how do they affect the distribution of earnings? What are the relative contributions of unequal variances of worker performances among sectors, correlation among sector performances and number of sectors to earnings inequality?

A final set of questions is related to explaining observed changes in the distribution of earnings. How are shifts in the mix of jobs and workers related to changes in the wage rates for high school and college graduates and the returns to education? How do wage or unemployment differentials reconcile so-called mismatches between supplies and demands for workers?

Assignment models offer the promise of incorporating the influence of demand on the distribution of earnings, accurately representing the relation between worker characteristics and earnings, and rigorously explaining changes in earnings inequality and wage differentials over time. This promise has been met only partially through applications of assignment models to many aspects of the distribution of earnings. While assignment models indicate the shortcomings of earnings function and human capital approaches, empirical work has only begun to provide a comprehensive alternative.

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